# A POWER SYSTEM LINEAR DECOUPLE CONTINGENCY ANALYSIS ALGORITHM

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ABSTRACT

This paper introduces a simple, linear, fast, decouple and non-iterative contingency analysis algorithm derived form the approximate D.C. loadflow technique. The linear relations between active and reactive powers injected to system buses and phase angles and magnitudes of nodal voltages of the system are coupled together to determine the power system state variables after changes in system network configuration resulting from contingent outages and/or planned switching of lines. The proposed technique takes into account second order term of changes in system states and parameters resulting from such contingencies. The drop of these changes in references [1], [2], [3], yields to a substantial amount of error in their output results, specially for lightly loaded power systems [2]. The consideration of these terms highly improved final output results of the proposed technique as compared with any previous technique developed on the same basis.

## INTRODUCTION

Automatic contingency analysis is an increasingly valuable analytical tool in many energy management system. It is predominantly used to predict steady state conditions following branch or generation outages. AC power flow methods have proved to be dispensable for any kind of steady state analysis in real time security monitoring owing to the prohibitive computer time and cost involved. Therefore, it is a common practice to sacrifice the accuracy for speed. Thus, considerable efforts [1-13] have been made to formulate techniques in linear form in order that a large number of system contingencies may be analysed swiftly on-or off-line.

Techniques [1,2,3] are dealing only with relation between bus injected active power and bus phase angles. They are simple, non-iterative and approximate DC load flow contingency analysis. They did not take into consideration the second-order terms of changes in system states and parameters resulting from system contingencies. Although the DC power flow methods [1], [2], [3] are simple and lend themselves to the use of superposition to evaluate the contingencies effects, their accuracy was not satisfactory. Therefore, the Z bus method [4] appeared to give better accuracy, that is, the results are more close to those obtained using standard AC power flow analysis. A modification was done in reference [5] to the DC load flow method to consider the second order terms of system phase angles and parameters. This modification led to a significant improvement in its performance and accuracy level for both single and multiple contingencies. However, all these methods [1-5] suffer from he inability to provide voltage and vars information.

To handle this problem, methods [6-7] were proposed to determine separately the changes in nodal voltages and phase angles by a linear iterative process. Another technique [8] had been proposed, it implemented a sensitivity matrix (inverted Jacobian) about contingencies which could be formulated during the iterative process. All these methods had been applied inspite of the possibility of no convergence exist. Other approximate AC techniques based on opposite current injections and modification of either bus impedance or Jacobian matrices were proposed in references [9-11]. A non iterative technique was proposed [12], to calculate the bus angle from DC load flow. These angles were used to determine the nodal voltage changes from nodal current equations. This technique gave unsatisfactory results due to the approximate nature of the angle calculation. A trial was made in reference [13] to determine effect of system network parameters on changes in system state variable. The nodal current changes are expressed in terms of changes in system nodal voltages by implementing the system nodal performance equation. It is iterative, not simple and its results were less accurate than the standard load flow techniques.

This paper introduces an extension of the modified algorithm [5] to determine power system state variables after contingencies. It is an approximate DC loadflow technique based on the relation between active and reactive power injected to system buses and phase angles and magnitudes of nodal voltages i.e a coupled, (p-$\delta$) and (Q-V) model. It takes into consideration the second-order terms of changes in system states and parameters involved in active and reactive power relations. These changes are due to power system network configuration contingencies.

The proposed technique is introduction and discussed through numerical examples to depict its validity and accuracy. It gives better results with an acceptable level of accuracy for both single and multiple contingencies.

## 2. MATHEMATICAL BASIS

### 2.1 Nonlinear Loadflow Equation

The general equation of the complex injected power and current to the i$^{th}$ bus of a power system having N buses, are;

$$\overline{S_i}^{*} = P_i - jQ_i = \overline{V_i}^{*} \cdot \overline{I_i} \qquad (1)$$

and

$$\overline{I_i} = \sum_{j=1}^{N} \overline{Y_{ij}}\,\overline{V_j}, \quad i=1,1,2,\ldots,N \qquad (2)$$

where $\overrightarrow{V_i}$ is the conjugate of the complex voltage vector $V_i$ of bus i,

$$\overline{V_i} = V_i \,|\delta_i$$

$\overline{Y_{ij}}$ is the element ij of the system bus admittance matrix.

$$= Y_{ij} \lfloor \Theta_{ij} = (G_{ij} + B_{ij})$$

Substituting equation (2) into equation (1), the later becomes:

$$P_i - j\,Q_i = \overline{V_i}^{*} \sum_{j=1}^{N} \overline{Y_{ij}}\,\overline{V_j} \qquad (3)$$

Equating real and imaginary parts of both sides of equation (3), thus:

$$P_i = \sum_{j=1}^{N} V_i\, Y_{ij}\, V_j \cos(\theta_{ij}+\delta_j-\delta_i) \qquad (4)$$

$$Q_i = -\sum_{j=1}^{N} V_i\, Y_{ij}\, V_j \sin(\theta_{ij}+\delta_j-\delta_i) \qquad (5)$$

