FRICTION HEAD LOSS IN BOX CULVERT HAVING DIFFERENT SIDE AND BED ROUGHNESS

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ABSTRACT

The friction head loss in box culvert having different bed and side roughness is experimentally studied for the cases when the whole culvert is running full, a part of it is full and as an open channel. In the case when the culvert is running full, a semi-empirical equation is developed in terms of side and bed roughness to express average roughness coefficient "Ks" and Hazen-Williams Coefficient "C". In the condition when the culvert is running partly full, an empirical equation is given to determine the coefficient of discharge "Cd". Further more, an empirical equation is developed to express Manning's coefficient "n" for the case of open channel flow.

NOTATIONS

A	cross section area
A_s	area corresponding to the sides
A_b	area corresponding to the bed
B	culvert width
C	Hazen-Williams coefficient
Cd	coefficient of discharge
d	pipe diameter
f	friction coefficient
f_b	bed friction coefficient
f_s	side friction coefficient
Fr	Froude number
g	gravitational constant
H	culvert height
h_f	friction head loss
Ks	equivalent roughness coefficient
Ks_b	bed roughness coefficient
Ks_s	side roughness coefficient
L	culvert length
n	Manning's coefficient
R	hydraulic radius
Re	Reynolds number
Y	depth of water
$Y_{U.S}$	headwater depth

INTRODUCTION

Discharge capacity of a box culvert is affected by the depth of headwater, entrance and outlets geometry, length and size of culvert, friction coefficient and depth of tailwater.

Several investigators (3,7,8,9) hydraulically classified the flow through culvert according to the following:

1- Inlet control flow, which is defined as flow through culvert with the depth of headwater controlled by the inlet as is usual for culverts on mild or steep slopes with tailwater below the crown of culvert at outlet. The area, shape and edge detail of the inlet face affect the depth of headwater. The operation in this case is analogous to an orifice. In inlet control, conditions of downstream, length, slope and roughness of culvert do not affect headwater depth.

2- Outlet control flow, which is defined as culvert operation with headwater controlled by conditions at the outlet. The depth of flow at the outlet, size, shape, roughness, length, slope and the inlet shape affect the headwater depth. This type of flow occurs when culvert inlet and outlet are submerged or when the slope is flat especially when downstream conditions cause the tailwater depth to be greater than the critical depth."

Herr and Bossy [3] gave a design procedure for highway culverts. They calculated the friction losses in the case of outlet control culverts from Manning's Strickler formula as follows

$$h_f = \left[\frac{8gn^2}{R^{0.33}}\right]\frac{v^2}{2g} \qquad (1)$$

The Manning's coefficient "n" was computed for culverts having uniform and homogeneous roughness along its walls.

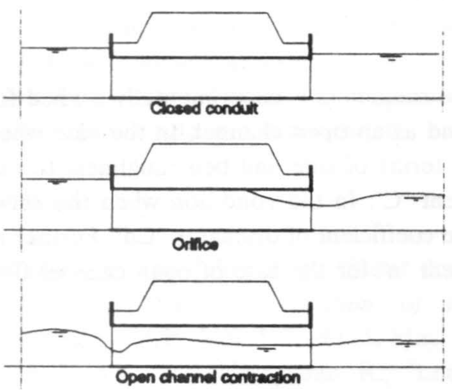

Figure 1. Types of culvert flow.

Chow [2] classified culvert flow into three main types (Figure 1):-

1- Full flow with submerged inlet and submerged or unsubmerged outlet (hydraulically long). In this case the culvert acts like a pipe.
2- Partly full flow with submerged inlet and unsubmerged outlet (hydraulically short culvert). When the headwater depth is greater than 1.5 the culvert's height, the culvert acts as an orifice. The discharge coefficient varies between 0.75 to 0.95
3- Open channel flow culvert in which the headwater depth is less than 1.5H and the tailwater level is less than the culvert outlet.

