POWER-LAW FLUID FLOW OF A HYDROMAGNETIC FREE JET

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ABSTRACT

A laminar two-dimensional viscous conducting free jet flow of a power-low non-Newtonian incompressible viscous fluid immersed in non-conducting space is studied. A constant magnetic field transverse to the axis of the jet is applied and the electrical conductivity is taken as an integral power of the axial velocity. The method of Sherbnin is used to solve the problem. Numerical values of the maximum velocity, the boundary-layer thickness and discharge rate of the jet are calculated and represented graphically.

1- INTRODUCTION

Our aim in the present work is to study the free jet flow of a viscous conducting power-law incompressible fluid subject to a transverse magnetic field and the electrical conductivity is taken as an integral power of the axial velocity.

Schlichting [1]. Andrade and Tsien [2] have solved corresponding problems to the one under consideration for a non-conducting Newtonian fluid.

Cenober and Sherbnin [3] and Peskin [4] extended the above mentioned work to include the effect of magnetic fields using expansion techniques. Smith and Cambel [5] have used the results in [3] and [4] to obtain an analytical solution using perturbation methods. Moreau [6] has obtained similar self - solutions to this problem.

The same problem was again solved by Sherbnin [7] who used the method of similarity solutions to obtain the exact solutions of the boundary-layer equations associated with this problem in a closed form.

Shelova and Sherbnin [8] studied the problem of magnetohydrodynamic free jet flow of a conducting Newtonian fluid with variable conductivity.

The problem of free jet flow of an incompressible power-law fluid was studied by Shulman and Berkovesky [9].

Sharikadza and Ezzat [10] solved an extension of the above problem which include the effect of magnetic fields. In their work the electrical conductivity was taken as a linear function of the axial velocity.

In the present analysis the electrical conductivity σ is assumed to have the following form

$$\sigma = \sigma_o u^m \tag{1.1}$$

where u is the axial velocity, σ_o is a constant and m is a positive real number.

2- THE BASIC EQUATIONS AND SIMILARITY SOLUTION.

let u and v be the components of velocity in the x and y directions taken along the axis of the jet and normal to it, respectively.

The boundary-layer equations for the two-dimensional free jet flow of a conducting power-law incompressible fluid with a cross magnetic field are given by

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = k\frac{\partial}{\partial y}\left[\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right] - \sigma_o B_o^2 u^{m+1}, \tag{1.2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,. \tag{2.2}$$

In above equations ρ is the fluid density, B_o the component of the electromagnetic induction, k and n are the fluid consistency and flow index of the power-law fluid respectively.

The boundary conditions are

$$\frac{\partial u}{\partial y} = o, v = o \quad \text{at } y = o, \qquad u = o \text{ as } y \to \infty . \tag{2.3}$$

Substituting from equation (2.2) into (2.1) and integrating the resulting equation with respect to y over the interval$(-\infty,\infty)$, we get upon using conditions (2.3), the following integral relation

$$\frac{d}{dx}\int_{-\infty}^{\infty} u^2 dy = -\frac{\sigma_o B_o^2}{\rho}\int_{-\infty}^{\infty} u^{m+1} dy . \tag{2.4}$$

The continuity equation (2.2) enables us to introduce a stream function Ψ defined by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} . \tag{2.5}$$

We assume that the stream function and similarity variable η have the form

$$\psi = \left(\frac{k}{\rho}\right)^{\frac{1}{2-n}} \phi^{\frac{1}{2-n}}(x) f(\eta), \qquad \eta = \frac{y}{[\delta(x)]^{\frac{1}{2n-1}}} , \tag{2.6}$$

where $\delta(x)$ is the thickness of a propagating layer.

Invoking the above assumption, the basic equations (2.1) and (2.4) can be expressed in the form

$$n|f''|^{n-1} f''' + \alpha f f'' - \beta f'^2 - N \phi^{\frac{m-n+1}{2-n}} \delta^{\frac{2n-m}{2n-1}} f'^{m+1} = 0, \tag{2.7}$$

and

$$\frac{d}{dx}\left(\frac{\phi^{\frac{2}{2-n}}}{\delta^{\frac{1}{2n-1}}}\right) = -\frac{Na}{b}\left(\frac{\phi^{\frac{m+1}{2-n}}}{\delta^{\frac{m}{2n-1}}}\right), \tag{2.8}$$

where

$$\alpha = \frac{1}{2-n}\,\delta\,\frac{d\phi}{dx},$$

$$\beta = \phi^{\frac{1-n}{2-n}} \delta^{\frac{2n}{2n-1}} \frac{d}{dx}\left(\frac{\phi^{\frac{1}{2-n}}}{\delta^{\frac{1}{2n-1}}}\right),$$

$$N = \left(\frac{k}{\rho}\right)^{\frac{m-1}{2-n}} \frac{\sigma_o B_o^2}{\rho} ,$$

$$a = \int_{-\infty}^{\infty} f'^{m+1}\, d\eta, \text{ and } b = \int_{-\infty}^{\infty} f'^2\, d\eta . \tag{2.9}$$

The boundary conditions (2.3) expressed in terms of f and η become

$$f'' = o, f = o \text{ at } \eta = o, \qquad f' = o \text{ as } \eta \to \infty , \tag{2.10}$$

where a dash denotes derivative with respect to η.