Equation (4) and (5) are the basic equation for conventional exact AC loadflow techniques. These equations can be linearized as shown in the next subsection

### 2.2 p-$\underline{\delta}$ Model[1]

The reactance, X, of high voltage overhead transmission lines is usually much more greater than their resistance, R, i.e

$$X/R >> 1$$

Thus, the conductive part $G_{ij}$ of the ij$^{th}$ element of the bus admittance matrix can be neglected, i.e

$$Y_{ij} \simeq jB_{ij} = B_{ij} \mid 90$$

Therefore, equation (4) can be rewritten as:

$$P_i = \sum_{j=1}^{N} V_i\, B_{ij}\, V_j \sin(\delta_i - \delta_j)$$

Also if $(\delta_i - \delta_j)$ is sufficiently small such that:

$$\sin(\delta_i - \delta_j) = (\delta_i - \delta_j)$$

then, the approximate linearized equation of the bus injected active power as a function of bus phase angles becomes: [1][5]

$$P_i = \sum_{j=1}^{i-1} K_{ij}\,\delta_j + K_{ii}\,\delta_i + \sum_{j=i+1}^{N} K_{ij}\,\delta_j \qquad (6)$$

where:

$$K_{ii} = \sum_{j=1,\, i\neq j} V_i V_j B_{ij} \qquad (7)$$

and

$$K_{ij} = - V_i\, V_j\, B_{ij} \qquad (8)$$

The linearized equation (6) can be rewritten in matrix form as [1][2],[5]:

$$[P] = [K]\,[\delta] \qquad (9)$$

where [K] is an (NxN) matrix, the diagonal and off diagonal elements of which are expressed by equations (7) and (8), respectively.

Equation (9) is the linear relation between active powers injections to system buses and phase angles for a given network configuration.

### *2.3 Q-V Model*

In a similar way, with the same assumptions proposed for the (P–δ) model, the following (Q–V) linear model can be deduced by linearizing the non-linear reactive power equation (5):

$$Q_i = - \sum_{j=1}^{N} V_i B_{ij} V_j \cos(\delta_i - \delta_j)$$

Since, $(\delta_i - \delta_j)$ is sufficiently small, then,

$$\cos(\delta_i - \delta_j) = 1$$

And the linear form of equation (5) becomes:

$$Q_i = - V_i^2 B_{ii} - V_i \sum_{j=1,\, j\neq i}^{N} B_{ij} V_{ij} V_j \qquad (10)$$

$$\text{for all } i = 1, 2, \ldots, N$$

Equation (10) can be expressed in matrix form as:

$$[Q] = [C]\,[V] \qquad (11)$$

where [C] is an (NxN) matrix, the diagonal elements of which are:

$$C_{ii} = - V_i\, B_{ii} \qquad (12)$$

and the off diagonal elements are:

$$C_{ij} = - V_i\, B_{ij} \qquad (13)$$

Since bus voltage magnitudes, for both slack and control buses, are specified and known, the voltages for load buses are unknown. Also the control and slack buses reactive powers are unknowns. Therefore, equation (11) should be partitioned into two sets of equations, one associated with load buses, and the other associated with both slack and control buses. As a results of this partitioning equation (11) becomes:

$$\begin{bmatrix} Q_g \\ --- \\ Q_L \end{bmatrix} = \begin{bmatrix} C_g \\ --- \\ C_L \end{bmatrix} \begin{bmatrix} V_g \\ --- \\ V_L \end{bmatrix} \qquad (14)$$

In this equation $Q_g$ for slack and control buses and $V_L$ for load buses are unknown, but $V_g$ for slack and control buses and $Q_L$ for load buses are known.

The lower part of equation (14) can be written as:

$$[Q_L] = [C_L] \begin{bmatrix} V_g \\ ---- \\ V_L \end{bmatrix} \qquad (15)$$

The matrix $[C_L]$ can in turn be partitioned to two submatrices $[C_1]$ and $[C_2]$ so that equation {15) can be changed to:

$$[Q_L] = [C_1\ C_2] \begin{bmatrix} V_g \\ V_L \end{bmatrix}$$

or

$$[Q_L] = [C_1]\,[V_g] + [C_2]\,[V_L] \qquad (16)$$

## 3. CONTINGENCY ANALYSIS ALGORITHM

The above two models can now be implemented for contingency analysis. The removal (addition) of a line or lines, from (to) the network can be simulated as follows:

If this happens, it will result in changing the matrices [K] and [$C_L$] by correction matrices [$\Delta K$] and [$\Delta C_L$] respectively, and in changing the vectors [$\delta$] and [$V_L$] by correction vectors [$\Delta\delta$] and [$\Delta V_L$]. Therefore, equations (9) and (16) can be rewritten after contingency as:

$$[P] = \{[K^o] + [\Delta K]\}\ [\delta^o + \Delta\delta] \tag{17}$$

and

$$[Q_L] = \{[C_1^o] + [\Delta C_1]\}[V_g] + \{[C_2^o] + [\Delta C_2]\}\ [V_L^o + \Delta V_L] \tag{18}$$

where [$K^o$], [$C^o$], [$\delta^o$] are the precontingency conditions.

