FORMATION OF SCOUR HOLE DOWNSTREAM BOX CULVERTS

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ABSTRACT

Scour of nonuniform sand mixture downstream box culverts is theoretically and experimentally studied. The effect of the geometric standard deviation of the sand mixture and its mean diameter and other flow parameters on the dimensions of the scour hole are introduced. A semi empirical and empirical equations are developed to determine the maximum depth of the scour hole, its length and the scour depth just downstream culvert.

NOMENCLATURE

A_p	wetted area between section I-I and Section II-II
C_d	jet diffusion coefficient
d	scour depth just downstream culvert
d_s	maximum scour depth
d_{50}	mean diameter of bed mixture
F	momentum force
Fr	culvert Froude number
F_*	unit force
g	gravitational acceleration
H	difference of elevation between pipe outlet and bed level
H_w	height of control weir
L_s	length of scour hole
P	hydrostatic force
Q	rate of flow
Re_*	grain Reynolds number
SD	geometric standard deviation of bed mixture
t	time in seconds
$Y_{U.S}$	water depth upstream culvert
$Y_{D.S}$	tailwater depth
v_1	flow velocity in culvert
v_2	flow velocity in flume
β'	jet angle near surface
γ	specific weight of water
γ_s	specific weight of sediments
τ_{av}	average tractive force
ω	grain fall velocity

INTRODUCTION

Local scour is defined as the removal of material composing the boundary through the action of water in motion. This phenomena may occur downstream culverts if the culvert water velocity is great enough in comparison to channel water velocity to form a submerged jet. The study of scour hole dimensions is important since culverts can be undermined leading to structural failure if not properly protected.

The scour due to two dimensional and three dimensional impinging on an erodible bed has been studied by several investigators for the case of submerged jump or for the local scour downstream spillways.

Rouse [3] conducted experimental studies on scour due to two dimensional jet. He expressed the depth of scour as a function of time "t", mean velocity "v" and particle fall velocity "ω" and the geometric standard deviation "SD", i.e.

$$\frac{d_s}{a} = f_1\left(\frac{\omega t}{a}, \frac{v}{\omega}, SD\right) \tag{1}$$

In which "a" is a characteristics length of boundary geometry.

Smith(4) investigated experimentally the scour below a cantilever pipe outlet. He analyzed the data on the basis of the relationship:

$$\frac{h_*}{H} = f_2\left(\frac{E_o}{\rho\,\omega^3 H^2}, \frac{Y}{H}, \frac{\omega t}{H}\right) \tag{2}$$

Where h_* is the cube root of volume of scour hole, H the difference of elevation between pipe outlet and bed level and E_o is the energy flux ($Q\rho v^2$) of the jet water at the point of impingement on the tail water surface.

Hassan and Narayanan(2) studied experimentally scour downstream rigid apron due to jet of water issuing through a sluice opening. They developed a semi empirical theory to predict the time rate of scour hole. They concluded that the parameter for dynamic similarity for local scour is the Froude number.

Bormann and Julien [1] investigated theoretically and experimentally local scour downstream of grade control structures. They conducted their experiments using large scale physical model with unit discharge up to 2.5 m^3/sec/m and scour depths exceeding 1.4 meter. They developed a semi empirical equation to predict the equilibrium scour depth.

$$d_s = \left[\left(\frac{\gamma \sin\phi}{\sin(\phi+\alpha) B (\gamma_s - \gamma) g}\right)^{0.8} \frac{C_d^2 Y_o^{0.6} v_1^{1.6}}{d_s^{0.4}} \sin\beta'\right] - H \tag{3}$$

Where B is the coefficient of friction ratio, C_d jet diffusion coefficient, Y_o jet thickness, Φ submerged angle of repose of bed sediment and β' jet angle near surface.

Steven and Ruff [5] studied experimentally the effects of culvert slope on the scour hole characteristics. They found out that a sloped culvert can increase the maximum dimensions of scour from 10 to 40% over the scour dimensions for a horizontal culvert.

