RESPONSE OF MULTISTORY BUILDINGS TO EARTHQUAKE TRANSLATIONAL EXCITATION

Abdelwahab M. Awad
Structural Engineering Department, Faculty of Engineering,
Alexandria University, Alexandria, Egypt.

ABSTRACT

An analytical study of the response of multistory buildings to earthquake translational excitation is presented. The lateral resisting elements of the buildings are columns and shear wall assumed to be elastic during the excitation. The earthquake excitation is represented by a design response spectrum. The study has led to the following main conclusions: the story shear decreases with the increase of building eccentricity, in the mean time; the story torque increases. The influence of the building eccentricity on the response is substantial when the uncoupled torsional and translational frequencies of the building get closer to each other.

INTRODUCTION

Multistory buildings subjected to earthquake often undergo torsional motion in addition to the translational motions. The torsional motion may arise owing to eccentricities between the centers of mass and resistance at various floors of the building.

The torsional response of a single story building has been studied by several researchers [1,2]. More recently, Hejal and Chopar [3] have suggested a simplified procedure for the analysis of a class of multistory buildings with frame type lateral resisting elements. In this paper, the response of multistory monosymmetric buildings with shear type lateral resisting elements to earthquake translational excitation is presented. The influence of building different characteristics on the response is considered. The earthquake translational excitation is represented by design response spectrum [4].

DESCRIPTION OF MULTISTORY BUILDING

The multistory building considered in this study is monosymmetric N story buildings as shown in Figure (1). The building floors are horizontal slabs having the same geometry and supported on columns and shear walls. The principal axes of resistance of all floors are oriented along x and y axes. The centers of mass of the floors lie on a vertical axis and the centers of resistance lie on another vertical axis. The building mass is concentrated at the floor levels and the floors have the same radius of gyration r about the centers of mass axis.

The building is symmetric about the y axis while it is unsymmetric about the x axis. It has N translational degrees of freedom u_x in the x direction and N rotational degrees of freedom u_θ about a vertical axis all considered at the centers of mass of various floor levels. The ratio $\beta = k_{\theta l}/(r^2\, k_{xl})$ is the same for all stories.

EQUATIONS OF MOTION

The multistory monosymmetric building shown in Figure (1) has 2N degrees of freedom; N of them are translational in the direction of the x axis and the other N are rotational about the vertical all acting at the centers of mass and measured relative to the ground. For floor l these are denoted by u_{lx} and $u_{l\theta}$ respectively. The building is subjected to earthquake translational acceleration $\ddot{u}_{gx}$ in the x direction.

The equations of motion of the multistory building shown in Figure (1) can be expressed as follows [5]:

$$[M]\{\ddot{u}\} + [K]\{u\} = \{P(t)\} \qquad (1\text{-a})$$

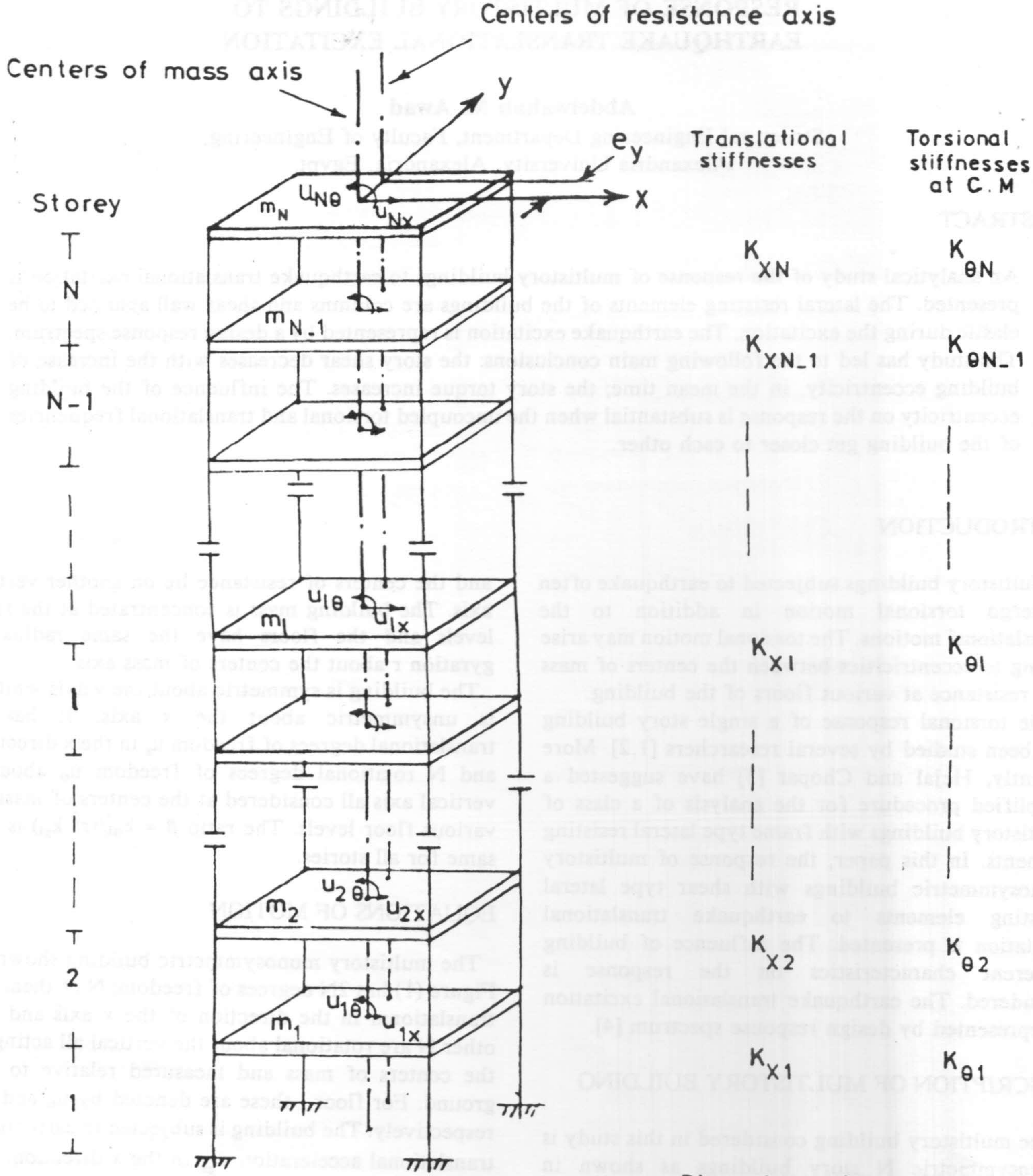

Figure 1. A Multistory building.

where [M] is the mass matrix, [K] is the stiffness matrix of the building, {P(t)} is the earthquake effective load vector acting on the building, {u} is the displacement vector relative to the ground and {ü} is the acceleration vector.

Equations (1a) can be expressed in the following expanded form:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{u}_x \\ \ddot{u}_\theta \end{Bmatrix} + \begin{bmatrix} K_x & -\frac{e_y}{r} K_x \\ -\frac{e_y}{r} K_x & K_t \end{bmatrix} \begin{Bmatrix} u_x \\ r u_\theta \end{Bmatrix} = - \begin{Bmatrix} m \; 1 \; \ddot{u}_{gx} \\ 0 \end{Bmatrix} \qquad (1\text{b})$$

in which the mass submatrix is

$$m = \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_N \end{bmatrix}$$

where m_l is the mass at level l.
The displacement subvectors are

$$u_x = \begin{Bmatrix} u_{1x} \\ u_{2x} \\ \vdots \\ u_{Nx} \end{Bmatrix}, \quad r u_\theta = \begin{Bmatrix} u_{1\theta} \\ u_{2\theta} \\ \vdots \\ u_{N\theta} \end{Bmatrix}$$

where r is the radius of gyration of the floor and the stiffness submatrix are:

$$K_x = \begin{bmatrix} K_{x1}+K_{x2} & -K_{x2} & & & \\ -K_{x2} & K_{x2}+K_{x3} & -K_{x3} & & \\ & -K_{x3} & K_{x3}+K_{x4} & -K_{x4} & \\ & & & \ddots & -K_{xN} \\ & & & -K_{xN} & K_{xN} \end{bmatrix}$$

and $$K_t = \beta K_x$$

where $\beta = \dfrac{K_{\theta l}}{r^2 K_{xl}}$,

1 is a vector of ones and **0** is a zero vector.

Equations (1b) are for undamped system. Damping will be introduced later as viscous damping ratio ξ in each mode of vibration.

The natural frequencies ω and the mode shapes α of the building are solutions of the following Eigenvalue problem of order 2N [5]:

$$\begin{bmatrix} K_x - \omega^2 m & -\frac{e_y}{r} K_x \\ -\frac{e_y}{r} K_x & K_t - \omega^2 m \end{bmatrix} \begin{Bmatrix} \alpha_x \\ \alpha_\theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (2)$$