Using (2.8), the constants α and β can be written as

$$\alpha = \frac{1}{3n}\frac{d}{dx}\delta\phi - \frac{a(2n-1)N}{3nb}\phi^{\frac{m-n+1}{2-n}}\delta^{\frac{2n-m}{2n-1}},$$

$$\beta = -\frac{1}{3n}\frac{d}{dx}\delta\phi - \frac{a(n+1)N}{3nb}\phi^{\frac{m-n+1}{2-n}}\delta^{\frac{2n-m}{2n-1}}. \tag{2.11}$$

Substituting from (2.11) into (2.7), we get

$$n|f''|^{n-1} f''' + \frac{1}{3n}(f'^2 + f f'')\frac{d}{dx}\delta\phi = N\phi^{\frac{m-n+1}{2-n}}\delta^{\frac{2n-m}{2n-1}} \left(f'^{m+1} + \frac{a(2n-1)}{3nb} f f'' - \frac{a(n+1)}{3nb} f'^2\right). \tag{2.12}$$

The similarity solution satisfies $d\delta\phi/dx$ = const. Without loss of generality, we take

$$\frac{d}{dx}\delta\phi = 1 . \tag{2.13}$$

Eliminating Φ between (2.8) and (2.13), we get the following equation satisfied by $\delta(x)$

$$\frac{x}{\delta}\frac{d\delta}{dx} = \frac{2(2n-1)}{3n} + \frac{a(2n-1)(2-n)N}{3nb} x^{\frac{m+1-n}{2-n}} \delta^{\frac{(n+1)(1-m)}{(2n-1)(2-n)}}, \tag{2.14}$$

The above equation has the general solution

$$\delta = D x^{\frac{2(2n-1)(1-m)}{3n}} \left(1 - \frac{a(n+1)(1-m)}{3nb} D^{\frac{(n+1)(1-m)}{(2n-1)(2-n)}} \int N(x)dx\right)^{\frac{-(2n-1)(2-n)}{(n+1)(1-m)}}, \tag{2.15}$$

Using assumptions similar to those used by Sherbnin [6] we obtain the solution of equation (2.12) in the form

$$f' = \left(\frac{1}{3n}\right)^{\frac{1}{2n-1}} \left(\frac{2n-1}{n+1}\right)^{\frac{n}{2n-1}} \left(|f(\infty)|^{\frac{n+1}{n}} - |f|^{\frac{n+1}{n}}\right)^{\frac{n}{2n-1}}, \quad n \neq \frac{1}{2} \tag{2.16}$$

provided that

$$a = \frac{1}{2n-1}\left(\frac{3n}{f(\infty)}\right)^{\frac{n+1}{n}}, \quad m = \frac{1}{n} - 1, \tag{2.17}$$

where

$$f(\infty)^{\frac{3n}{2n-1}} = \frac{1}{2}(3n)^{\frac{1}{2n-1}} \left(\frac{n+1}{2n-1}\right)^{\frac{n}{2n-1}} \frac{\Gamma\left(\frac{n}{n+1} + \frac{3n-1}{2n-1}\right)}{\Gamma\left(\frac{2n+1}{n+1}\right).\Gamma\left(\frac{3n-1}{2n-1}\right)}, \quad b = 1 \tag{2.18}$$

(Γ is the gamma function)

If the magnetic field is uniform, i.e $N(x)$ = const, then the impulse at the start of the flow J_o is given by

$$J_o = \lim_{x \to o} \rho\left(\frac{1}{\rho}\right)^{\frac{2}{2-n}} \phi^{\frac{2}{2-n}} \delta^{\frac{1}{2n-1}}. \tag{2.19}$$

Using (2.19) in (2.15), we have

$$\delta = D x^{\frac{2(2n-1)}{3n}} \left(1 - \frac{N}{3n-1}\left(\frac{3n}{f(\infty)}\right)^{\frac{n+1}{n}} D^{\frac{n+1}{n(2-n)}} x^{\frac{(3n-1)(n+1)}{3n^2}}\right)^{\frac{-n(2-n)}{n+1}} \tag{2.20}$$

where

$$D = \left(\frac{\rho}{J_o}\left(\frac{k}{\rho}\right)^{\frac{2}{2-n}}\right)^{\frac{(2-n)(2n-1)}{3n}}. \tag{2.21}$$