Abt, Ruff, Doehring and Donnell [6] investigated experimentally the effect of the square, arch and rectangular culvert shapes on the dimensions of scour hole such as depth, width and volume. They developed empirical equations to express the scour hole characteristics. These equations are valid for uniform mixtures.

The aim of the present study is to investigate the effect of sand gradation; as well as other flow parameters, on the dimensions of the scour hole downstream box culverts.

THEORETICAL ANALYSIS

Scour downstream culverts is assumed to be affected by hydraulic and sediment parameters. The hydraulic parameters which influence the formation of scour hole are the velocity through culvert, the tailwater depth, the velocity in the downstream channel and the cross sectional dimensions of the culvert. The sediment characteristics are the mean diameter, the geometric standard deviation of the bed mixture and the submerged unit weight of the grains. The velocity in culvert "v_1" is normally greater than the velocity in channel "v_2". This change in water velocity will create a momentum force which will eventually increase the boundary tractive force acting on the sediment bed causing the formation of a scour hole downstream the culvert.

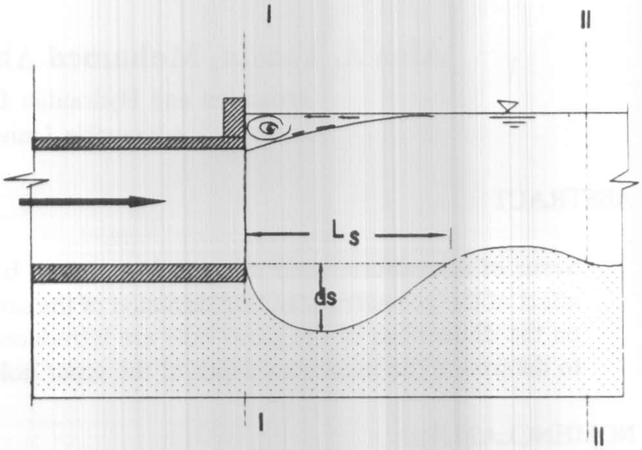

Figure 1. definition sketch for scour downstream culvert

Applying the momentum equation between section I-I and section II-II (Figure 1) the following is obtained:

$$\frac{\gamma}{g} Q(v_2 - v_1) = P_1 - P_2 - \tau_{av} A_p \tag{4-a}$$

$$\frac{\gamma}{g} Q(v_1 - v_2) - (P_2 - P_1) = \tau_{av} A_p \tag{4-b}$$

The left hand side of equation (4-b) is defined as a total force "F". At the threshold condition, F will equal its critical value F_{cr}. It is assumed that as the total force exceeds F_{cr}, the boundary tractive force increases causing erosion of bed mixture and the formation of a scour hole. As F is further increased, more sediments are removed and a larger scour hole is expected. In other words, it may be assumed that the submerged weight of the sediment mixture removed from the scour hole is some function of the total force F. According to the foregoing assumption, the weight of the eroded sediment can be expressed as follows:

$$F = \varphi(W) \tag{5}$$

In which W is the weight of eroded sediments and is given as

$$W = \varphi_1(L_s, d_s, (\gamma_s - \gamma), \sigma_g)$$

i.e. equation (5) can be written as follows

$$F = f_1(L_s, d_s, (\gamma_s - \gamma), \sigma_g) \tag{6}$$

Applying the Backingham π-theory, equation (6) can be written in a dimensionless form as follows:

$$\frac{F}{(\gamma_s - \gamma) d_{50}^3} = f_2\left(\frac{L_s}{d_{50}}, \frac{d_s}{d_{50}}, \sigma_g\right) \qquad (7)$$

The left hand side of equation (7) is defined as a unit force F_*. In order to define the function in equation (7), experimental study are required.