The mode shapes of the building can be expressed in the following matrix form:

$$\alpha = \left[\alpha_1 \; \alpha_2 \; \ldots \; \alpha_{2N} \right]$$

and the n-th. mode shape α_n is expressed as

$$\alpha_n = \begin{Bmatrix} \alpha_{jxn} \\ \\ \alpha_{j\theta n} \end{Bmatrix} \qquad j = 1,2,\ldots,N \quad n = 1,2,\ldots,2N$$

and the natural frequencies can be expressed as:

$$\omega = \left\{\omega_1 \ \omega_2 \ . \ . \ . \ . \ \omega_{2N}\right\}$$

The displacement component of the building, relative to the ground, are then defined in terms of modal coordinates Y_n as follows

$$\begin{Bmatrix} u_x \\ u_\theta \end{Bmatrix} = \sum_{n=1}^{2N} Y_n \begin{Bmatrix} \alpha_{jxn} \\ \alpha_{j\theta n} \end{Bmatrix} \qquad j = 1,2,...,N \qquad (3)$$

and the uncoupled equations of motion of equations (1b) are given by (5).

$$\ddot{Y}_n(t) + 2\,\xi\,\omega_n\,\dot{Y}_n(t) + \omega_n\,Y(t) = \frac{P_n(t)}{M_n} \qquad (4)$$

where

$Y_n(t)$ is the modal coordinate for the n-th mode at time t

ξ is the damping ratio, assumed to be the same for all modes of vibration

ω_n is the natural frequency for the n-th mode

$M_n = \alpha_n^T[M]\alpha_n$ is the generalized mass for the n-th mode, and [M] is the mass matrix of the building

and

$P_n(t) = \alpha_n^T P(t)$ is the generalized load for the n-th mode

For earthquake translational excitation in the x direction only the generalized load for the n-th mode can be expressed as:

$$P_n(t) = -\alpha_n^T\,[M]\begin{Bmatrix} 1 \\ 0 \end{Bmatrix}\ddot{u}_{gx}$$

Using response spectrum, the absolute maximum value of the modal coordinate for the n-th mode is given by [5]

$$Y_n(t)_{max} = \frac{L_n}{M_n\,\omega_n^2}\,S_{ax}\left(\omega_n,\xi\right) \qquad (5)$$

where $L_n = \alpha_n^T\left[M\right]\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is the model earthqua excitation factor in the x direction.

and $S_{ax}\left(\omega_n,\xi\right)$ is the translational spect acceleration.

The relative displacement model vector is th given by

$$U_n(t) = \alpha_n\,Y_n(t)_{max}$$

$$U_n(t) = \frac{\alpha_n}{M_n\,\omega_n^2}L_n\,S_{ax}\left(\omega_n\,,\,\xi\right) \qquad ($$

The maximum modal forces are finally obtained a expressed as:

$$f_n = \begin{Bmatrix} f_{jxn} \\ f_{j\theta n} \end{Bmatrix} = \omega_n^2\left[M\right]u_n(t)$$

$$f_n = \left[M\right]\alpha_n\,\frac{L_n}{M_n}\,S_{ax}\left(\omega_n,\xi\right) \qquad \begin{matrix} j = 1,2,...,N \\ n = 1,2,...,2N \end{matrix} \qquad ($$

The maximum value of any desired story for resultant due to the n-th mode can be computed standard method of statics from the external mod forces obtained from equations (7). Referring Figure (2); the resultant modal shear and t resultant modal torque at floor l are expressed follow:

$$V_{lxn} = \sum_{j=l}^{N} f_{jxn} \qquad (8-$$

j = l, l+1, ...,

$$T_{l\theta n} = r\sum_{j=l}^{N} f_{j\theta n} \qquad (8-$$

The resultant model torque can be converted abo the center of resistance as follows:

$$\left(T_R\right)_{l\theta n} = T_{l\theta n} + e_y\,V_{lxn} \qquad ($$

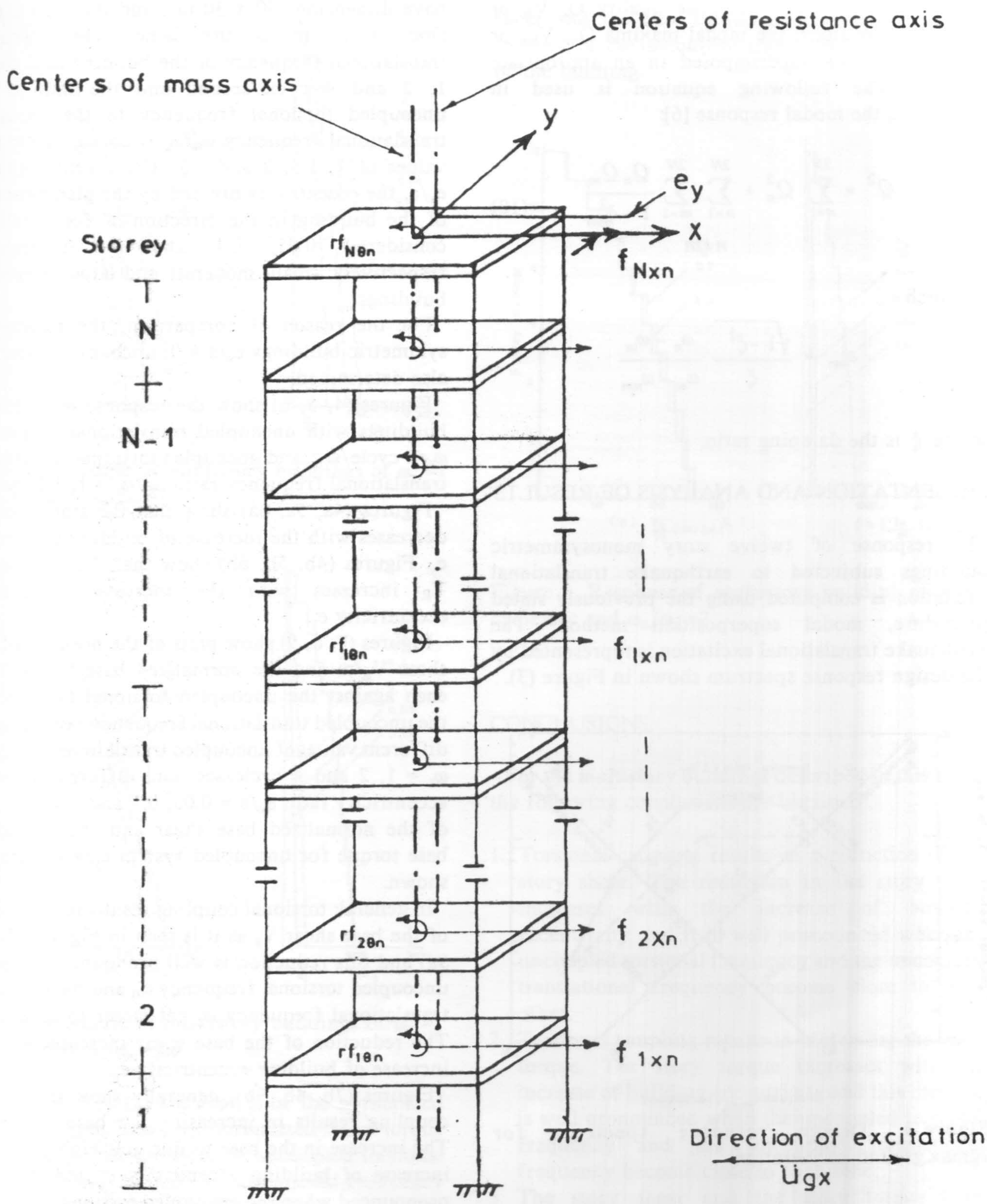

Figure 2. Modal forces acting on a multistory building.

To get the resultant response quality Q, V_{lx} or $(T_R)_{l\theta}$, at any story; the modal maxima Q_n, V_{lxn} or $(T_R)_{l\theta n}$, are then superimposed in an appropriate manner. The following equation is used in combining the modal response [6]:

$$Q^2 = \sum_{n=1}^{2N} Q_n^2 + \sum_{n=1}^{2N} \sum_{m=1}^{2N} \frac{Q_n Q_m}{1 + \varepsilon_{nm}^2} \qquad n \neq m \tag{10}$$

in which

$$\varepsilon_{nm} = \frac{\sqrt{1 - \xi^2}}{\xi} \; \frac{\omega_n - \omega_m}{\omega_n + \omega_{nm}}$$

where ξ is the damping ratio.

PRESENTATION AND ANALYSIS OF RESULTS

The response of twelve story monosymmetric buildings subjected to earthquake translational excitation is computed using the previously stated procedure, modal superposition method. The earthquake translational excitation is represented by the design response spectrum shown in Figure (3).

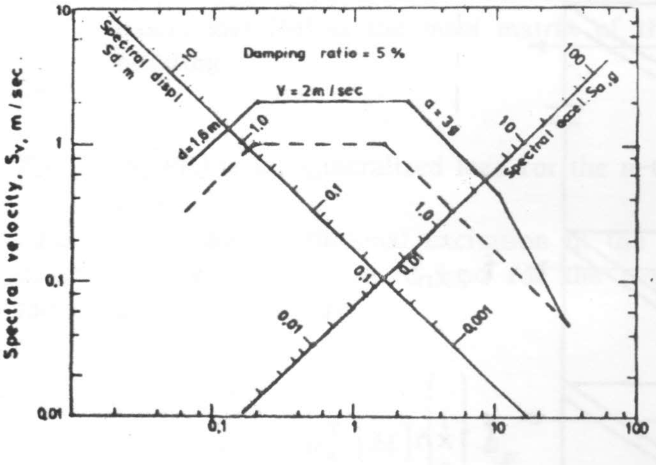

Figure 3. Design response spectrum for 1gmax.ground acceleration.