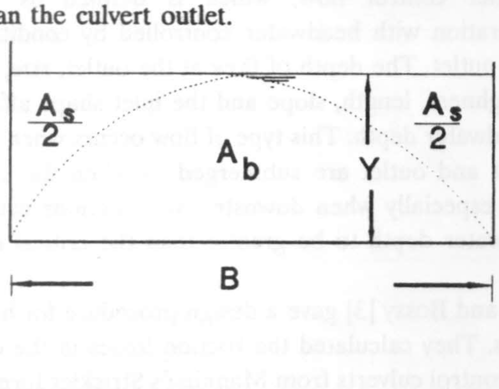

Figure 2.

Vanoni and Brooks [6] developed a procedure to determine the friction coefficient "f" in bed and banks of an open channel. Their analysis based on the assumption that the velocity is uniformly distributed over the whole cross section. Assuming that the total flow area can be divided into area corresponding to the bed and sides (Figure 2) one writes

$$A = A_s + A_b \qquad (2)$$

In which A_s is the area corresponding to the sides, A_b the area corresponding to the bed and "A" is the whole cross section area.

Using Karmen-Prandtl equation, the hydraulic radius of the sides and bed can be written as follows:

$$\frac{v^2}{8gS} = \frac{R_s}{f_s} = \frac{R_b}{f_b} = \frac{R}{f} \qquad (3)$$

Where R is the hydraulic radius, S is the slope of the total energy line and the subscript s and b indicates sides and bed, respectively.

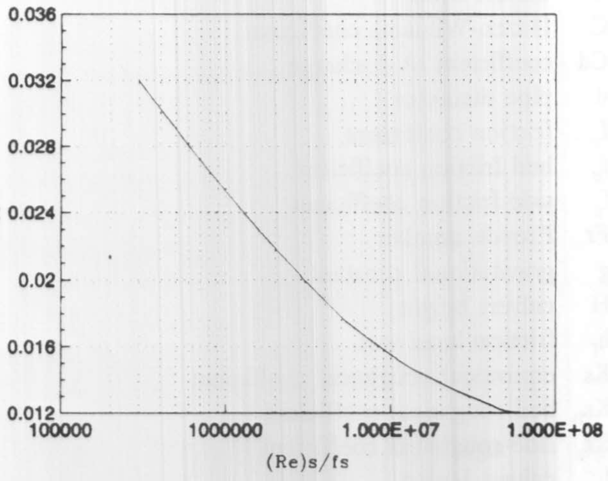

Figure 3. variation of Re_s/f_s with side friction factor.

In case of smooth sides Vanoni and Brooks [6] plotted a graph (figure 3) between f_s and the ratio Re_s/f_s which can be used to find f_s by assuming that Re_s/f_s = Re/f. Where Re_s is sides Reynolds number.

All equations used for computing the friction head loss along closed conduits consider that the roughness inside the cross section is homogeneous. However, in some box culverts the bed roughness differs than that of the sides because sediment particles and derbies are deposited on the bed.

The aim of this study is to develop a method to calculate the friction head loss in box culverts taking into consideration the effect of side and bed roughness for different types of flow in box culverts.

THEORETICAL APPROACH

Flow through culverts are hydraulically classified into three types:

i- Open channel flow

In the condition when a culvert is running as an open channel the friction losses can be determined by using Manning's equation. The Manning's coefficient of the culvert is assumed to be a function of culvert width, bed and side roughness, culvert water depth, and the culvert Froude number, i.e:

$$n = f(n_s, Fr, Y, B, (Ks)_b, (Ks)_s) \qquad (4)$$

The function of equation (4) will be determined from experiments.