EXPERIMENTAL SETUP AND PROCEDURE

For the purpose of observing the effect of hydraulic and sediment parameters on the geometry of the scour hole downstream box culverts, a set of experiments is performed in the hydraulic laboratory, faculty of engineering, Alexandria University. In all experimental runs, a wooden square culvert of 10x10 cm cross section and 2.5 meters long is installed in a testing flume 50 cm wide and 10 meters long. The flume bed is filled with nonuniform sand mixture to the culvert invert level (25 cm depth). Figure (2) shows a schematic diagram of the experimental setup.

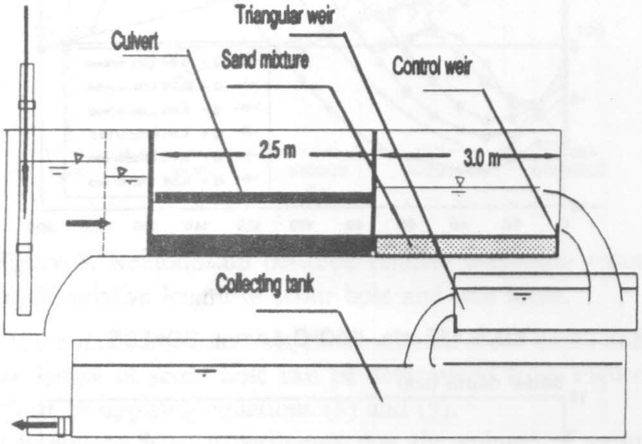

Figure 2. Schematic diagram of the experimental set-up.

The test runs have been undertaken with two erodible sand mixtures, the first sand mixture have mean diameter (d_{50}) equal 1.00 mm and geometric standard deviation "SD" equal 3.27 while, d_{50} of the second sand mixture is 0.44 mm and SD = 1.95. A wooden weir is installed at the downstream end of the flume to control tail water depth. For each type of sand mixture, experimental runs have been conducted considering three different weir heights and various discharges are allowed to pass, the details of which are shown in Table (I).

For each run the rate of flow is gradually increased to the required discharge in a way that no local scour take place during this process. Water is then allowed to recirculate for a sufficient period of time, till equilibrium stage is practically attained in which the scour hole reached its equilibrium stage and practically no further sediments are removed from it. At this stage, the flow is gradually reduced, the flume is drained and the scour depth along the flume centerline is measured. Water depths upstream and downstream the culvert, downstream depth in the undisturbed flow zone are also measured.

ANALYSIS OF RESULTS

All experiments show that a submerged jet is formed just downstream the culvert and dissipated after a certain distance. The length of dissipation depends on the velocity in culvert and in flume. An anticlockwise vortex is formed just downstream the culvert, which is responsible of scouring bed mixture in the vicinity of the culvert.

The volume of the scour hole increases with time till it reaches its final equilibrium form. The equilibrium scour hole is build in a way so that the influence of jet velocity on the bed is disappeared. A sample of the contour lines of a scour hole is shown in Figure (3), while variation of scour depth along centerline of the flume is shown in Figure (4).

All experimental runs have revealed that the scour hole has always a specific shape as shown in Figure (4). The scour starts immediately downstream the culvert in which the scour depth is denoted as "d". Moving farther in the downstream direction, the scour depth is gradually increased until its maximum value "d_s" is reached. Afterwards the scour depth is gradually reduced and the scour hole is terminated at some distance "L_s" from the culvert outlet. The maximum scour depth "d_s" is located at a distance equal to 35→40% of the scour hole length measured downstream the culvert outlet. When the tailwater depth is less than the culvert height, the downstream slope of the scour hole is larger than the upstream slope. However, both slopes are almost of the same order when the culvert is running under outlet control. Downstream the scour hole sediments are accumulated.

