ANALYSIS OF MULTI-CELL STRUCTURES THE COLUMN ANALOGY METHOD

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ABSTRACT

The column analogy method which is applicable for the analysis of single-cell structures is modified and applied in the analysis of multi-cell structures. The multi-cell structures, in this work, is divided into a number of single cells by dividing the common members into two halves. Each cell is, then, analysed by the column analogy method for the applied external loads and for the redundants which restore continuity of the sliced members. The equations of compatibility at the ends of each common member are written and solved for the unknown redundants and the bending moments are superposed. Numerical examples are given.

1- INTRODUCTION

The column analogy method for the analysis of single-cell structures, in which the axial displacements can be ignored, is a powerful and common tool. But, for the application of the method in multi-cell structures, two approaches can be found in the literature. In the first, [ref. 1], a single cell is chosen as a main system, Figure (1-b), for the virtual work method to be applied. The bending moments for the external loads and redundants are calculated by the column analogy method. The displacements at the released constraints are calculated and equations of displacements conditions are written which yield as many equations as the number of redundants. Solution of those equations gives the unknown redundants and the bending moments are superposed.

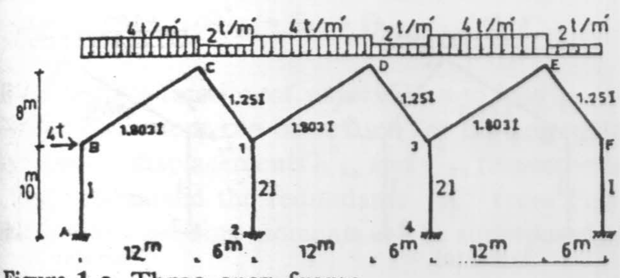

Figure 1-a. Three-span frame.

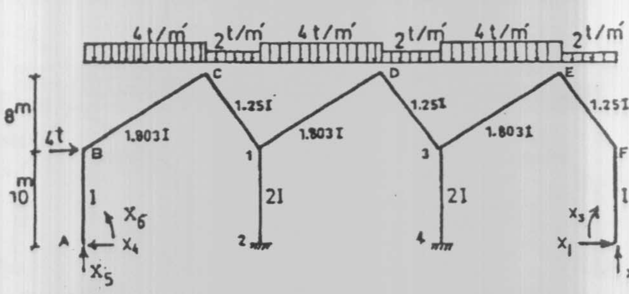

Figure 1-b. Main system, load 'O' & redundants.

In the second approach, [refs. 2,3], the multi-cell structures is divided into a number of single cells by dividing the common members into two halves, as shown in Figure (2-a). Each cell can then be analysed by the column analogy method for the external applied loads. Next, a correction moment diagram, positive or negative, is introduced for each half of the sliced members, if their bases are fixed, as shown in Figure (2-b), and the corresponding bending moment diagram, for each cell, is obtained by the column analogy method. The continuity conditions at each end of the sliced member necessitate that, [see ref.2] :

$$M^{*}_{ko} + \Sigma X_n . M^{*}_{kn} = 0 \qquad (1)$$

in which

$$M^*_{ko} = M_{ko} + M_{k'o}$$

is the sum of bending moments at k and k' ends of the sliced member due to external loads. (positive bending moments produce tension inside the frame).

$$X_n \text{ ,n=1,2,....}$$

is the correction factor of a correction moment diagram n.
and,

$$M^*_{kn} = M_{kn} + M_{k'n}$$

is the sum of bending moments at k an k' ends of the sliced member due to correction moment diagram n.

Equations (1), which are applicable only for fixed base members, yield as many equations as the number of unknown correction moment diagrams or equal to twice the number of sliced members, [ref. 2], (one equation at top and the other at bottom).

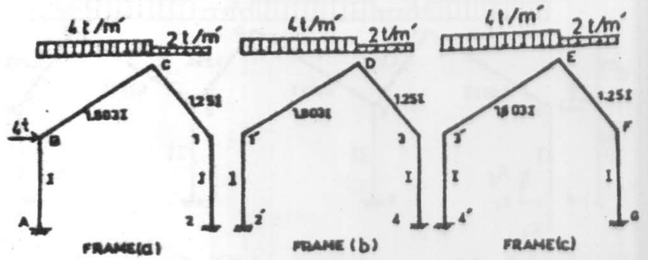

Figure 2-a. Divided frames of Figure (1-a) & load 'O'.