The correction matrices [$\Delta K$] and [$\Delta C_L$] should be found firstly for both single and multiple contingencies before using equation (17) and (18) to derive a linear, approximate, fast and decoupled contingency analysis algorithm.

### 3.1 *Branch Outage Simulation*

If a single contingency occurs such as the removal of a line pq, four changes will occur in both matrices [K] and [$C_L$]. The only elements of [K] matrix that will be changed [1] are $K_{pp}$, $K_{pq}$, $K_{qp}$ and $K_{qq}$. Thus the non-zero elements of the correction matrix [$\Delta K$] for the case of p-q line removal are [1][5]:

$$\Delta K_{pp} = K_{pq} + y_{sp},\ \Delta K_{pq} = -K_{pq},\ \Delta K_{qp} = -K_{pq}$$

$$\text{and } \Delta K_{qq} = K_{pq} + y_{sq}$$

Thus the matrix [$\Delta K$] can be represented as follows:

$$[\Delta K] = \begin{array}{c c} & \begin{array}{ccccc} & p & & q & \end{array} \\ \begin{array}{c} \\ \\ p \\ \\ q \\ \\ \\ \end{array} & \begin{bmatrix} 0.. & 0 & & & ..0 \\ ... & ... & ... & ... & ... \\ 0.. & K_{pq}+y_{sp} & 0 & -K_{pq} & ..0 \\ 0.. & 0 & 0 & 0 & ..0 \\ 0.. & -K_{pq} & 0 & K_{pq}+y_{sq} & ..0 \\ ... & ... & ... & ... & ... \\ 0.. & 0 & 0 & 0 & ..0 \end{bmatrix} \end{array} \tag{19}$$

Similarly the changes in matrix [$C_L$] due to a single contingency are the elements $C_{pp}$, $C_{pq}$, $C_{qp}$ and $C_{qq}$. Therefore, the non-zero element of correction matrix [$\Delta C_L$] are:

$$\Delta C_{pp} = C_{pq} + y_{sp},\ \Delta C_{pq} = -\Delta C_{qp},\ \Delta C_{qp} = -C_{pq} \text{ and } \Delta C_{qq} = C_{pq} + y_{sq}$$

and

$$\Delta C_L = \begin{array}{c c} & \begin{array}{ccccc} & p & & q & \end{array} \\ \begin{array}{c} \\ \\ p \\ \\ q \\ \\ \\ \end{array} & \begin{bmatrix} 0.. & 0 & 0 & 0 & ..0 \\ ... & ... & ... & ... & ... \\ 0.. & C_{pq}+y_{sp} & 0 & -C_{pq} & ..0 \\ 0.. & 0 & 0 & 0 & ..0 \\ 0.. & -C_{pq} & 0 & C_{pq}+y_{sq} & ..0 \\ ... & ... & ... & ... & ... \\ 0.. & 0 & 0 & 0 & ..0 \end{bmatrix} \end{array} \tag{20}$$

where $y_{sp}$ and $y_{sq}$ are the shunt admittances of line p-q at buses p and q respectively. Or

$$[\Delta C_L] = [\Delta C_1\ \Delta C_2]$$

where, [$\Delta C_1$], [$\Delta C_2$] are the right-and left hand partitions of [$\Delta C_L$] according to vectors $V_g$ and $V_L$ respectively.

Note that in case of line addition both [$\Delta K$] and [$\Delta C_L$] will have opposite signs.

Similarly, if there is multiple contingencies occurrence, such as, for example, the removal of the two lines pq and ij, eight changes will occur in both matrices [K] and [$C_L$]. Similar to the single contingency case, the only elements that will be changes in both matrices are:

pp, pq, qp, qq, ii, ij, ji, and jj

### 3.2 *Fast Decoupled Algorithm*

A fast linear contingency analysis algorithm can be developed by expanding equations (17) and (18). Its objective is to determine the changes of the system state variables $\Delta\delta$ and $\Delta V_L$ resulting from the contingencies under consideration.

The expansion of equation (17) is:

$$[P] = [K^o][\delta^o]+[\Delta K][\delta^o]+\{[K^o]+[\Delta K]\}\Delta[\delta] \qquad (21)$$

Equations (21), after some rearrangements, can be written as:

$$[K'] \, [\Delta\delta] = [\Delta P] \qquad (22)$$

where

$$[K'] = \{[K^o] + [\Delta K]\} \qquad (23)$$

and

$$[\Delta p] = [p] - [p^o] - [\Delta K] \, [\delta^o]$$

The vector [$\Delta\delta$] is the correction vector of the unknown phase angles of nodal voltages. It takes into account the change in network configuration due to contingencies an in assigned power injections.