The scour hole can be described by the maximum scour depth and the length of scour hole. Using the available data, the following empirical equation is developed to express the relationship between the maximum scour depth and the length of scour hole:

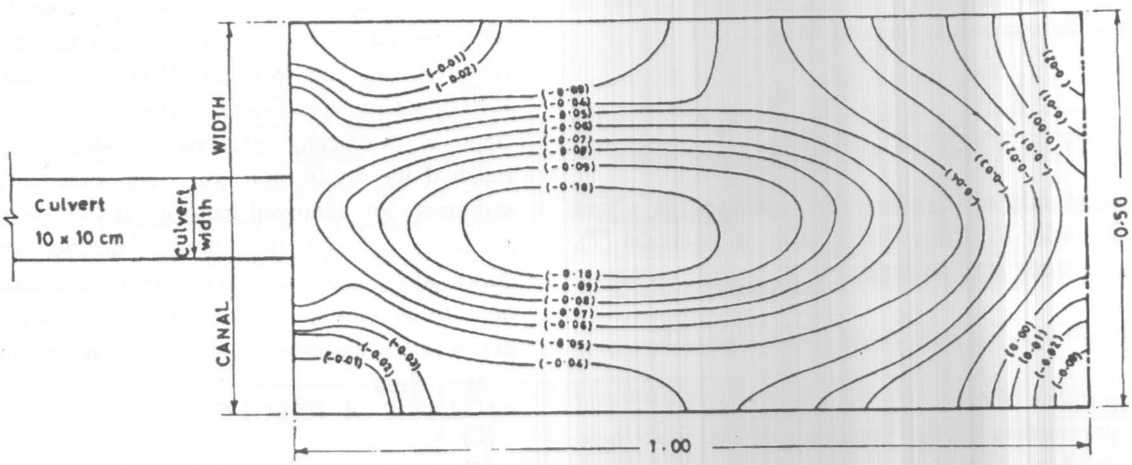

Figure 3. Contour line of scour hole for run No. 29.