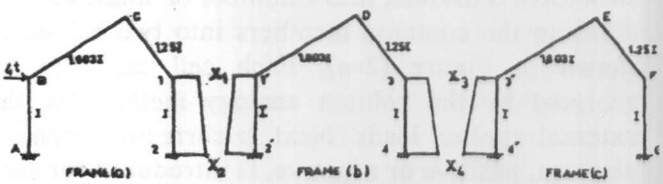

Figure 2-b. Divided frames & correction moments diags.

In case of hinged base members, [ref. 3], two corrections are taken as the moments resulting from two equal and opposite forces "X_1" and two equal and opposite couples "X_2", both acting at the nonhinged ends of the sliced members, Figure (3-b). The continuity conditions, in this case, are: [see ref. 3]

$$M^*_{ko} + \Sigma X_n . M^*_{kn} = 0 \quad (1)$$

for the nonhinged end; and for the hinged end:

$$r^*_{ko} + \Sigma X_n . r^*_{kn} = 0 \quad (2)$$

where;

$$r^*_{ko} = r_{ko} + r_{k'o}$$

is the rotation at hinges k and k' of the sliced member due to external loads.
and;

$$r^*_{kn} = r_{kn} + r_{k'n}$$

is the rotation at hinges k and k' due to load X_n.

Although the above methods, (Eqs.1 and Eqs.1&2), produced accurate results for the examples solved, they should be applied with care if the common members, themselves, are loaded as shown in examples 2 and 3 of this work. Therefore, the above methods are modified in order to be applied generally regardless of the way of loading as described in the next articles.

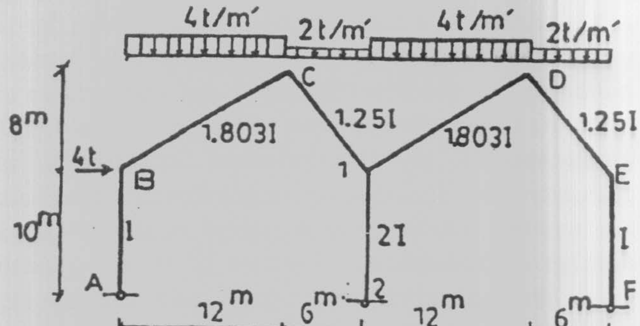

Figure 3-a. Two span hinged frame.

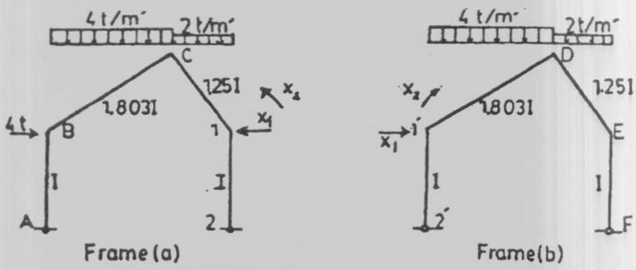

Figure 3-b. Divided frames, load 'O' & redundants.

3- METHOD OF SOLUTION

As explained above, the multi-cell structure is divided into single cells by dividing the common members into two halves, as shown in Figures. (4-b & 5-b). Each cell is then analysed by the column analogy method for the applied external loads. Next, a redundant "X" corresponding to each possible displacement, (rotation and translation), of the ends of the sliced members, is applied as two equal and opposite moments or forces as shown in Figures (4-b & 5.b). The corresponding bending moments diagram for each cell is obtained by the column analogy method. The displacements at the ends of the sliced members are calculated using the Elastic Load method, [ref. 4], or the Virtual Work method, [refs. 1 & 5]. The equations of compatibility, which necessitate that relative displacements of the corresponding ends of each sliced member are zeros, can be written for frame of Figure (4-a) as:

$$\theta^{\bullet}_{10} + X_1\,\theta^{\bullet}_{11} + X_2\,\theta^{\bullet}_{12} + X_3\,\theta^{\bullet}_{13} = 0 \qquad (3\text{-a})$$

$$h^{\bullet}_{10} + X_1\,h^{\bullet}_{11} + X_2\,h^{\bullet}_{12} + X_3\,h^{\bullet}_{13} = 0 \qquad (3\text{-b})$$

$$v^{\bullet}_{10} + X_1 v^{\bullet}_{11} + X_2 v^{\bullet}_{12} + X_3\,v^{\bullet}_{13} = 0 \qquad (3\text{-c})$$

