A STUDY OF PUMPING FROM A PARTIALLY PENETRATING WELL IN AN ANISOTROPIC UNCONFINED AQUIFER

Hosam El-Din Mohamed Moghazi
Irrigation and Hydraulics Department, Faculty of Engineering,
Alexandria University, Alexandria, Egypt.

ABSTRACT

A finite element model has been designed to demonstrate the effect of partial penetrating of a water well in an anisotropic unconfined aquifer on the characteristics of steady state groundwater flow in the neighborhood of the well. This includes the determination of the rate of pumping and the corresponding seepage face developed at the well for a wide range of drawdown in the well. Particular attention is paid to the rate of flow entering the well along the seepage face. The results are non-dimensionally presented in chart forms.

NOTATIONS

$\{F\}$	global nodal force vector
h_s	seepage face height measured from the impermeable bed
h_w	water depth in fully penetrating well
H_o	saturated thickness of unconfined aquifer
k	permeability coefficient of isotropic soil
k_r	permeability coefficient in the radial direction
k_z	permeability coefficient in the vertical direction
$[k]$	global stiffness matrix
P	penetration depth of well in unconfined aquifer
Q	yield from fully penetrating well
Q_p	yield from partially penetrating well
Q_{seep}	flow enters well along the seepage face
r	radial distance from well axis
r_w	well radius
R_o	well radius of influence
s	distance from well bottom to impermeable bed
s_s	water depth outside well measured from the original water table
s_w	drawdown of water in well
t	depth of water in partially penetrating well
ϕ	potential function or total head
$\{\phi\}$	global vector of unknown head

INTRODUCTION

Producing wells frequently do not completely penetrate the aquifer from which they are pumping. In such cases the flow lines towards a well bottom are no longer horizontal but have a substantial vertical velocity component (Fig. 1). The curvatures induced near a well reduce the discharge for the same drawdown or increase the drawdown for the same discharge as compared to a fully penetrating well. Methods have been developed by various investigators to correct for the yield from a partially penetrating well in an isotropic unconfined aquifer such as Forchheimer [2], Kozeny [5] and Boreli [2]. Although most geologic mechanisms forming soils tend to make deposits in layers, problems of anisotropy have received scant attention due to their mathematical difficulty. However, in highly stratified soils the error in analysis the flow towards wells can be so great as to render the analysis useless, unless judgmental adjustments are made. Dagan [4], Lakshminarayama [7] and Neuman [8] investigated unsteady flow towards a partially penetrating well in an anisotropic unconfined aquifer to determine its hydraulic properties. However, some of these studies neglected the seepage face developed at the well casing in their investigations. Bear [1], Brauns [3] and Zienkiewicz [12] showed the accuracy and the flexibility of the finite element method to handle the free surface groundwater flow problem in anisotropic soil.

The main objective of this research is to investigate the effect of anisotropy in permeability on the seepage face height developed at the well casing and the yield of a partially penetrating well in an unconfined aquifer using the finite element method. The study also includes an investigation of the effect of anisotropy on the quantity of flow entering the well along the seepage face. The results will be presented in non-dimensional chart forms suitable for practical use.

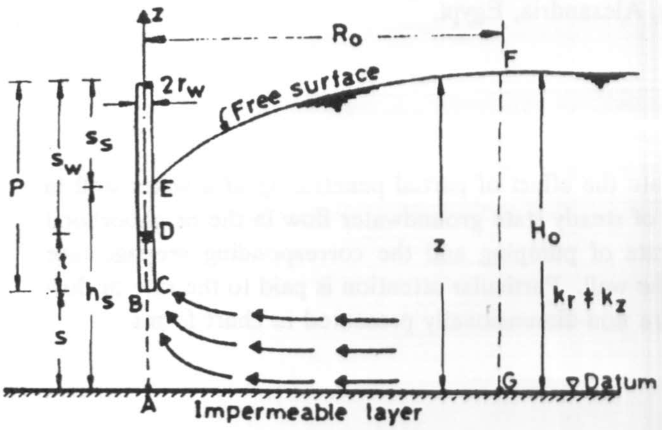

Figure 1. Geometry of the partially penetrating well-unconfined aquifer configuration.