The following expression can be developed in a similar way from equation (18) to find the correction vector [$\Delta V_L$] for nodal voltages of load buses.

$$[C_2'] \, [\Delta V_L] = [\Delta Q_L] \qquad (24)$$

where,

$$[C_2'] = \{[C_2^o] + [\Delta C_2]\}$$

and

$$[\Delta Q_L] = [Q_L] - \{[C_L^o] + [\Delta C_L]\} \begin{bmatrix} V_g \\ V_L^o \end{bmatrix} \qquad (25)$$

Equations (22) and (24) are the basic equations of the contingency analysis algorithm. They can be coupled together to determine both $\Delta\delta$ and $\Delta V_L$ simultaneously as follows:

$$\begin{bmatrix} K' & 0 \\ 0 & C' \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V_L \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q_L \end{bmatrix}$$

Therefore, the general equation after n sequential network configuration contingencies equivalent to simultaneous outage of n lines in a multiple contingency (of the $n^{th}$ order) is:

$$\begin{bmatrix} K'^{n} & 0 \\ 0 & C_2'^{n} \end{bmatrix} \begin{bmatrix} \Delta\delta^n \\ \Delta V_L^n \end{bmatrix} = \begin{bmatrix} \Delta P^n \\ \Delta Q_L^n \end{bmatrix} \qquad (26)$$

The coefficient matrices [K'] and [$C'_2$] can be evaluated from the matrices [K] and [$C_2$] after the simulation due to prior knowledge of the network configuration changes. These matrixes are symmetrical and have dominant diagonal elements. Their sparsity structure is identical to the system nodal admittance matrix. Therefore, the sparse matrix inversion technique [14], [15] can be implemented to obtain [$\Delta\delta^n$] and [$\Delta V_L^n$] where:

$$\begin{bmatrix} \Delta\delta^n \\ \Delta V_L^n \end{bmatrix} \begin{bmatrix} K'^{n} & 0 \\ 0 & C_2'^{n} \end{bmatrix} = \begin{bmatrix} \Delta P^n \\ \Delta Q_L^n \end{bmatrix} \qquad (27)$$

Thus, the bus phase angles and load bus voltage magnitudes vectors after (n) contingencies are:

$$[\delta^n] = [\delta^{n-1}] + [\Delta\delta^n]$$

$$[V_L^n] = [V_L^{n-1}] + [\Delta V_L^n] \qquad (28)$$

As the $V_L^n$ vector is calculated, the reactive power injection to control buses can be calculated from the upper part if equation (14) as:

$$[Q_g^n] = \{[C_g^{n-1}] + [\Delta C_g^n]\} \, [V^n] \qquad (29)$$

where

$$[V^n] = \begin{bmatrix} V_g \\ V_L^n \end{bmatrix}$$

Therefore, a linear non-iterative algorithm is introduced. Its output results will be the power system post contingency state variables which are the system bus voltage magnitudes and phase angles in addition to the reactive power modifications necessary to keep the voltages of control buses constant. The algorithm will be applied for both single and multiple contingencies to investigate its validity and accuracy level. The algorithm computational flow chart is shown in Figure (1).