and, for frame of Figure (5-a) as:

$$\theta^{\bullet}_{10} + X_1\,\theta^{\bullet}_{11} + X_2\,\theta^{\bullet}_{12} + X_3\,\theta^{\bullet}_{13} = 0 \qquad (4\text{-a})$$

$$h^{\bullet}_{10} + X_1\,h^{\bullet}_{11} + X_2\,h^{\bullet}_{12} + X_3\,h^{\bullet}_{13} = 0 \qquad (4\text{-b})$$

and;

$$\theta^{\bullet}_{20} + X_1\,\theta^{\bullet}_{21} + X_2\,\theta^{\bullet}_{22} + X_3\,\theta^{\bullet}_{23} = 0 \qquad (4\text{-c})$$

in which;

$$\theta^{\bullet}_{kn} = \theta_{kn} + \theta_{k'n}$$

is the relative rotation of point k due to load n. Similar definitions can be written for the horizontal and vertical displacements $h^{\bullet}_{kn}$ and $v^{\bullet}_{kn}$ respectively.

Having obtained the redundants "X" from Eqs.3 or Eqs.4, the bending moments can be superposed as:

$$M_k = M_{ko} + X_1\,M_{k1} + X_2\,M_{k2} + \ldots\ldots \qquad (5)$$

where the subscript k of Eq.5 refers to any point, but the positive side of the common member, whether it is left or right (top or bottom), should be observed during summation of terms in Eq.5.

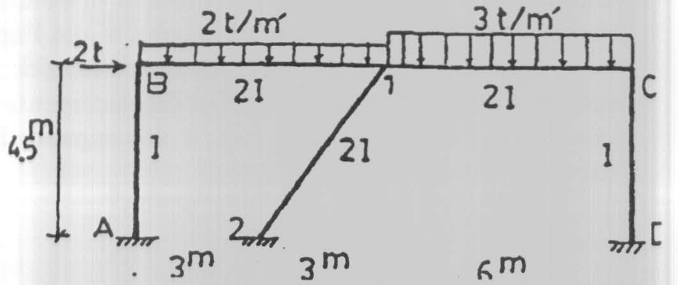

Figure 4-a. Two-span frame.

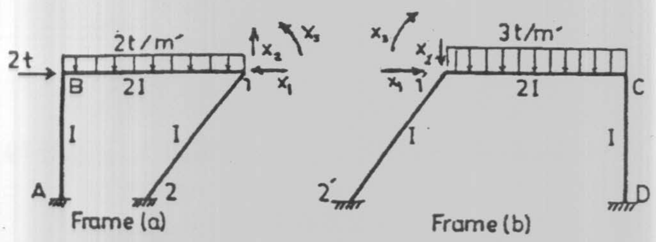

Figure 4-b. Divided frames, load 'O' & redundants.

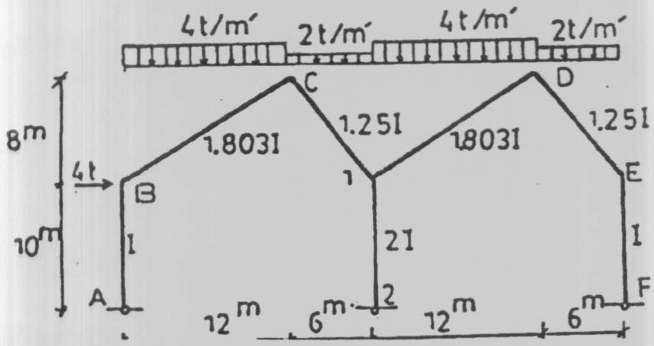

Figure 5-a. Two-span hinged frame.

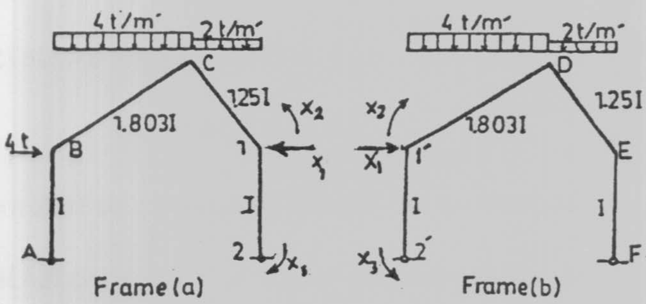

Figure 5-b. Divided frames, load 'O' & reundants.