A damping ratio ξ = 5% is considered in the analysis. The floors of the buildings are identical and have dimensions 30 x 30 ms, and the mass at each floor level m is the same. The uncoupled translational frequency of the building ω_x is chosen 1, 2 and 4 cycle/second and the ratio of the uncoupled torsional frequency to the uncoupled translational frequency ω_θ/ω_x is considered to have values of 1, 1.5, 2 and 2.5. The eccentricity ratio e_y/a, the eccentricity divided by the plan dimension of the building in the direction of eccentricity, is considered 0.05, 0.1 and 0.4 representing respectively; small, moderate and large eccentricity buildings.

For the reason of comparison, the response of symmetric buildings $e_y/a = 0$, uncoupled systems, is also determined.

Figures (4, 5, 6) show the response of multistory buildings with uncoupled translational frequencies ω_x=1 cycle/sec. and uncoupled torsional frequency to translational frequency ratio ω_θ/ω_x = 1, 1.5 and 2.

Figures (4a, 5a, 6a) show that the story shear V_x decreases with the increase of building eccentricity e_y. Figures (4b, 5b, 6b) show that the story torque T_R increases with the increase of building eccentricity e_y.

Figures (7, 8, 9) show plots of the normalized base shear V_x/m and the normalized base torque T_R/m each against the uncoupled torsional frequency to the uncoupled translational frequency ratio ω_θ/ω_x for different values of uncoupled translational frequency ω_x = 1, 2 and 4 cycle/sec. and different values of eccentricity ratio e_y/a = 0.05, 0.1 and 0.4. The plots of the normalized base shear and the normalized base torque for uncoupled system e_y/a = 0 are also shown.

In general; torsional coupling results in a reduction of the base shear V_x as it is seen in Figures (7a, 8a, 9a) and this reduction is well pronounced when the uncoupled torsional frequency ω_θ and the uncoupled translational frequency ω_x get closer to each other. The reduction of the base shear increases with the increase of building eccentricity e_y.

Figures (7b, 8b, 9b), generally show that torsion coupling results in increasing the base torque T_R. The increase in the base torque gets higher with the increase of building eccentricity e_y and it is well pronounced when the uncoupled torsional frequency w_θ and the uncoupled translational frequency ω_x get closer to each other.

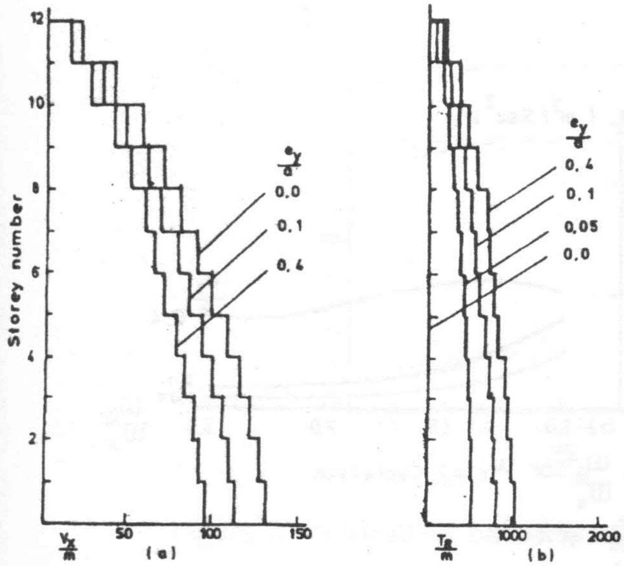

Figure 4. Response of multistory buildings of ω_x =1 cycle/sec and ω_Θ/ω_x =1

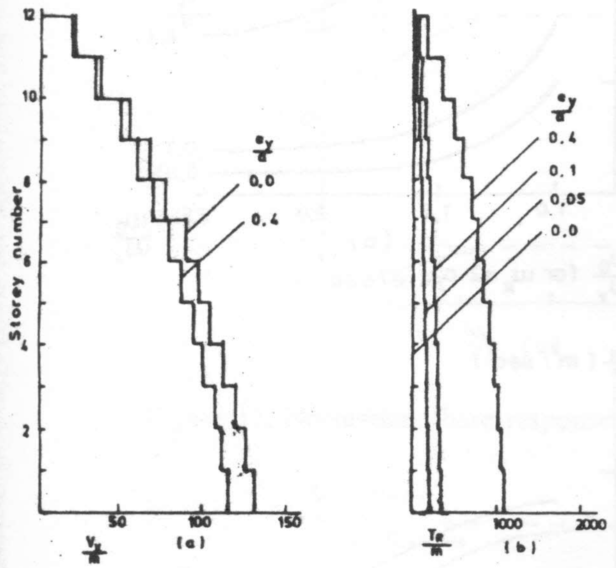

Figure 5. Response of multistory buildings of ω_x =1 cycle/sec and ω_Θ/ω_x =1.5

Figures (10, 11, 12) show plots of the normalized base shear v_x/m and the normalized base torque T_R/m each against the ratio of uncoupled torsional frequency to the uncoupled translational frequency ω_Θ/ω_x for uncoupled translational frequency ω_x = 1, 2 and 4 cycle/sec.