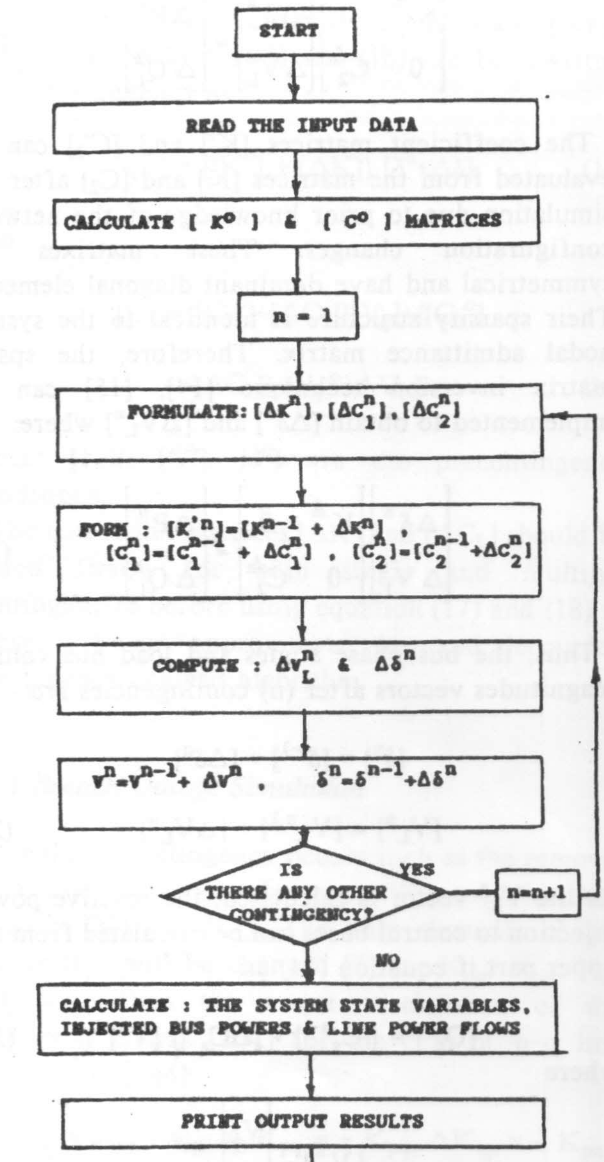


**Figure 1**. Proposed algorithm flow chart.

4. NUMERICAL EXAMPLES

For the purpose of illustration, the IEEE thirty bus system shown in Figure (2), is used as a test sample. The operating condition of the system is computed first, with all its network parameters in service by the conventional Newton Raphson load flow method. All buses are considered as (P-Q) buses except busbar (1) as a slack bus. The output results are considered as base data the contingency analysis. The proposed technique is applied for two cases of study. The first is single and the second is a simultaneous multiple contingencies. The output results of the decoupled algorithm are compared with those obtained by the separate runs for (P-$\delta$) and (Q-V) models are applied individually using equations (22), (24), to determine the effect of decoupling. Both results are compared with conventional Newton-Raphson AC loadflow method with the selected lines out of service.

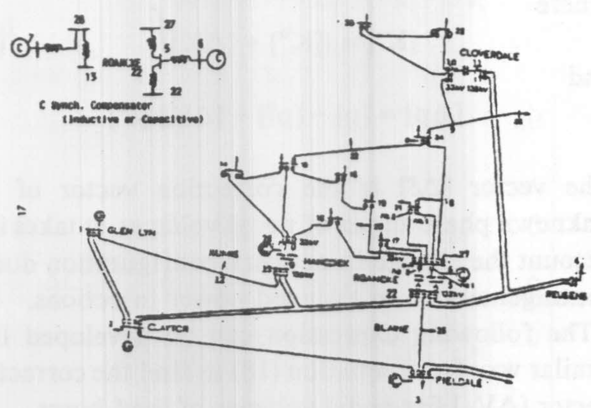


**Figure 2**. IEEE thirty bus system.

4.1 *Single Contingency*

It is assumed, a decision is taken to remove line (15-18), or a forced outage occurs to it. The proposed technique is implemented to determine system state variables due to such outage. The output results are given in Table (1), (2). They indicate that, inspite of the fact that the proposed technique is approximate and non-iterative, it is valid for single contingency, with accuracy level very close to AC load flow. The decoupled algorithm improves such level. It is clear that, the deviations for the nodal

**Table 1**. Computer results. (.Single contingency).

| Bus code | non coupled algor | | conventional L.F. | | Proposed algor | |
|---|---|---|---|---|---|---|
| | V | δ | V | δ | V | δ |
| 1 | 1.0600 | 0.00 | 1.0600 | 0.00 | 1.0600 | 0.00 |
| 2 | 1.0590 | -5.95 | 1.0590 | -5.66 | 1.0590 | -5.66 |
| 3 | 1.0547 | -14.26 | 1.0547 | -14.35 | 1.0547 | -14.34 |
| 4 | 1.0707 | -12.79 | 1.0707 | -12.58 | 1.0707 | -12.55 |
| 5 | 1.0663 | -14.47 | 1.0663 | -14.80 | 1.0663 | -14.64 |
| 6 | 1.0663 | -14.47 | 1.0663 | -14.80 | 1.0663 | -14.64 |
| 7 | 1.0852 | 15.52 | 1.0852 | -15.31 | 1.0860 | -15.57 |
| 8 | 1.0707 | -16.85 | 1.0707 | -17.04 | 1.0671 | -16.84 |
| 9 | 1.0759 | -16.98 | 1.0759 | -17.03 | 1.0821 | -17.02 |
| 10 | 1.0775 | -16.11 | 1.0775 | -16.29 | 1.0713 | -16.27 |
| 11 | 1.0644 | -12.42 | 1.0644 | -12.45 | 1.0533 | 12.43 |
| 12 | 1.0623 | -8.53 | 1.0623 | -8.36 | 1.0666 | -8.37 |
| 13 | 1.0590 | -10.06 | 1.0590 | -10.05 | 1.0659 | -10.07 |
| 14 | 1.0797 | -16.77 | 1.0797 | -16.26 | 1.0726 | -16.64 |
| 15 | 1.0771 | -16.99 | 1.0771 | -16.34 | 1.0814 | -16.79 |
| 16 | 1.0784 | -16.13 | 1.0784 | -16.02 | 1.0709 | -16.27 |
| 17 | 1.0736 | -15.81 | 1.0736 | -16.67 | 1.0586 | -16.51 |
| 18 | 1.0720 | -17.16 | 1.0720 | -18.96 | 1.0847 | -17.45 |
| 19 | 1.0689 | -17.50 | 1.0683 | -18.72 | 1.0703 | -17.62 |
| 20 | 1.0710 | -17.30 | 1.0710 | -18.21 | 1.0776 | -17.31 |
| 21 | 1.0710 | -16.85 | 1.0710 | -17.09 | 1.0746 | -16.88 |
| 22 | 1.0625 | -11.76 | 1.0625 | -11.77 | 1.0688 | -11.76 |
| 23 | 1.0733 | -17.29 | 1.0733 | -17.03 | 1.0857 | -17.29 |
| 24 | 1.0703 | -17.66 | 1.0703 | -17.43 | 1.0633 | -17.43 |
| 25 | 1.0619 | -13.48 | 1.0619 | -13.50 | 1.0622 | -13.48 |
| 26 | 1.0753 | -17.85 | 1.0753 | -17.98 | 1.0758 | -17.97 |
| 27 | 1.0694 | -16.10 | 1.0694 | -16.40 | 1.0725 | -16.15 |
| 28 | 1.0841 | -15.56 | 1.0841 | -15.31 | 1.0841 | -15.58 |
| 29 | 1.0699 | -17.85 | 1.0841 | -17.70 | 1.0770 | -17.58 |
| 30 | 1.0642 | -18.76 | 1.0639 | -18.60 | 1.0543 | -18.57 |