4.NUMERICAL EXAMPLES

Example 1:

The Three-Span frame, Figure (1-a), given in ref. [2], is used here to explain the method of solution. Figure (2-a) shows the divided frames, while Figure (6) shows the bending moments diagrams for the applied loads and redundants. The displacements are integrated, [refs. 1,5], and Eqs. 3 are expanded as follows:

$\theta_1 = 0$

$$69.7765 + 3.2889\, X_1 + 9.5909\, X_2 - 0.4924 X_3 - 5.3193\, X_4 = 0 \quad (a)$$

$h_1 = 0$

$$-1164.7979 + 9.5909\, X_1 + 165.9009\, X_2 - 6.4579\, X_3 -53.8029\, X_4 = 0 \quad (b)$$

$\theta_3 = 0$

$$95.611 - 0.4924\, X_1 - 6.4579\, X_2 + 3.2889\, X_3 + 9.5909\, X_4 = 0 \quad (c)$$

$h_3 = 0$

$$-949.5459 - 5.3193\, X_1 - 53.8029\, X_2 + 9.5909\, X_3 + 165.9009\, X_4 = 0 \quad (d)$$

Solution of the above equations gives:

$X_1 = -43.5661$, $X_2 = 11.23146$, $X_3 = -44.23675$, $X_4 = 10.52654$

The bending moments are calculated as follows:

$$M_A = 16.08 + (-43.5661) * .289 + (11.23146) * 2.16438 = 27.7986 \text{ t.m.}$$

$$M_B = -39.901 + (-43.5661) * -.19053 + (11.23146) * -1.1005 = -43.9606 \text{ t.m.}$$

$$M_E = 5.5591 + (-44.23675) * -.1332 + (10.52654) * -1.4319 = -3.622 \text{ t.m.}$$

$$M_F = -42.5374 + (-44.23675) * -.1222 + (10.52654) * -.6541 = -43.9223 \text{ t.m.}$$

$$M_D = 5.5591 + (-43.5661) * -.1332 + (11.23146) * -1.4319 + (-44.23675) * -.05232 + (10.52654) * -1.02744 = -13.221 \text{ t.m.}$$

$$M_{12} = (-45.125 + 52.6771) + (-43.5661) * (-.4078 + .3323) + (11.23146) * (2.927 - 3.1907) + (-44.23675) * .19053 + (10.52654) * 1.1005 = 11.036 \text{ t.m.}$$

$$M_{34} = (-42.5374 + 52.6771) + (-43.5661) * -.1222 + (11.23146) * -.6451 + (-44.23675) * (-.4078 + .3323) + (10.52654) * (2.927 - 3.1907) = 8.782 \text{ t.m.}$$

and so the other points which give the bending moment diagram as shown in Figure (7).

Example 2

The Two-Span frame shown in Figure (8), is solved using Eqs. (1), which give the results declared between brackets in Figure (10). The other results are obtained using the method proposed here, Eqs. (3). The discrepancies between results are because the horizontal load on the common member 1-2, is taken on frame (b), member 1'-2', For calculation of M_o- Diagram. Accurate results, using Eqs. (1), are obtained when the horizontal load is divided as the common member. Figures (8 & 9) explain the proposed method and Eqs. (3) can be expanded, for this example, as follows:

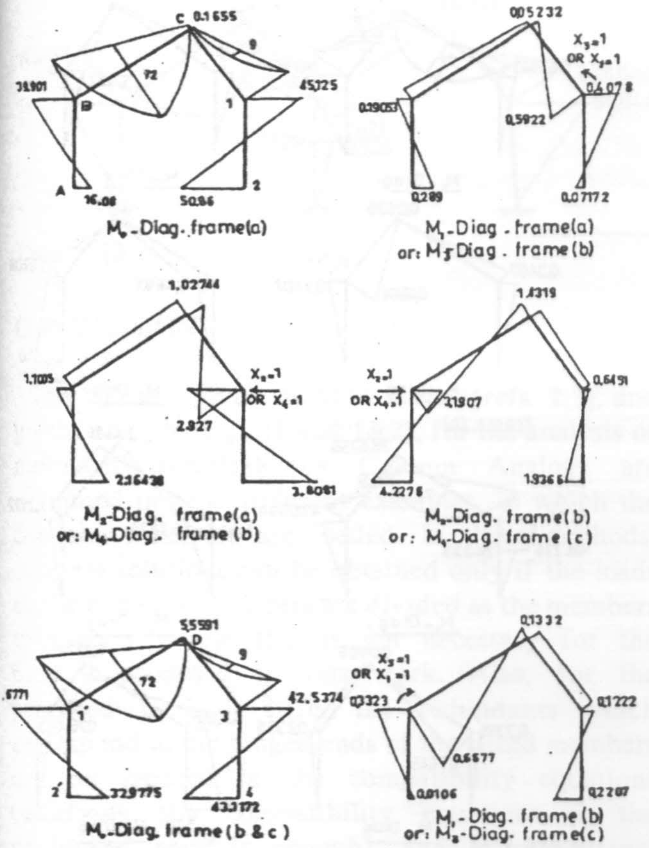

Figure 6.

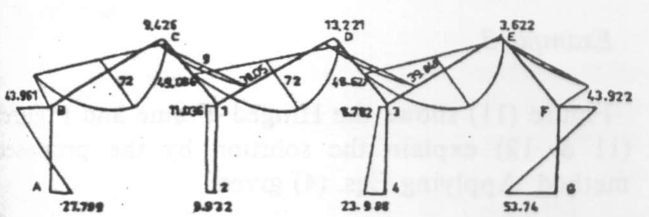

Figure 7. B.M.D.

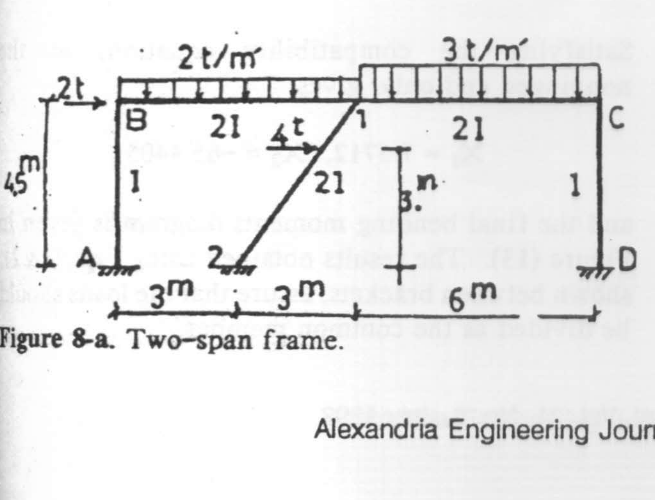

Figure 8-a. Two-span frame.

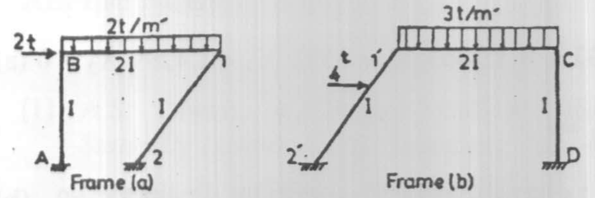

Figure 8-b. Divided frames.

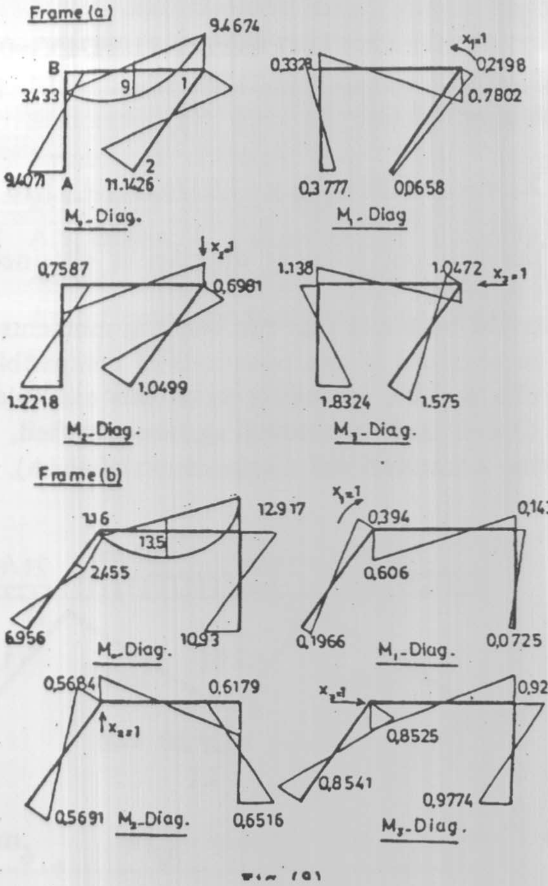

Figure 9.