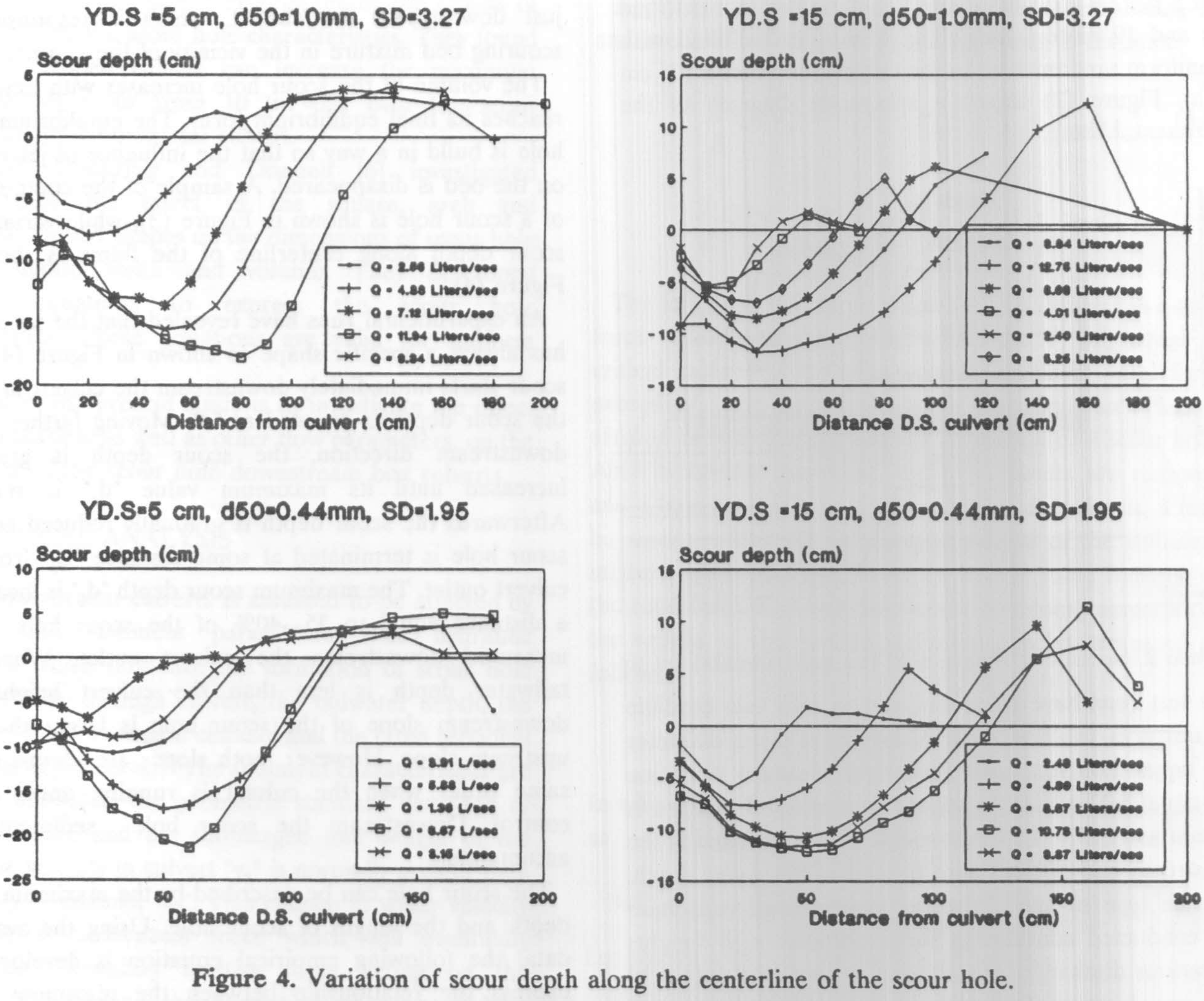

Figure 4. Variation of scour depth along the centerline of the scour hole.

$$d_s = 0.46723 L_s^{0.6866} \qquad (8)$$

The correlation coefficient of equation (8) is 0.895.
Furthermore, an empirical equation is developed to describe the relationship between "F.", L/d_{50} and d_s/d_{50}. This equation is expressed as follows:

$$\frac{L_s}{d_{50}} = 12.9236 F_*^{0.3525} \left[\frac{d_s}{d_{50}}\right]^{0.0019} \qquad (9)$$

The correlation coefficient of equation (8) is 0.93. Equation (9) shows that the final relative length and relative maximum depth of scour hole increases with the increase of unit force Figure (5).

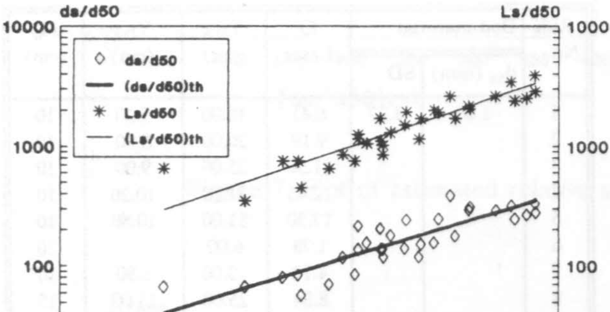

Figure 5. Relationship between relative maximum scour depth, relative length of scour hole and unit force.

If the unit force is known, the maximum scour depth and the length of scour hole can be determined from Figure (5) or by applying equations (8) and (9).
It is noticed from experiments that the volume of scour hole increases with the increase of the culvert Froude number as shown in Figure (6), which illustrates the relationship between relative scour depth "d_s/d_{50}" and the culvert Froude number.

It is obvious from Figure (6) that the depth of scour depends not only on culvert Froude number but also on the geometric standard deviation and the mean diameter of the bed mixture.