**Table 2.** Computer results (power flows for single contingency).

| Bus code from | | Proposed algorithm | | Conventional AC L.F | |
|---|---|---|---|---|---|
| | | active | reactive | active | reactive |
| 1 | 2 | 1.76684 | −0.50616 | 1.78089 | 0.20828 |
| 1 | 12 | 0.84468 | −0.17919 | 0.84287 | 0.13583 |
| 2 | 3 | 0.82397 | −0.10874 | 0.81796 | 0.18907 |
| 2 | 13 | 0.44291 | −0.16795 | 0.46153 | −0.00257 |
| 2 | 22 | 0.60801 | −0.22305 | 0.62920 | 0.04964 |
| 3 | 25 | −0.14758 | −0.00836 | −0.15644 | −0.09654 |
| 4 | 11 | 0.01572 | −0.08787 | 0.00945 | −0.05102 |
| 4 | 22 | −0.33750 | 0.14635 | −0.29054 | −0.24895 |
| 5 | 6 | 0.00 | −0.00000 | 0.00 | 0.00 |
| 5 | 22 | −0.27571 | −0.00584 | −0.30595 | 0.10612 |
| 5 | 27 | 0.27438 | 0.05609 | 0.30595 | −0.10612 |
| 7 | 28 | −0.00 | −0.06747 | −0.00 | 0.00 |
| 8 | 21 | −0.10651 | −0.29060 | −0.01926 | −0.05803 |
| 8 | 24 | 0.05617 | −0.01350 | 0.05441 | 0.03934 |
| 8 | 27 | −0.8925 | 0.00516 | −0.07364 | 0.01873 |
| 9 | 10 | −0.03377 | 0.07407 | −0.03219 | 0.01286 |
| 9 | 24 | 0.04524 | 0.03587 | 0.00329 | −0.08057 |
| 9 | 26 | 0.04346 | −0.01077 | 0.03547 | −0.02229 |
| 10 | 11 | −0.19116 | 0.05502 | −0.16671 | −0.04866 |
| 10 | 29 | 0.04724 | −0.03904 | 0.06213 | −0.01152 |
| 10 | 30 | 0.07168 | −0.00640 | 0.07104 | −0.01046 |
| 11 | 22 | −0.27479 | −0.30308 | −0.27624 | −0.07309 |
| 12 | 13 | 0.79970 | −0.24322 | 0.78927 | 0.07019 |
| 13 | 22 | 0.73603 | −0.27598 | 0.74056 | 0.15244 |
| 13 | 28 | 0.43851 | −0.09887 | 0.41434 | −0.08235 |
| 14 | 15 | 0.01651 | −0.02905 | 0.00709 | 0.00191 |
| 14 | 28 | −0.10456 | −0.04133 | −0.06909 | 0.01409 |
| 15 | 23 | 0.03161 | −0.03806 | 0.07042 | 0.00535 |
| 15 | 28 | 0.19612 | −0.00816 | 0.14535 | 0.02154 |
| 16 | 17 | 0.04534 | 0.04925 | 0.05019 | −0.00550 |
| 16 | 28 | −0.10783 | −0.07623 | −0.08519 | 0.02350 |
| 17 | 27 | −0.13107 | −0.12321 | −0.04002 | 0.05203 |
| 18 | 19 | 0.06850 | 0.08700 | 0.03200 | 0.00901 |
| 19 | 20 | −0.11821 | −0.05645 | −0.12709 | 0.04290 |
| 20 | 27 | −0.08309 | 0.06480 | −0.14970 | 0.04869 |
| 21 | 27 | 0.14929 | 0.10150 | 0.15199 | −0.04357 |
| 22 | 25 | 0.40487 | −0.03914 | 0.39033 | −0.03024 |
| 22 | 27 | 0.15804 | −0.00098 | 0.17498 | −0.04818 |
| 23 | 24 | 0.04384 | 0.06843 | 0.03794 | 0.02038 |
| 29 | 30 | 0.05306 | −0.02489 | 0.03716 | −0.00437 |