ASSUMPTIONS

The idealized aquifer chosen for the study may be described as follows (1) The aquifer is composed of homogeneous and anisotropic unconfined aquifer and it is of sufficient areal extent so that the effects of boundaries can be neglected. (2) Steady state conditions are established over a large area around the well. (3) The soil is fully saturated and the compressibility of both water and soil are neglected. (4) The effect of capillary flow in the zone above the free surface is neglected.

THEORETICAL CONSIDERATIONS

The general governing partial differential equation for steady state flow in an anisotropic and homogeneous porous continuum can be described as

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial \phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial \phi}{\partial z}\right) = 0 \quad (1)$$

where ϕ is the potential head and k_x, k_y and k_z are the permeability coefficients in the x, y and z coordinates respectively.

Since the domain and the flow are symmetrical about the well axis the cartesian coordinates in equation (1) are transformed to cylindrical coordinates (r, z):

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k_r\, r \frac{\partial \phi}{\partial r}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial \phi}{\partial z}\right) = 0 \quad (2)$$

where k_r and k_z are the permeability coefficients in the radial and vertical directions respectively and r is the radial distance from the well axis (see Figure (1)).

By applying the Galerkin [12] residual approach to equation (2) yields set of simultaneous equations.

$$[K]\,\{\phi\} = \{F\} \quad (3)$$

where [K] is the global stiffness matrix, $\{\phi\}$ is the global vector of unknown head to be determined and {F} is the global nodal force vector. From equation (3), the final solution can be obtained after applying the boundary conditions. More details about the finite element equations are available elsewhere such as Hinton [6] and are beyond the purpose of this research.

BOUNDARY CONDITIONS

Referring to the geometry of the model in Figure (1), the boundary conditions associated with the flow towards a partially penetrating well are as follows :

Water boundaries

These constitute the faces BCD and FG. The total potential head, Φ, along these faces equal to the elevation of the water face above the datum :

$$\Phi = t + s \text{ (along BCD)}, \quad \Phi = H_o \text{ (along FG)} \quad (4)$$

Phreatic surface

Along EF the total head equal the elevation above the datum and the flow across this boundary is nil :

$$\left.\begin{aligned} \phi &= z \\ \frac{\partial \phi}{\partial n} &= 0 \end{aligned}\right\} \text{(along EF)} \quad (5)$$

Seepage face

At which the pressure is atmospheric and the total head equal the elevation head :

$$\phi = z \quad \text{(along DE)} \tag{6}$$

Impervious boundaries

Along the surface AG, $\partial\Phi/\partial n = 0$. Also, along the boundary AB, and due to symmetry condition $\partial\Phi/\partial n = 0$.

MODEL DIMENSIONS

The permeability coefficient in the radial direction, k_r, was assumed equal to 0.001 m/sec and values of the anisotropy ratio, k_r/k_z, were chosen equal to 10, 3, 1 and 1/3 respectively, where k_v is the permeability coefficient in the vertical direction. The saturated thickness of the aquifer, H_o, and the well radius, r_w, were assumed equal to 50.0 and 0.15 m respectively. Four cases of well penetration ratio, P/H_o, are studied (P/H_o = 1, 0.8, 0.6 and 0.4) where P is the penetration depth of the well to the aquifer. Different values of the drawdown ratio, t/P are studied (t/P = 0.8, 0.6, 0.4, 0.2 and 0.0), where t is depth of water in the well Figure (1). There are various formulae have been derived to estimate the value of the well radius of influence, R_o, in an isotropic soil. The Sichart [10] empirical formula is the most common used formula in the dewatering purposes and is highly recommended by many authors (Power [9] and Somerville [10]) :

$$R_o = 3000\, s_w \sqrt{k} \tag{7}$$

where

s_w drawdown of water in well (m)
k permeability coefficient of an isotropic aquifer (m/sec)

In the Dupuit [11] equation dewatering volume varies inversely as a log function of R_o for a fully penetrating well in an unconfined aquifer

$$\frac{Q}{\pi k} = \frac{H_o^2 - h_w^2}{\ln\left(\frac{R_o}{r_w}\right)} \tag{8}$$

where

Q well yield from a fully penetrating well
h_w water depth in the well.