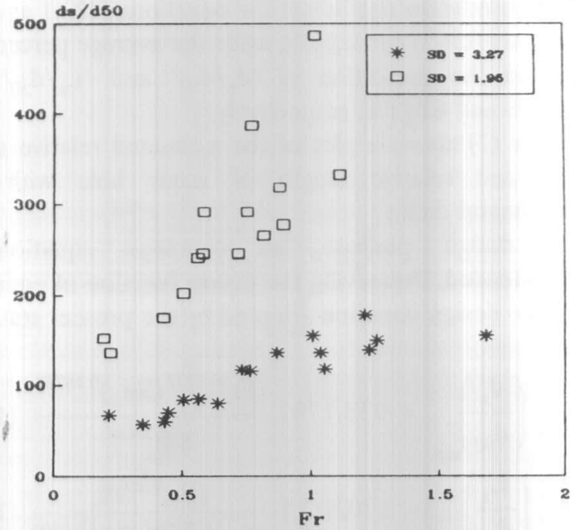

Figure 6. variation of relative scour depth with culvert Froude number.

It is also noticed that the relative scour depth increases with the increase of the grain Reynolds number Re. which is defined as

$$Re_* = \frac{V_2 d_{50}}{\nu}$$

Based on the forgoing findings, it may be assumed that the dimensions of the scour hole which is expressed by L_s and d_s, are a function of geometric standard deviation, mean diameter of bed mixture, culvert Froude number and grain Reynolds number, i.e.:

$$\frac{d_s}{d_{50}} = f_3(Re_*, Fr, SD)$$
$$\frac{L_s}{d_{50}} = f_4(Re_*, Fr, SD) \qquad (11)$$

In order to determine the best function of equation (11), a comprehensive multiple regression analysis is carried out using the experimental data. The obtained best fit equations are:

$$\left(\frac{d_s}{d_{50}}\right)_{est} = 546.03182 \frac{(Fr)^{0.2977}(Re_*)^{0.26758}}{SD^{2.436}} \qquad (12\text{-a})$$

$$\left(\frac{L_s}{d_{50}}\right)_{est} = 14502.28706 \frac{(Fr)^{0.75646}}{(Re_*)^{0.01896}(SD)^{2.0709}} \qquad (12\text{-b})$$

The correlation coefficient for equations (12-a) and (12-b) are 0.9675 and 0.9678, while the average percentage error in the prediction of "d_s/d_{50}" and "L_s/d_{50}" are 10.052% and 8.883%, respectively.

Figure (7) shows a plot of the estimated relative scour depth and relative length of scour hole with the experimental data.

For design purpose, an envelope equation is recommended. Preserving the power function of equation (12), the design equation adopted by the present study is:

$$\left(\frac{d_s}{d_{50}}\right)_{env} = 933.1781\,\frac{(Fr)^{0.2977}(Re_*)^{0.26758}}{SD^{2.436}} \quad (13\text{-a})$$

$$\left(\frac{L_s}{d_{50}}\right)_{env} = 17020.15\,\frac{(Fr)^{0.75646}}{(Re_*)^{0.01896}(SD)^{2.0709}} \quad (13\text{-b})$$

Three extreme experimental data points are excluded in the formation of equation (13). A comparison of equation (12) and (13) with experimental data are shown in Figure (8).

Equation (12) and (13) show that the scour volume or scour hole dimensions decreases with the increase of the geometric standard deviation and increases with the increase of culvert Froude number. Meanwhile, the maximum scour depth increases with the increase of grain Reynolds number. On the other hand, the length of scour hole decreases with the increase of grain Reynolds number. However, the effect of Re* is very slight in the latter case and the relative scour hole length is primarily affected by "Fr" and "SD".

It is observed from experiments and the results shown in Figure (4), that a scour depth formed just downstream the culvert apron "d" is a function of the volume of the scour hole. An empirical equation is developed to predict the scour depth "d" in terms of the maximum scour depth "d_s" as follows:

$$d = 0.72\,d_s^{0.92} \quad (14)$$

The correlation coefficient of equation (14) is 0.7564. All equations given by the present study are valid for the range of experimental data given in Table (1). However, to obtain more accurate design equations for "L_s", "d_s", and "d" more experiments are recommended to cover wider ranges of culvert size and shape, flow parameters and other bed mixtures of different mean diameter and geometric standard deviation.