ii- Closed conduit flow

The friction head loss in closed conduits can be determined by using Darcy-Weisbach equation:

$$h_f = f\frac{L}{R}\frac{v^2}{8g} \qquad (5)$$

Where L is the conduit length, R is the conduit hydraulic radius, V is the mean velocity and f is the friction factor. Nikuradse studied the effect of the roughness coefficient on the flow in pipes and found that for Laminar flow the friction factor f = 64/Re. For turbulent flow with smooth boundary, the friction factor is independent of roughness height "Ks" and can be obtained from the following equation given by Prandtl:

$$\frac{1}{\sqrt{f}} = 2\,\mathrm{Log}(Re\sqrt{f}) - 0.8 \qquad (6)$$

For fully turbulent flow, the roughness coefficient "f" is independent of Reynolds number and is expressed by Von Karmen as follows:

$$\frac{1}{\sqrt{f}} = 2\log\left[\frac{d}{Ks}\right] + 1.14 \qquad (7)$$

Colebrook and White combined equations (6) and (7) to produce an equation covering both smooth and rough boundaries as well as transition zone. Colebrook and White equation is expressed as follows:

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{Ks}{3.71\,d} + \frac{2.51}{Re\sqrt{f}}\right] \qquad (8)$$

Stephenson [5] stated that C - W equation yields satisfactory results for various commercially pipes.

The roughness height in all equations is called the equivalent roughness Ks. The friction head loss for box culvert running full and having homogeneous roughness can be calculated using equations 5, 6, 7 and 8. The pipe diameter "d" in equation (8) is assumed to be equal four times the hydraulic radius "R".

Deposition occur only on the bottom of culvert. It is assumed that the roughness height of the top of box culvert equal to the average roughness height of the side. Then equivalent roughness height of the whole box culvert can be expressed as follows

$$(Ks)_{th} = \frac{2H(Ks)_s + B(Ks)_b + B(Ks)_s}{2B + 2H} \qquad (9)$$

The friction factor "f" is then calculated using Colebrook and White equation. The value of "Ks" determined from equation (9) will be examined from experiments.

Friction head loss in closed conduits can also be calculated by applying the Hazen-Williams equation, which in S.I. units is:

$$h_f = 10.77\frac{L}{(4R)^{4.865}}\left(\frac{Q}{C}\right)^{1.852} \qquad (10)$$

Where C is Hazen-Williams coefficient which depends only on culvert roughness. In the condition where bed roughness differs than side roughness, Hazen-Williams Coefficient is assumed herein to be a function of the relative roughness $(Ks)_b/H$

iii- Orifice culvert

In the case when the culvert entrance is submerged and the culvert outlet is not submerged, the culvert acts as an orifice. The coefficient of discharge Cd depends on the entrance shape, the length of culvert in which the flow is running full, and the roughness coefficient of the culvert. For box culverts in which the bed roughness differs from the side roughness, the coefficient of discharge is assumed by the present study to be a function of bed roughness $(Ks)_b$, side roughness $(Ks)_s$, headwater depth "$Y_{U.S.}$", culvert height "H" and culvert Reynolds Number "Re".

$$Cd = \phi((Ks)_b, (Ks)_s, Y_{U.S.}, H, Re) \qquad (11)$$

The function in equation (11) will be determined experimentally.

EXPERIMENTAL SETUP AND PROCEDURE

For the purpose of determining the function of equations (4) and (11) and to examine the value of the theoretical average roughness determined from equation (9), a set of experiments is conducted in the hydraulic laboratory, Faculty of Engineering, Alexandria University. A plexiglass square culvert of 12.5x12.5 cm cross section and 3.00 meters long is constructed specially for these experiments. Thirty piezometer tubes are mounted to the culvert to determine the pressure distribution along the culvert. The culvert is installed in a testing flume 10 meters long and 50 cm wide. A tail gate is located at the exit of the testing flume to adjust the tailwater depth. The culvert bed roughness is changed by covering and fixing uniform sediment mixture over a thin steel plate 12.5x300 cm. This plate is installed on the culvert bed. The test runs have been undertaken with smooth walls and five different bed roughness. The mean diameter of each mixture is 0.78, 1.015, 1.77, 3.555 and 11.1125 mm. For each bed roughness, six different discharges are allowed to flow 6.279, 9.175, 10.411, 12.342, 16.811 and 20.444 liters/sec. All discharges except the smallest one are allowed to pass through the culvert under two conditions: i) with submerged entrance and exit ii) with submerged entrance and unsubmerged exit. The culvert entrance and exit are not submerged when the discharge is 6.279 L/sec.