Thus errors in estimating R_o are not significant in the estimate of well yield. Therefore, equation (7) may be used to estimate the well radius of influence in an anisotropic soil by replacing the maximum value of k_r or k_z instead of k.

A typical finite element mesh used to investigate the problem, is shown in Figure (2). Since the flow lines gradients will be highest near and underneath the withdrawal well, the smallest elements are used at these locations and the mesh is allowed to coarsen as radial distance increases. The Taylor and Brown [11] iterative technique was used to determine the steady position of the free surface and consequently the corresponding well yield.

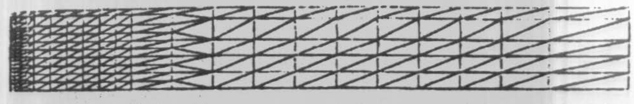

Figure 2. A finite element mesh for a partialy penetrating well No. of elements = 405 no of modes = 238.

RESULTS AND DISCUSSIONS

1. Seepage face

Figures (3) to (5) show the relationship between s_s/s_w, t/P and k_r/k_z for P/H_o = 0.8, 0.6 and 0.4 respectively, where s_s and s_w are the drawdowns outside and inside the well measured from the original water table respectively and t/P is the drawdown ratio. The seepage face height, h_s, then can be obtained by subtracting s_s from the saturated thickness of the aquifer, H_o. Generally speaking, it can notice from Figures (3) to (5) that the seepage face height is a function of the ratio k_r/k_z. If $k_r/k_z > 1$, the free surface is higher than for isotropic conditions and the greater k_r/k_z the higher the seepage face height. This can be explained by sketches as shown in Figure (6) where the greater k_r/k_z the flatter the free surface gradient and consequently the higher the seepage face at the well. Thus, the efficiency of partially penetrating well to drawdown the water table in unconfined aquifer decreases as the increase of the anisotropy ratio k_r/k_z. Referring to Figure (5) and for instance at k_r/k_z = 10 the maximum possible drawdown outside the well is about 0.40 - 0.42 the penetration depth of the well when the well is fully drained (t = 0.0), while for k_r/k_z = 1/3 the corresponding percentage of drawdown is about 0.55 - 0.60 at the same pumping conditions.

A comparison between the corresponding ratios of k_r/k_z in Figures (3), (4) and (5), at the same drawdown ratio, t/P, indicates that the penetration ratio, P/H_o, has an insignificant effect on the ratio s_s/s_w.

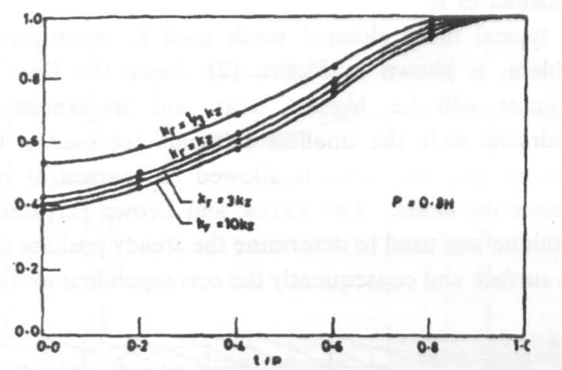

Figure 3. Relationship between the seepage face and the anisotropy ratio ($P/H_o=0.8$), ($H_o=50$m, $r_w=0.15$m).

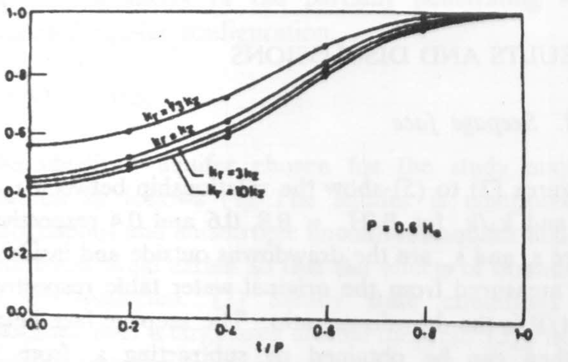

Figure 4. Relationship beteen the seepage face and the anisotropy ratio ($P/H_o=0.6$), ($h_o=50$m, $r_w=0.15$m).