It is seen from Figures (10, 11, 12) that the base shear and the base torque both increase with the increase of the uncoupled translational frequency ω_x of the building.

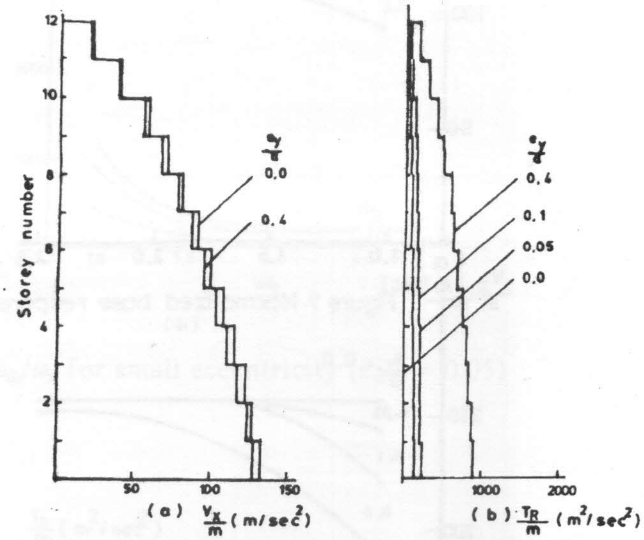

Figure 6. Response of multistory building of ω_x =1 cycle/sec and ω_Θ/ω_x =2

CONCLUSIONS

For the multistory buildings described in this paper the following conclusions are obtained:

1. Torsional coupling results in a reduction of the story shear. The reduction in the story shear increases with the increase of building eccentricity and it is well pronounced when the uncoupled torsional frequency and the uncoupled translational frequency become close to each other.
2. Torsional coupling results in increasing the story torque. The story torque increases with the increase of building eccentricity and this increase is well pronounced when the uncoupled torsional frequency and the uncoupled translational frequency become close to each other.
3. The story shear and the story torque both increase with the increase in the uncoupled translational frequency of the building.

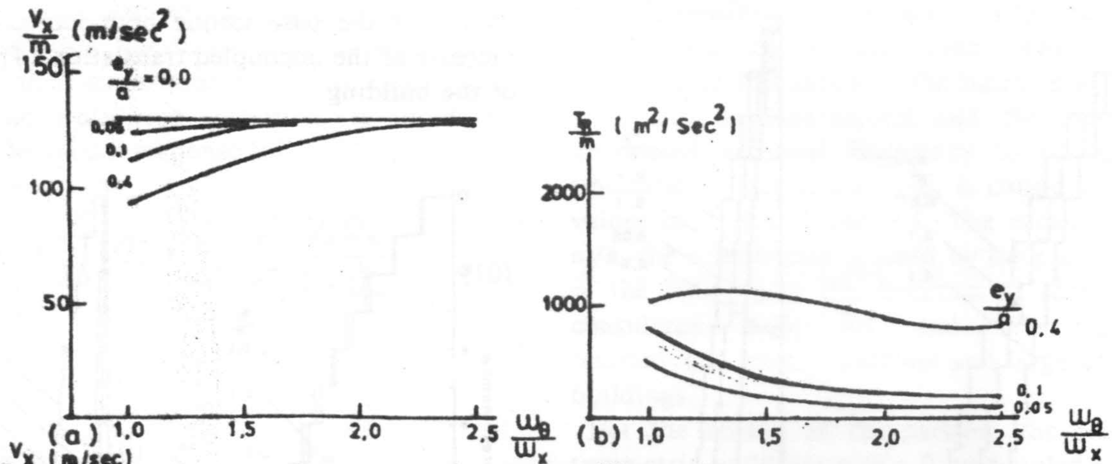

Figure 7. Normalized base response VS. $\frac{\omega_\theta}{\omega_x}$ for ω_x = 1 Cycle/sec