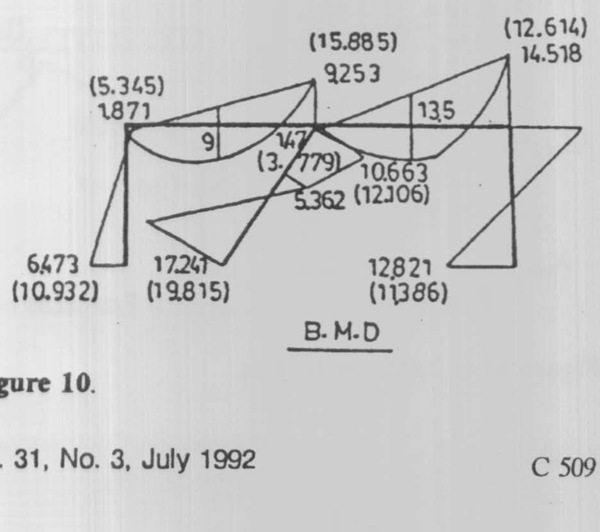

Figure 10.

$\Theta_1 = 0$

$$.3318 + 1.3063\,X_1 - .9533\,X_2 + 1.4299\,X_3 = 0 \quad (a)$$

$v_1 = 0$

$$14.5859 - .9533\,X_1 + 5.3324X_2 - 7.9985X_3 = 0 \quad (b)$$

$h_1 = 0$

$$-21.8775 + 1.4299X_1 - 7.9985X_2 + 11.9971X_3 = 0 \quad (c)$$

which give:

$$X_1 = -2.587984,\ X_2 = -7.9376,\ X_3 = -3.16$$

and the bending moment diagram is obtained as given in Figure (10).

It should be noted that the bending moments can also be obtained if two equations of compatibility, Eqs. (3.a & 3.b), excluding x_2 of this example, or Eqs. (3.a & 3.c), excluding x_3, are satisfied, (i.e. ignoring a translational displacement of them).

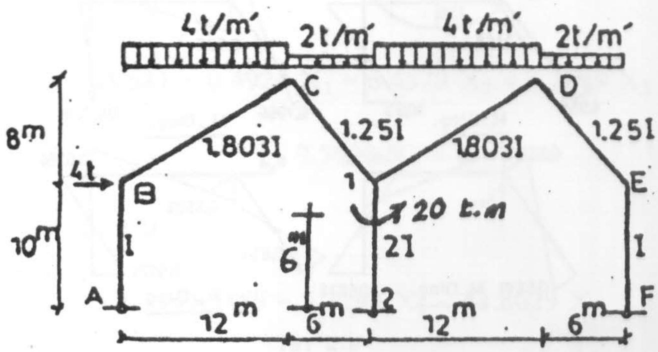

Figure 11-a. Two-span hinged frame.

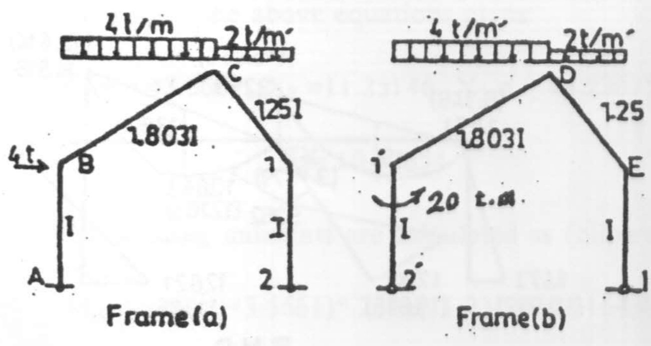

Figure 11-b. Divided frames.