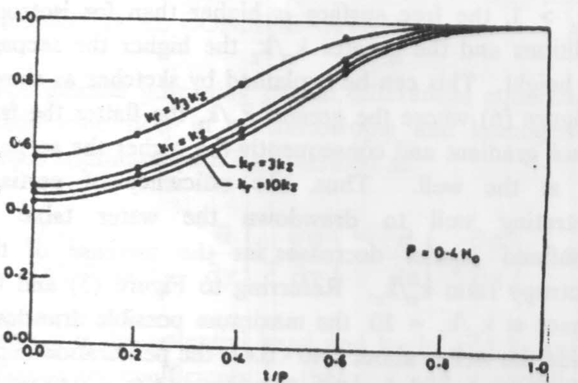

Figure 5. Relationship beteen the seepage face and the anisotropy ratio ($P/H_o=0.4$), ($h_o=50$m, $r_w=0.15$m).

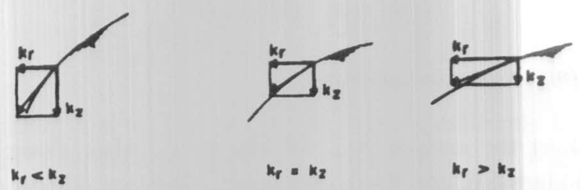

Figure 6. Variation of the surface gradient with k_r/k_z.

2. Well Yield

The relationships between $Q_p/k_{av} H_o^2$, t/P and k_r/k_z for P/H_o = 1, 0.8, 0.6 and 0.4 are shown in Figures (7) to (10) respectively, where Q_p is the yield of a partially penetrating well and k_{av} is the average permeability coefficient = $\sqrt{k_r k_z}$. It can notice that a partially penetrating well will discharge less than a completely penetrating well if they are operated at the same pumping level and Q_p decreases as the decrease of the penetration ratio P/H_o. Moreover, pumping at the same level, the yield of a partially penetrating well in anisotropic aquifer will increase with increasing k_r/k_z. This is attributed to the fact that close to a partially penetrating well the flow percolates upward vertically to the well bottom and thus more head is lost and if k_r/k_z increases the effect of this vertical upward movement will become more or less radial, except at the well bottom, and becomes purely radial when k_z completely vanishes.

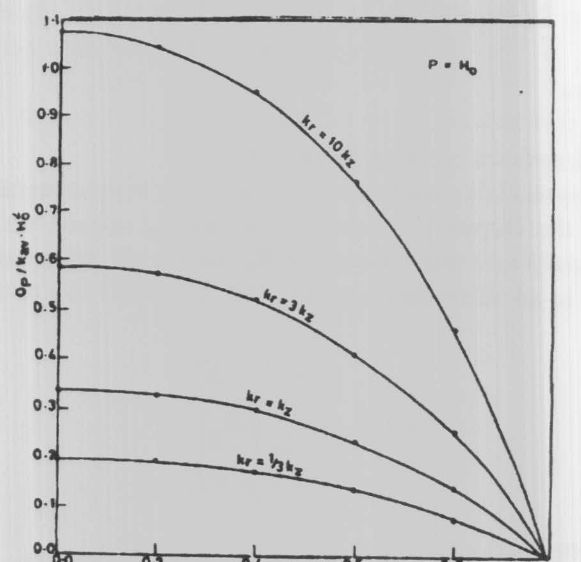

Figure 7. Relationship between the well yield and the anisotropy ratio (P/H_o), ($H_o=50$m, $r_w=015$m).