PROTECTION AGAINST LOCAL SCOUR DOWNSTREAM CULVERT

The scour hole formed just downstream the culvert endanger its stability. Therefore, it is important to predict the dimensions of the scour hole to take precautions to avoid or minimize the influence of the scour hole on the stability of the culvert. It is recommended to protect the downstream bed by extending the length of culvert apron to a minimum distance equal to the length of scour hole. This distance can be determined from equation (13-b).

Table (1). Scheme of experiment runs.

Run No	Bed material		Q L/sec	$Y_{U.S}$ (cm)	$Y_{D.S}$ (cm)	H_w (cm)
	d_{50} (mm)	SD				
1	1.00	3.27	6.41	15.00	7.20	10
2			9.19	20.00	8.00	10
3			11.50	25.00	9.00	10
4			12.45	31.00	10.20	10
5			17.30	51.00	10.80	10
6			1.78	6.00		10
7			4.12	10.00	6.80	10
8			8.84	25.00	15.00	15
9			12.75	35.00	15.00	15
10			6.69	20.00	14.40	15
11			4.01	15.00	13.60	15
12			2.40	13.80	12.80	15
13			5.25	18.00	14.20	15
14			2.56	7.00	3.60	5
15			4.33	10.00	4.70	5
16			7.12	16.00	6.90	5
17			9.34	20.00	8.80	5
18			11.94	26.80	10.00	5
19	0.44	1.95	2.48	13.50	12.70	15
20			4.89	17.00	13.50	15
21			8.67	24.50	15.00	15
22			10.78	30.50	15.00	15
23			9.87	28.50	15.00	15
24			3.91	9.50	5.00	5
25			5.88	13.00	6.00	5
26			1.36	5.20	5.00	5
27			8.67	18.20	7.50	5
28			3.15	8.30	4.00	5
29			4.44	10.30	6.00	10
30			7.42	15.50	10.00	10
31			8.67	18.50	10.00	10
32			10.97	23.00	10.00	10
33			5.75	13.50	10.00	10.0