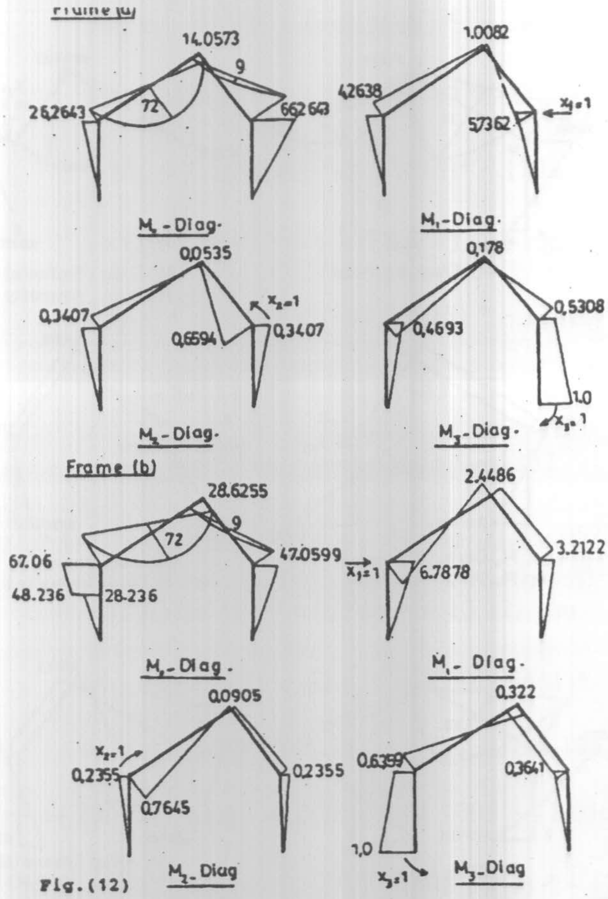

Figure 12.

Example 3

Figure (11) shows the Hinged–Frame and Figures (11 & 12) explain the solution by the proposed method. Applying Eqs. (4) gives:

$$X_1 = 8.08868,\ \ X_2 = -67.23261,\ \ X_3 = 3.3722$$

Satisfying the compatibility equations at the nonhinged end only, gives:

$$X_1 = 7.5712,\ \ X_2 = -65.4405$$

and the final bending moments diagram is given in Figure (13). The results obtained using Eqs. (1&2), shown between brackets, assure that the loads should be divided as the common member.

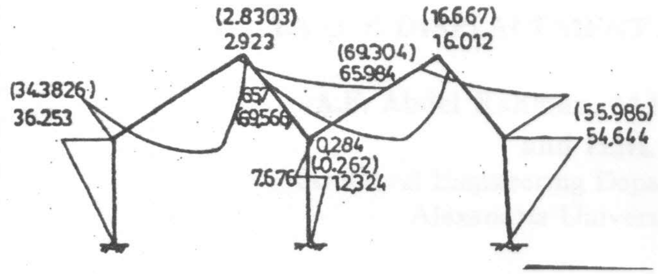

Figure 13.

CONCLUSION

The methods given by M.H. Badir, [refs. 2,3], and given, here, by Eqs. (1 and 1&2), for the analysis of multi-cell structure by Column Analogy are examined in new different examples, in which the common members are loaded. In those methods, accurate solutions can be obtained only if the loads on the common members are divided as the members themselves, while this is not necessary for the method proposed in this work. Also, for the proposed method, here, the redundants which correspond to the hinged ends of the sliced members can be ignored in the compatibility equations (satisfying the compatibility equations at the nonhinged ends is enough), and a translational displacement of an end will satisfy the other translational displacement of the same end, excluding one of them in the compatibility equations.

REFERENCES

[1] A.S. Diwan, A. Fahmy and M. Rasheed, *Statically Indeterminate Structures*, Dar-Elrateb Al-Jamiah, Lebanon, Part-I, 1988.

[2] M.H. Badir, "Column Analogy in Multi-Cell Structures (1)", *First Alexandria Conference on Struct. and Geotech. Eng.*, Alex., Egypt, pp. 199-213, 1-3 Dec. 1990.

[3] M.H. Badir, "Column Analogy in Multi-Cell Structures (2)", *First Alexandria Conference on Struct. and Geotech. Eng.*, Alex., Egypt, pp. 215-220, 1-3 Dec. 1990.

[4] A.S. Diwan, A. Fahmy and M. Metwally, *Stress Analysis and Theory of Structures*, in Arabic, Arab Center for Publ. and Dist., Alex., Egypt, part-II, 1980.

[5] M.H. Badir, *Statically Indeterminate Structures*, Lectures notes, Faculty of Eng., Alex. Univ., Alexandria, Egypt.