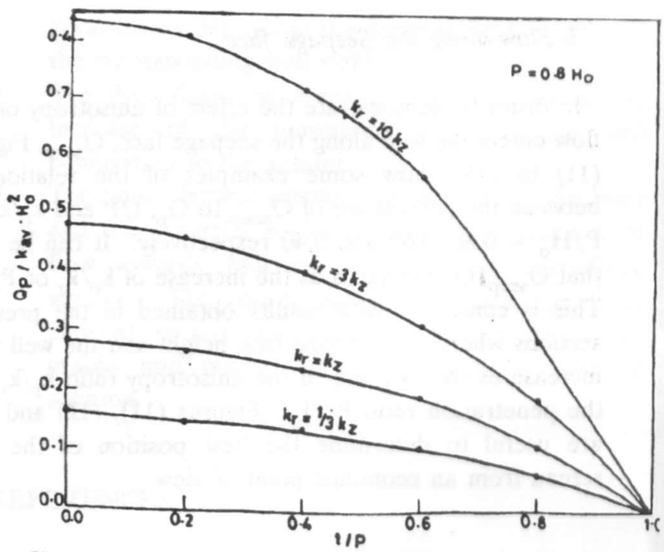

Figure 8. Relationship between well yield and the anisotropy ratio (P/ = 0.8 H_o).

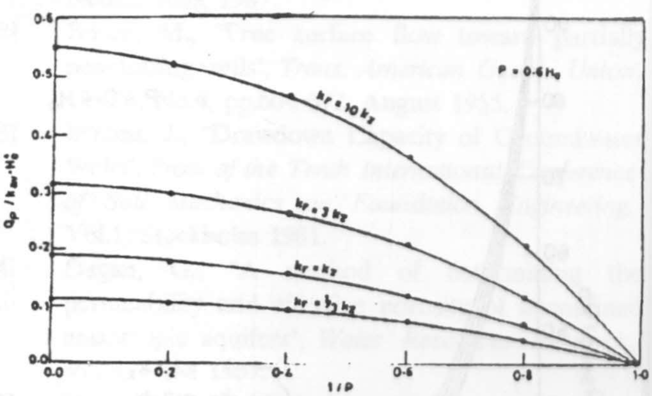

Figure 9. Relationship between welll yield and the anisotropy ratio (P = 0.6 H_o).

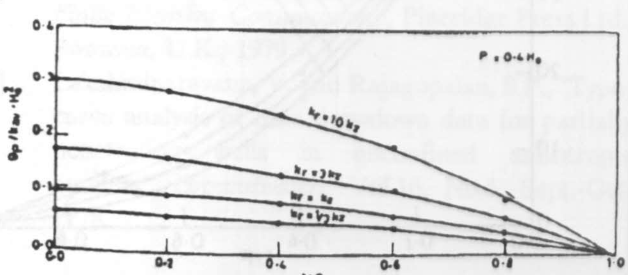

Figure 10. Relationship between well yield and the anisotropy ratio (P = 0.4 H_o).

In order to demonstrate the accuracy of the finite element method (FEM) to handle this problem, a comparison between the results and other available previous studies can only be made for isotropic soil (k_r/k_z) as listed in Table (1). The comparison is made with the Forchheimer [2], Kozeny [5] and Boreli [2] formulae (see Appendix I) for a partially penetrating well, while it is made with the Dupuit equation (Eq. 8) for a fully penetrating well. Generally speaking, a big difference is noticed between the Boreli results and the others. This is mainly attributed to the assumption made in deriving his formula, where the well radius of influence was kept as a constant and equal to 115 well radius and independent on the drawdown in the well. A slight difference is also noticed between the finite element and other studies at deep drawdowns in the well. This may be due to the neglect of the seepage face at the well in their studies.