ANALYSIS OF RESULTS

When the headwater level is less than the culvert entrance, the culvert acts like a contraction in an open channel. The water level drops at the culvert entrance and rise again after a certain distance. The water surface profile in the culvert shows undulations (Figure 4-a).

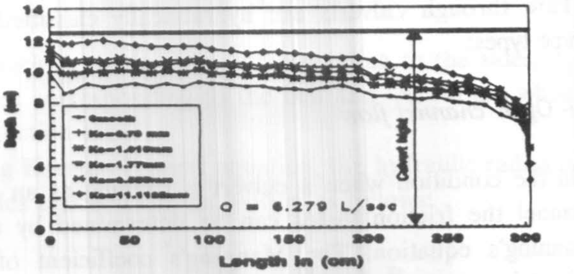

Figure (4-a), open channel contraction culvert

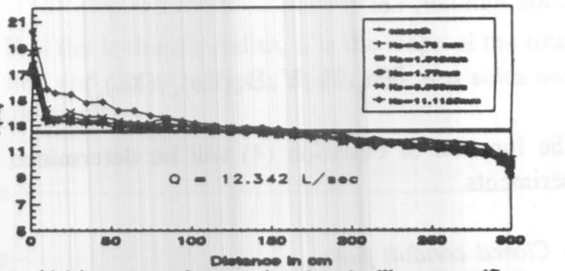

Figure (4-b), case when culvert acts like an orifice

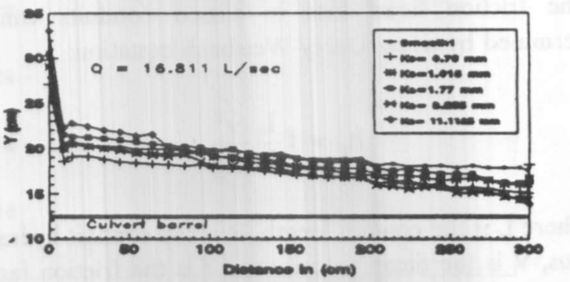

Figure (4-c), closed conduit culvert

Figure 4. variation of H.G.L. along culvert.

Under the same flow conditions, the depth of flow in culvert increases with the increase of bed roughness. The friction head loss in the middle portion of the culvert is computed by determining the slope of the total energy line using the least square method. The relationship between bed roughness and the energy slope is shown in Figure (5). In order to determine the best function in equation (4), a comprehensive multiple regression analysis is carried

out using the experimental data.
The obtained best fit equation is:-

$$\frac{n}{n_s} = 0.7864\left(\frac{Y}{(Ks)_b}\right)^{0.03086}\left(\frac{B}{Y}\right)^{0.253773}\left(\frac{(Ks)_b}{(Ks)_s}\right)^{0.182462} Fr^{0.06149} \qquad (12)$$

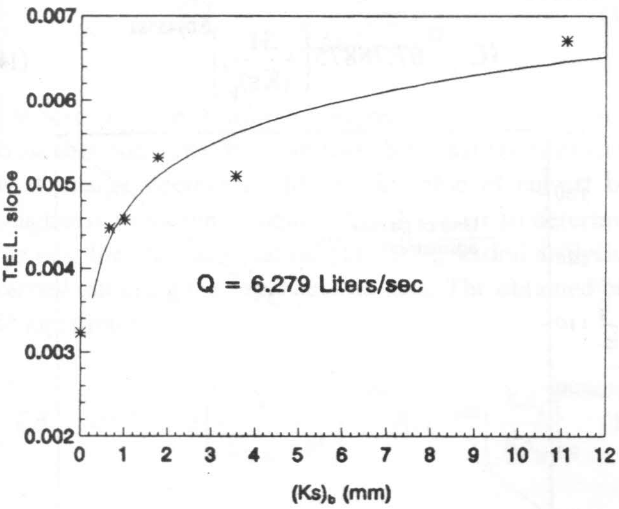

Figure 5. variation of energy slope with bed roughness for open channel culvert.