The maximum velocity u_m and the volume rate of discharge per unit length of the jet Q are given by

$$u_m = A x^{-\frac{1}{3n}} \left(1 - BNx^{\frac{(3n-1)(n+1)}{3n^2}}\right)^{\frac{n}{2n-1}}, \tag{2.22}$$

$$Q = C x^{\frac{1}{3n}} \left(1 - BN\,x^{\frac{(3n-1)(n+1)}{3n^2}}\right)^{\frac{n}{n+1}}, \tag{2.23}$$

where

$$A = \left(f_{(\infty)}^{n+1}\left(\frac{2n-1}{n+1}\right)^n\right)^{\frac{1}{2n-1}} \frac{J_o^{n+1}}{\rho^n k},$$

$$B = \frac{1}{3n-1}\left(\frac{3n}{f(\infty)}\right)^{\frac{n+1}{n}} \left(\frac{\rho}{J_o}\left(\frac{k}{\rho}\right)^{\frac{2}{2-n}}\right)^{\frac{(n+1)(2n-1)}{3n^2}},$$

$$C = 2f(\infty)\left(k\frac{J_o^{2n-1}}{\rho^{2n}}\right)^{\frac{1}{3n}}.$$

CONCLUSIONS

The boundary-layer thickness δ is plotted against x in Figure (1) for a value N = 0.5 of the magnetic field number and for three different values of the flow index n. n = 1.3 (dilatant fluid), n = 1 (Newtonian fluid) and n = 0.7 (pseudoplastic fluid). This figure shows that δ in increases with the decrease of n. For all values of n the boundary layer thickness δ increases with x.

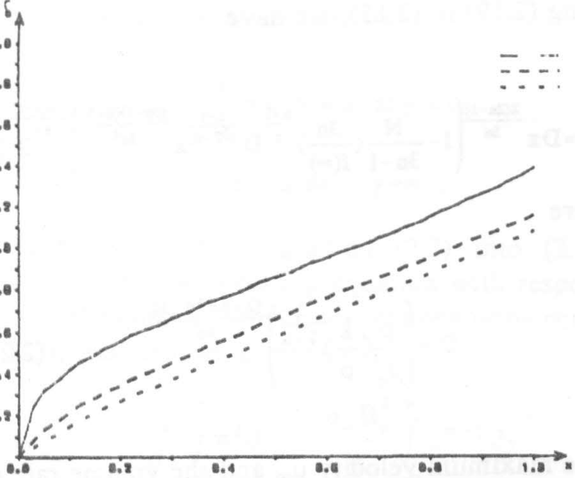

Figure 1. The boundary layer thickness for N=0.5.

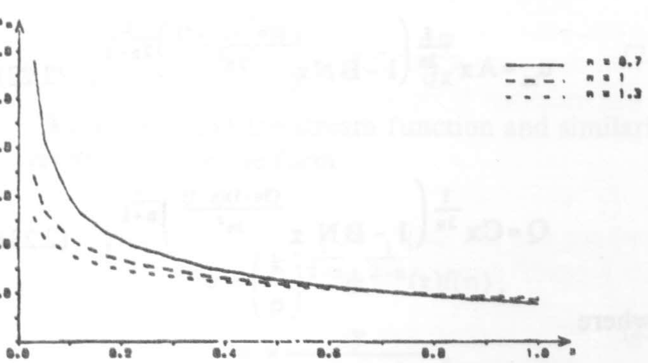

Figure 2. The maximal axial velocity for N=0.5.

Figure (2) represents the relation between the maximal axial velocity u_m and x for the same values of the parameters mentioned above. The maximal velocity decreases with x starting with infinite value at x = o until it reaches zero at the end of the boundary layer. It was found that u_m decreases with increasing n for small values of x ($x < 0.7$). For $x > 0.7$, u_m increases with the increase of n.

In Figure (3) the discharge rate of the jet Q is shown against x for the same values of the parameters as before. The discharge rate increases with x until it reaches a maximal value and then it decreases with increasing x. Q increases with n for small values of x ($x < 0.75$). For $x > 0.75$ Q decreases with n.

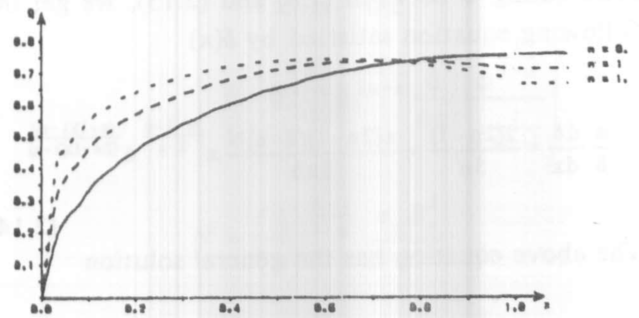

Figure 3. The discharge rate of the jet for N=0.5.

It is interesting to observe that putting n = 1 we get the solution of the problem of magnetohydrodynamic free jet flow with a constant conductivity $\sigma = \sigma_0$ obtained in [7].

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