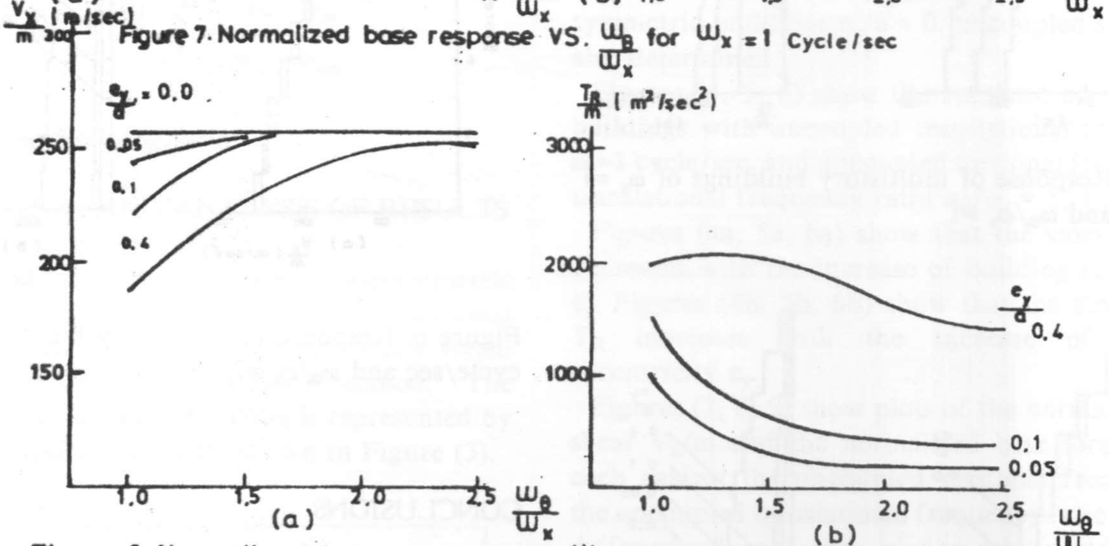

Figure 8. Normalized base response VS. $\frac{\omega_\theta}{\omega_x}$ for ω_x = 2 Cycle/sec.

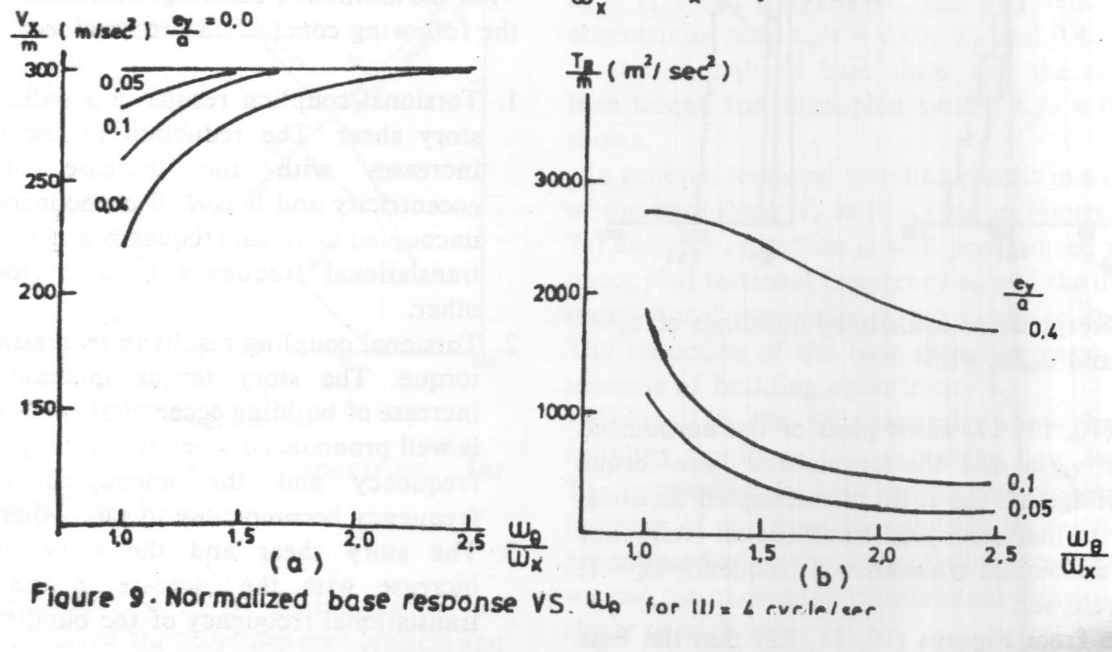

Figure 9. Normalized base response VS. ω_θ for ω = 4 cycle/sec

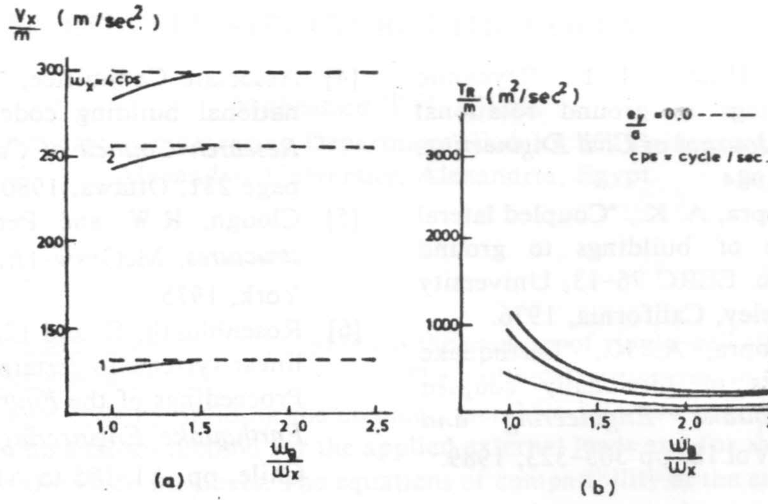

Figure 10. Normalized base response vs ω_θ/ω_x for small eccentricity (e_y/a = 0.05)