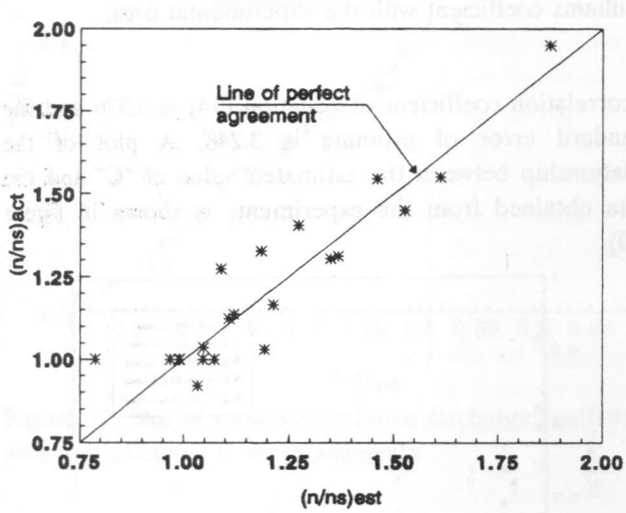

Figure 6. relationship between the estimated relative roughness coefficient and the experimental data.

The correlation coefficient of equation (12) is 0.935 and the standard error of estimate is 0.08269. Figure (6) shows a plot of the estimated relative Manning's coefficient "n/n_s" with the values obtained from experimental data.

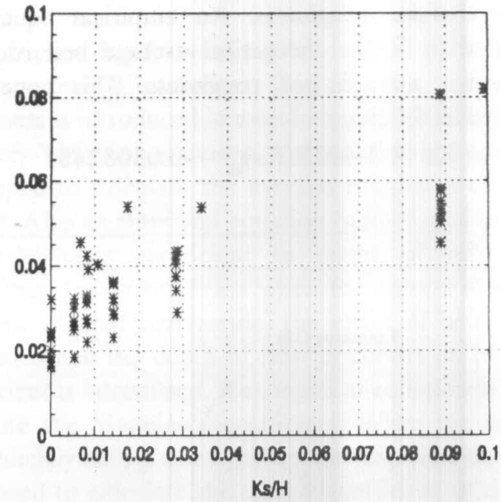

Figure 7. variation of relative bed roughness with friction coefficient.

In the case of submerged entrance and exit, and the headwater depth is greater than 1.5H, the culvert is found to be running full. However, a contraction zone appears at the culvert entrance where the water surface separates from the culvert soffit. The water level rises after a certain distance then the culvert is running full. It is noticed from experiments that the separation distance decreases with the increase of headwater depth. The variation of piezometric head along the culvert with different bed roughness is shown in fig (4-c). The energy loss along the culvert is calculated. The slope of the total energy line in the middle portion of the culvert is computed and is considered to be equal to the friction head loss per meter run along the culvert. For each bed roughness, five runs are performed considering different flow rates. The value of the friction factor "f" for different bed roughness is computed using Darcy-Weisbach equation. The Colebrook and White equation is applied to determine the roughness height for each run, and a mean value for the culvert roughness is obtained "Ks". Figure (7) shows the variation of the relative bed roughness "Ks_b/H" with the friction coefficient "f". Similarly, the average absolute roughness "$(Ks)_s$" is determined for the case when the culvert barrels are smooth (i.e. no artificial material are fixed on the bed, $Ks = (Ks)_b = (Ks)_s$). The theoretical average roughness "$(Ks)_{th}$" is calculated from equation (9) and is compared with the obtained experimental equivalent roughness "Ks". It is found that there is a discrepancy between the average roughness computed from equation (9) and the average roughness obtained from the experiments data. This discrepancy come from the assumption that the bed roughness coefficient $(Ks)_b$ is equal to the mean diameter of the artificial roughness. An empirical equation is developed to fit the theoretical average bed roughness with actual average bed roughness. This equation is expressed as follows:

$$Ks = 1.4732\,(Ks)_{th} + 0.03082481 \qquad (13)$$

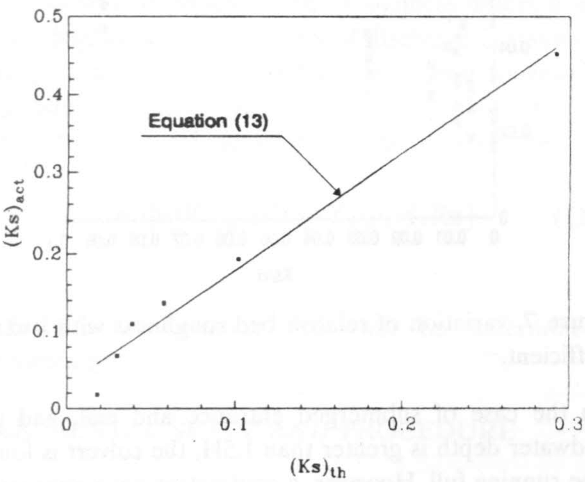

Figure 8. relationship between estimated roughness coefficient and experimental data.

In which $(Ks)_{th}$ is the theoretical roughness coefficient determined from equation (9) and the units of equation (13) is in millimeters. The correlation coefficient of equation (13) is 0.98. The relationship between the estimated average bed roughness and the experimental data is shown in figure (8).

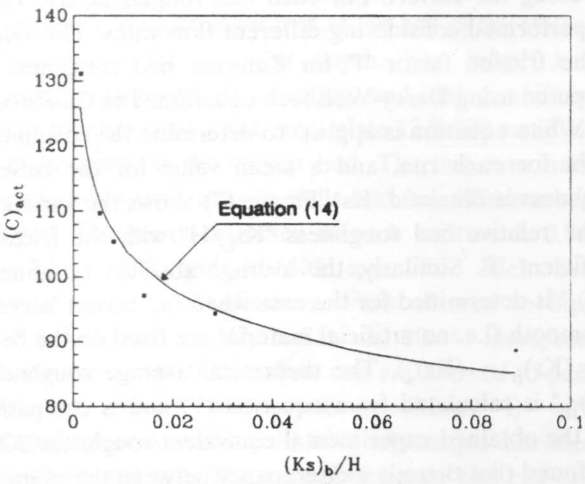

Figure 9. relationship between relative bed roughness with Hazen-Williams coefficient.

The Hazen-Williams coefficient "C" is also computed in the condition of closed conduit culvert flow. The experiments shows that "C" decreases with the increase of relative bed roughness "Ks_b/H" as shown in figure (9). An empirical equation is developed to determine Hazen Williams coefficient in term of Ks_b/H which is expressed as follows

$$C = 67.78875\left(\frac{H}{(Ks)_b}\right)^{0.0945581} \qquad (14)$$

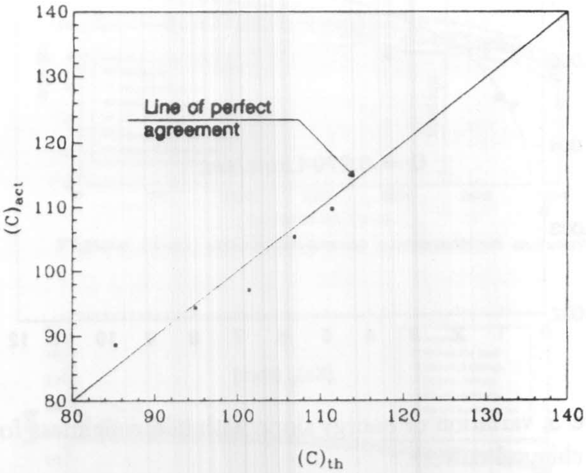

Figure 10. variation of estimated values Hazen-Williams coefficient with the experimental data.

correlation coefficient of equation (14) is 0.976 and the standard error of estimate is 3.248. A plot of the relationship between the estimated value of "C" and the data obtained from the experiments is shown in figure (10).