Table 1. Comparison between the finite element method and other methods to determine the well yield from a partially penetrating well (k_r = k_z)

P (m)	t (m)	Ro (m)	Q (FEM) (m3/s)	Q (Forchheimer) (m3/s)	Q (Kozeny) (m3/s)	Q (Boreli) m3/s)
40	32	750	0.276	0.250	0.267	0.210
	24	15002	0.462	0.411	0.450	0,350
	16	2200	0.590	0.510	0.570	0.440
	8	3035	0.663	0.520	0.630	0.488
	0	3800	0.687	0.080	0.640	0.500
30	24	570	0.177	0.175	0.167	0.124
	18	1150	0.310	0.284	0.273	0.203
	12	1700	0.400	0.344	0.343	0.255
	6	2280	0.446	0.332	0.380	0.283
	0	2800	0.475	0.050	0.388	0.290
20	16	380	0.094	0.103	0.082	0.058
	12	750	0.171	0.170	0.135	0.095
	8	1140	0.217	0.198	0.169	0.119
	4	1520	0.258	0.185	0.187	0.132
	0	1900	0.271	0.030	0.190	0.133

P (m)	t (m)	Ro (m)	Q (FEM) (m3/s)	Q(Dupuit) (m3/s)
50	40	950	0.363	0.323
	30	1900	0.600	0.532
	20	2850	0.744	0.670
	10	3800	0.825	0.744
	0	4740	0.840	0.760

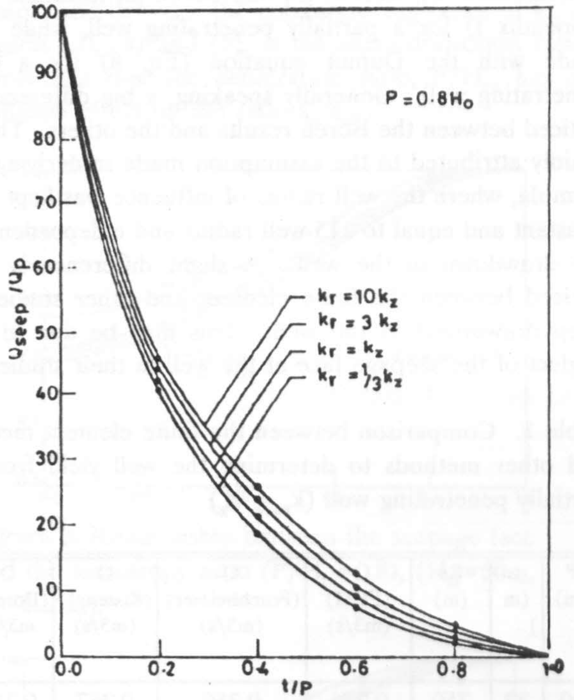

Figure 11. Relationship between the flow along the seepage face and well yield ($P = 0.8\ H_o$).

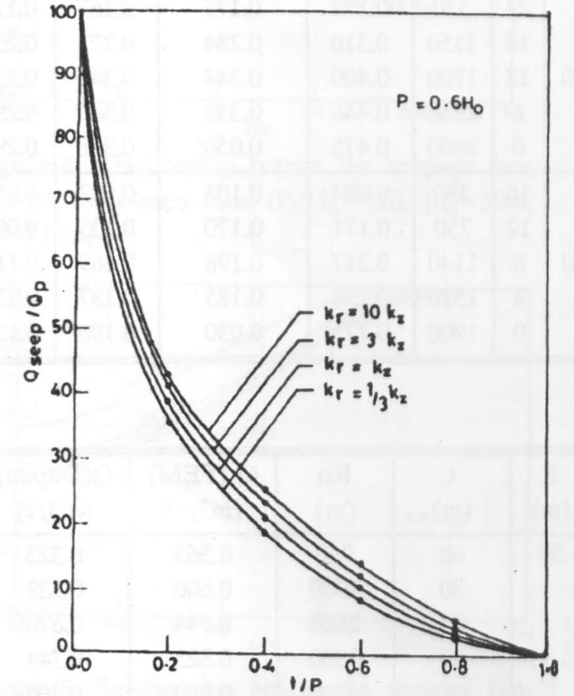

Figure 12. Relationship between the flow along the seepage face and well yield ($P = 0.6\ H_o$).

3. Flow along the Seepage face

In order to demonstrate the effect of anisotropy on the flow enters the well along the seepage face, Q_{seep}, Figures (11) to (13) show some examples of the relationship between the percentage of Q_{seep} to Q_p, t/P and k_r/k_z for P/H_o = 0.80, 0.60 and 0.40 respectively. It can be seen that Q_{seep}/Q_p increases as the increase of k_r/k_z or P/H_o. This is consistent with results obtained in the previous sections where the seepage face height and the well yield increase as the increase of the anisotropy ratio k_r/k_z and the penetration ratio P/H_o. Figures (11), (12) and (13) are useful to determine the best position of the well screen from an economic point of view.