**Table 3**. Computer results. (Multiple contingency)

| Bus code | non coupled algor | | conventional L.F. | | Proposed algor | |
|---|---|---|---|---|---|---|
| | V | δ | V | δ | V | δ |
| 1 | 1.0600 | 0.00 | 1.0600 | 0.00 | 1.0600 | 0.00 |
| 2 | 1.0590 | -12.22 | 1.0590 | -5.66 | 1.0590 | -5.66 |
| 3 | 1.0547 | -22.92 | 1.0547 | -15.69 | 1.0547 | -14.33 |
| 4 | 1.0707 | -21.04 | 1.0707 | -15.75 | 1.0707 | -12.57 |
| 5 | 1.0663 | -12.38 | 1.0663 | -17.71 | 1.0663 | -14.63 |
| 6 | 1.0663 | -19.39 | 1.0663 | -17.71 | 1.0663 | -14.63 |
| 7 | 1.0837 | 7.32 | 1.0727 | -18.25 | 1.0827 | -15.57 |
| 8 | 1.0710 | -20.46 | 1.0587 | -19.88 | 1.0758 | -16.85 |
| 9 | 1.0755 | -11.50 | 1.0621 | -20.10 | 1.0758 | -17.02 |
| 10 | 1.0781 | -21.83 | 1.0637 | -19.38 | 1.0846 | -16.27 |
| 11 | 1.0647 | -7.16 | 1.0497 | -15.53 | 1.0643 | 12.43 |
| 12 | 1.0604 | 6.76 | 1.0497 | -10.15 | 1.0653 | -8.37 |
| 13 | 1.0584 | -10.09 | 1.0468 | -12.23 | 1.0565 | -10.06 |
| 14 | 1.0790 | -25.44 | 1.0682 | -19.37 | 1.0808 | -16.64 |
| 15 | 1.0770 | -10.48 | 1.0655 | -19.58 | 1.0835 | -16.80 |
| 16 | 1.0784 | -10.69 | 1.0668 | -19.08 | 1.0827 | -16.26 |
| 17 | 1.0733 | -28.20 | 1.0614 | -19.49 | 1.0658 | -16.51 |
| 18 | 1.0723 | -14.66 | 1.0604 | -20.34 | 1.0390 | -17.46 |
| 19 | 1.0688 | -11.17 | 1.0582 | -20.56 | 1.0702 | -17.61 |
| 20 | 1.0695 | -21.72 | 1.0580 | -20.28 | 1.0745 | -17.31 |
| 21 | 1.0713 | -24.61 | 1.0589 | -19.92 | 1.0708 | -16.88 |
| 22 | 1.0625 | -14.55 | 1.0507 | -14.91 | 1.0554 | -11.75 |
| 23 | 1.0743 | -17.51 | 1.0623 | -20.18 | 1.0742 | -17.22 |
| 24 | 1.0710 | -13.02 | 1.0581 | -20.44 | 1.0723 | -17.42 |
| 25 | 1.0619 | -12.04 | 1.0506 | -15.92 | 1.0618 | -13.48 |
| 26 | 1.0749 | -24.21 | 1.0618 | -21.07 | 1.0687 | -17.96 |
| 27 | 1.0694 | -7.89 | 1.0573 | -19.18 | 1.0645 | -16.15 |
| 28 | 1.0841 | -17.13 | 1.0841 | -18.25 | 1.0841 | -15.58 |
| 29 | 1.0704 | -24.69 | 1.0560 | -20.83 | 1.0745 | 17.68 |
| 30 | 1.0644 | -12.70 | 1.0497 | -21.75 | 1.0667 | 18.58 |