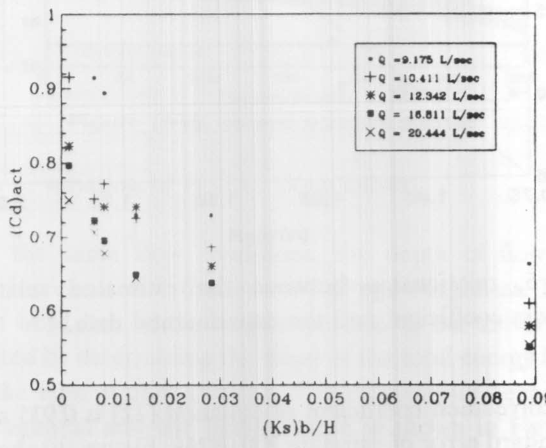

Figure 11. variation of coefficient of discharge with bed roughness.

In the case when the flow through culvert is running with submerged entrance and unsubmerged exit, the coefficient of discharge is calculated for different bed roughness. The culvert coefficient of discharge is defined here as follows:

$$Cd = \frac{Q}{A\sqrt{2g(Y_{U.S.} - H)}} \tag{15}$$

Where $Y_{U.S.}$ is headwater depth and A is the culvert cross sectional area. Experiments show that the coefficient of discharge decreases with the increase of culvert bed roughness as shown in figure (11). In order to determine the best function of equation (11), a regression analysis is carried out using the experimental data. The obtained best fit equation is:

$$Cd = 0.003098\left(\frac{Y_{U.S}}{Y_{U.S.} - H}\right)^{0.444} Re^{0.4365}\left(\frac{Ks_s}{Ks_b}\right)^{0.04068} \tag{16}$$

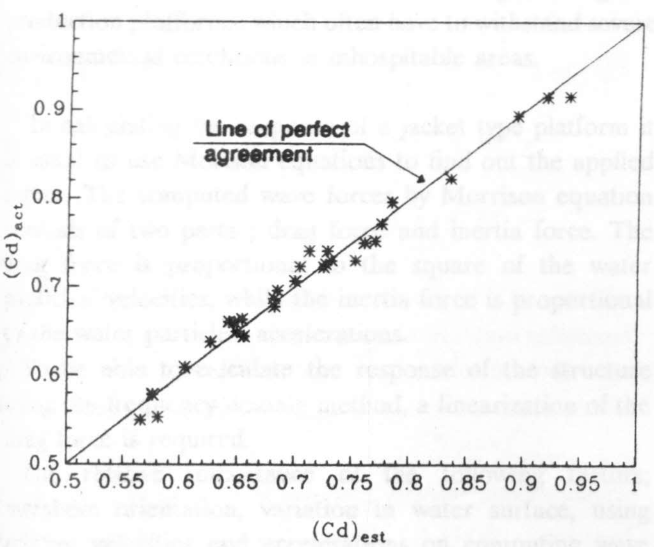

Figure 12. plot of estimated relative discharge coefficient with that obtained from experiments.

The correlation coefficient of equation (16) is 0.993 and the standard error of estimate is 0.01347. Figure (12) shows a plot of relationship between the estimate relative coefficient of discharge with the experimental data. All equations given by the present study are valid for the range of experimental data.

CONCLUSION

The friction head loss along box culvert is theoretically and experimentally studied. The effect of bed and side roughness is introduced in calculating the friction factor "f" in Darcy-Weisbach equation. A semi-empirical equation is developed to calculate the average roughness of the box culvert. Also an empirical equation is developed to express Hazen-Williams coefficient in terms of relative bed roughness. In the condition when the culvert is acting as an open channel contraction, the effect of bed and side roughness and the depth of flow in culvert on Manning's coefficient is introduced. An empirical equation is given to compute the Manning's coefficient. When the culvert is hydraulically acting as an orifice, an empirical equation is developed to calculate the culvert coefficient of discharge in terms of bed and side roughness.

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