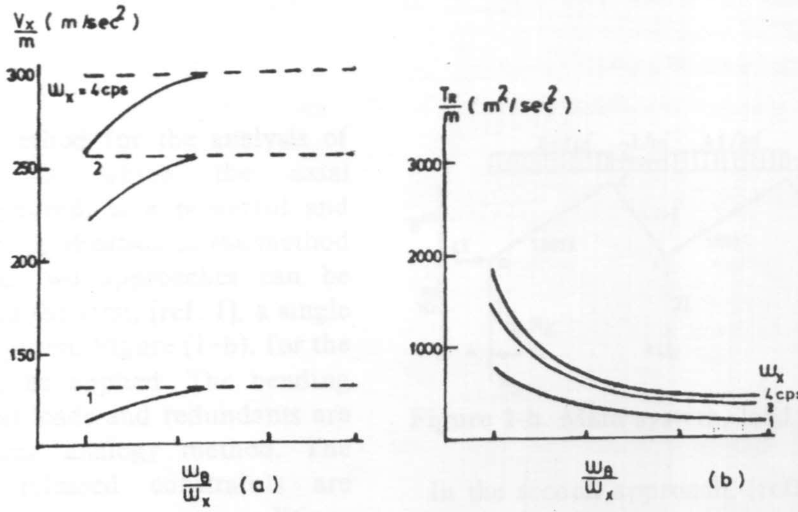

Figure 11. Normalized base response vs ω_θ/ω_x for moderate eccentricity (e_y/a = 0.1)

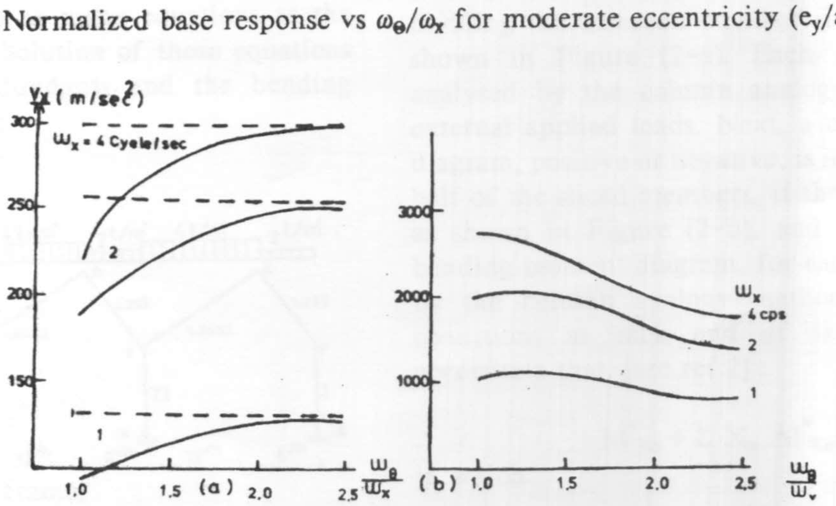

Figure 12. Normalized base response vs ω_θ/ω_x for large eccentricity (e_y/a = 0.4)

REFERENCES

[1] Awad, A. M. and Humar, J. L., "Dynamic response of buildings to ground rotational motion", *Canadian Journal of Civil Engineering*, Vol.11, pp 48-56, 1984.

[2] Kan, C. L. and Chopra, A. K., "Coupled lateral torsional/ response of buildings to ground shaking", Report No. EERC 76-13, University of California, Berkeley, California, 1976.

[3] Hejal, R. and Chopra, A. K., "Earthquake analysis of a class of torsionally coupled buildings", *Earthquake Engineering and structural dynamic*, Vol.18, pp 305-323, 1989.

[4] Associate Committee, "The supplement to the national building code of Canada", *National Research Council of Canada*, Commentary K. page 231, Ottawa, 1980.

[5] Clough, R.W. and Penzien, J., *Dynamics of structures*, McGraw-Hill Book Company, New York, 1975.

[6] Rosenblueth, E. and Elorduy, J., "Response of linear systems to certain transient disturbance", Proceedings of the *Fourth World Conference on Earthquake Engineering*, Volume 1, Santiago, Chile, pp. A1-185 to A1-196, 1969.