**Table 4**. Computer results (power flows for single contingency).

| Bus code from | | Proposed algorithm active p.u | Proposed algorithm reactive p.u | Conventional AC L.F active p.u | Conventional AC L.F reactive p.u |
|---|---|---|---|---|---|
| 1 | 2 | 1.76605 | −0.50569 | 1.62317 | 0.31481 |
| 1 | 12 | 0.84543 | −0.17177 | 1.03746 | 0.26441 |
| 2 | 3 | 0.82356 | −0.10871 | 0.99779 | 0.27286 |
| 2 | 13 | 0.45605 | −0.11564 | 0.76131 | 0.08490 |
| 3 | 25 | −0.15495 | −0.00554 | 0.00626 | −0.08576 |
| 4 | 22 | −0.25137 | −0.46459 | −0.30000 | −0.29997 |
| 5 | 6 | 0.0 | 0.0 | 0.0 | 0.0 |
| 5 | 22 | −0.27255 | 0.06296 | −0.27666 | 0.11148 |
| 7 | 27 | 0.27279 | 0.02132 | 0.27666 | −0.011148 |
| 8 | 28 | 0.0 | −0.0 | 0.0 | 0.0 |
| 8 | 21 | 0.11045 | 0.17202 | 0.01138 | −0.05129 |
| 8 | 24 | 0.05586 | −0.01474 | 0.06735 | 0.02814 |
| 9 | 27 | −0.04287 | −0.10248 | −0.07871 | 0.02312 |
| 9 | 10 | −0.07610 | −0.00518 | −0.02901 | 0.09838 |
| 9 | 24 | 0.02341 | −0.00200 | −0.00650 | −0.07614 |
| 10 | 26 | −0.04383 | 0.00867 | 0.03551 | −0.02224 |
| 10 | 11 | −0.19552 | 0.06210 | −0.16376 | −0.04137 |
| 10 | 29 | 0.06521 | −0.00717 | 0.06225 | −0.01132 |
| 11 | 30 | 0.07429 | 0.00559 | 0.07188 | −0.01021 |
| 12 | 22 | −0.16361 | −0.20690 | −0.16377 | −0.05488 |
| 13 | 13 | 0.86124 | −0.04082 | 0.96683 | 0.12796 |
| 13 | 22 | 0.74532 | −0.17306 | 1.14263 | 0.20414 |
| 14 | 28 | 0.43028 | −0.09167 | 0.46379 | −0.06837 |
| 14 | 15 | 0.01630 | −0.01481 | 0.01780 | −0.00173 |
| 15 | 28 | −0.07331 | −0.02334 | −0.07980 | 0.01774 |
| 15 | 18 | 0.13516 | −0.15472 | 0.06356 | −0.01463 |
| 15 | 23 | 0.05394 | −0.02339 | 0.06064 | 0.00910 |
| 16 | 28 | −0.15241 | −0.07709 | −0.18848 | 0.02873 |
| 16 | 17 | 0.05700 | −0.07062 | 0.04450 | 0.00231 |
| 17 | 28 | 0.05911 | 0.02250 | −0.07950 | 0.01569 |
| 18 | 27 | 0.06777 | −0.04266 | −0.04567 | 0.05992 |
| 18 | 19 | −0.08103 | −0.12067 | 0.03109 | −0.00660 |
| 18 | 20 | −0.08404 | 0.21667 | −0.06399 | 0.02728 |
| 20 | 27 | −0.07273 | −0.08518 | −0.08617 | 0.03393 |
| 21 | 27 | −0.12320 | 0.14910 | −0.16364 | 0.06063 |
| 22 | 25 | 0.35064 | 0.19062 | 0.22363 | −0.05044 |
| 22 | 27 | 0.15510 | −0.01134 | 0.15823 | −0.05235 |
| 23 | 24 | 0.01545 | −0.00018 | 0.02825 | 0.02431 |
| 29 | 30 | 0.03874 | 0.00225 | 0.03720 | 0.00508 |

voltage magnitudes, between the proposed technique and AC load flow, has a maximum value of 2.3% while for the angles is about 0.5%. This accuracy level can be considered acceptable since the permissible errors for such approximate linearized techniques are about 4-8% (see discussions on references [3], [6], [10] and [12]).

### 4.2 *Multiple Contingencies*

The proposed technique is rerun, in this case, with lines (2-22) and (4-11) out of service simultaneously. The output results are given in Tables (3) and (4). They show that, the proposed technique is still valid with less accuracy level. The maximum deviation of voltage magnitudes is about 2.1%, but for the phase angles the deviation are high (=4%).

## 5. CONCLUSION

A simple, linear and fast contingency analysis algorithm is introduced in this paper. Its output results indicate its validity to determine the power system state variables, after contingent changes in its network parameters, specially those resulting from single contingencies. If multiple contingencies occur, it could be run sequentially to obtain output results close to the AC load flow techniques. Therefore it can be used, for such cases, instead of conventional AC techniques with an acceptable accuracy level and time saving. When AC load flow is to be used to study contingencies, the speed of solution and the number of cases to be studied are critical. If the contingency alarm comes too late for the operator, they are worthless [2], [17].

The proposed technique is based on linearizing the basic AC loadflow non-linear equations. It takes into consideration second order change terms of state variables and network parameters effects. The decoupling of both (P-$\delta$) and (Q-V) models improves the proposed technique accuracy level. The technique results depicts that, it could be implemented for single contingency successfully. In multiple contingency analysis, the algorithm can b\deal with the system on the basis of sequential single failure events. The lines to be outaged can be taken one after the one in sequence until all credible outages has been studied [2], [17]. For each outage tested the contingency analysis check all lines and nodal voltages in the network against respective limits. The proposed technique will be a good guide for system dispatcher and planner to have a quick view for the power system under emergency conditions. It could be used to check the most important lines in the network whose outages may cause the system monitoring to collapse.

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