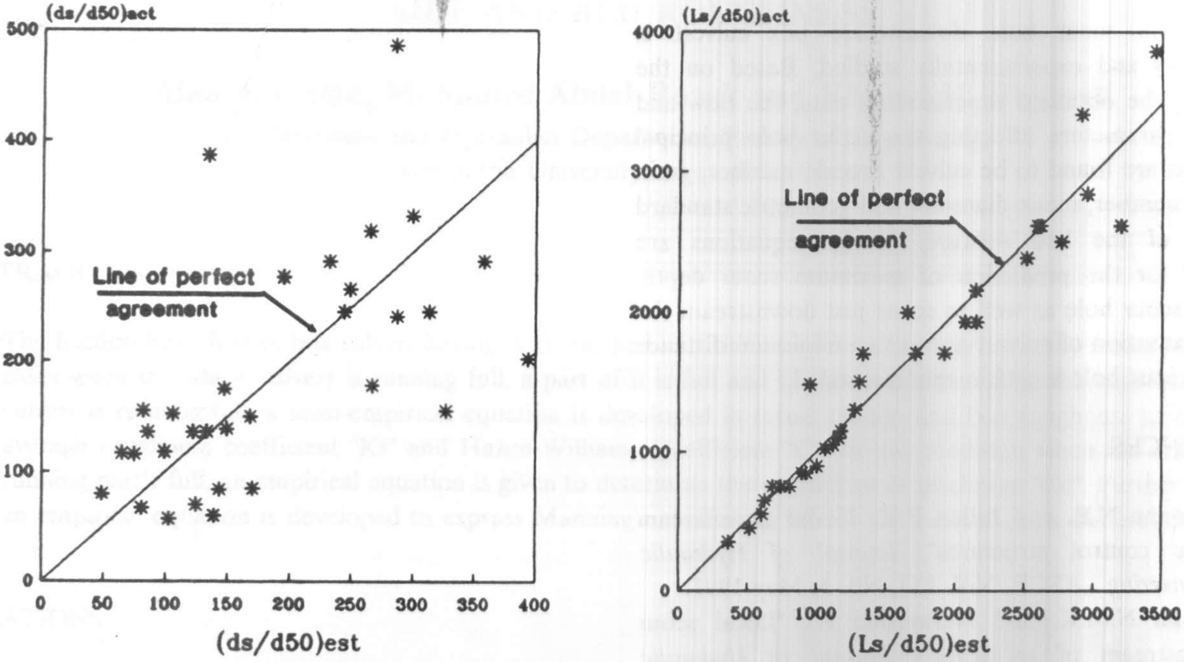

Figure 7. Plot of estimated relative scour depth & length with experimental data.

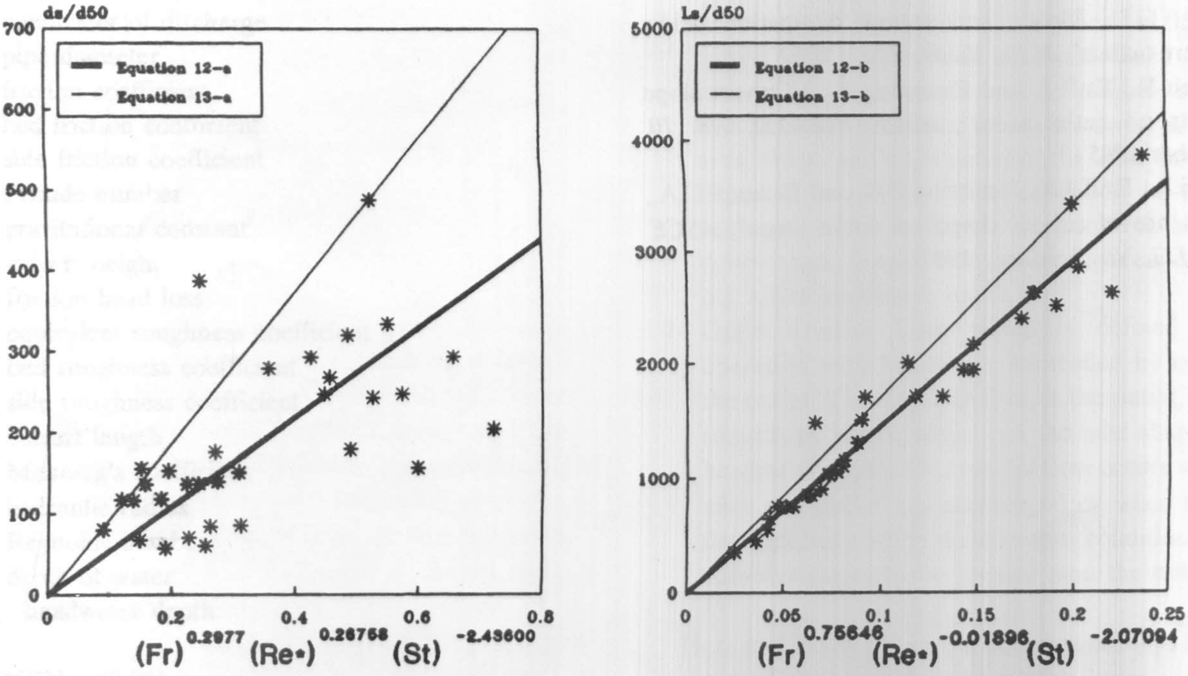

Figure 8. Comparison between estimate and experimental data.

CONCLUSION

Equilibrium scour hole downstream box culvert is theoretically and experimentally studied. Based on the analysis of the obtained experimental data, the flow and sediment parameters affecting the scour hole principal dimensions are found to be culvert Froude number, grain Reynolds number, mean diameter and geometric standard deviation of the bed mixture. Design equations are presented for the prediction of maximum scour depth, length of scour hole as well as scour just downstream the culvert. Extension of culvert apron to a minimum distance equal to scour hole length is recommended.

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