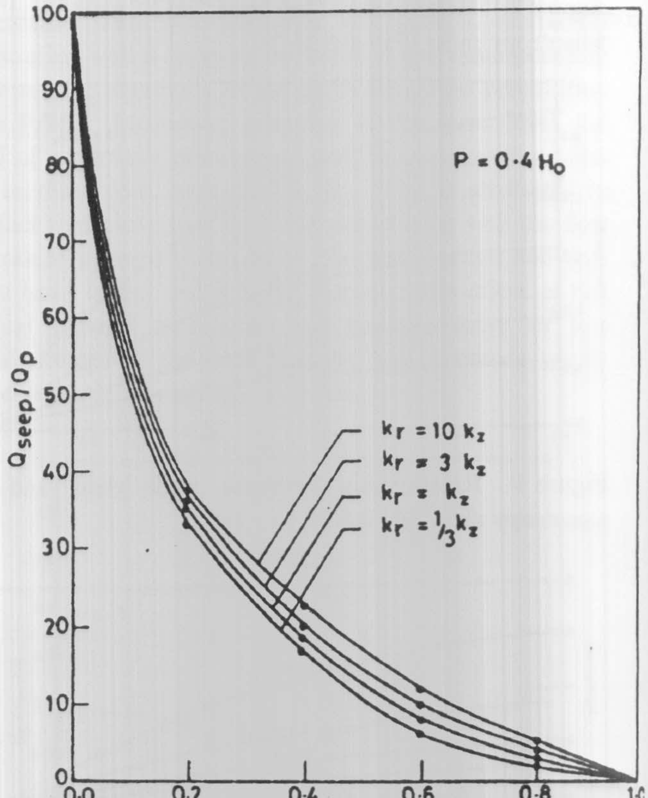

Figure 13. Relationship between the flow along the seepage face and the well yield ($P = 0.4\ H_o$).

CONCLUSIONS

1. The developed seepage face at a partially penetrating well in an anisotropic unconfined aquifer and the corresponding well yield increase as the increase of the anisotropy ratio k_r/k_z. Families of curves are

obtained to determine the seepage face height and the corresponding well yield.

2. The flux along the seepage face increases as the increase of the anisotropy ratio or the well penetration to the aquifer.
3. The finite element method has showed the flexibility and the powerful to handle easily the problem of flow towards well in an anisotropic unconfined aquifer. The results demonstrate how the finite element method can be used to prepare design graphs and charts to provide a more practical solution.

REFERENCES

[1] Bear, J. and Verruijt, A., "Modeling Groundwater Flow and Pollution", *D. Reidel Publ. Company*, The Netherlands, 1987.

[2] Boreli, M., "Free surface flow toward partially penetrating wells", *Trans. American Geoph. Union*, Vol.36, No.4, pp.664-672, August 1955.

[3] Brauns, J., "Drawdown Capacity of Groundwater Wells", *Proc. of the Tenth International Conference of Soil Mechanics on Foundation Engineering*, Vol.1, Stockholm 1981.

[4] Dagan, G., "A method of determining the permeability and effective porosity of unconfined anisotropic aquifers", *Water Resources Research*, Vol.3, No.4, 1967.

[5] Harr, M. E., *Groundwater and Seepage*, McGraw-Hill, Inc., 1962.

[6] Hinton, E. and D.R.J. Owen, *An Introduction to Finite Element Computations*, Pineridge Press Ltd., Swansea, U.K., 1979.

[7] Lakshminarayana, V. and Rajagopalan, S.P., "Type-curve analysis of time drawdown data for partially penetrating wells in unconfined anisotropic aquifers", *Groundwater*, Vol.16, No.5, Sept.-Oct. 1978.

[8] Neuman, S., "Effect of partial penetration on flow in unconfined aquifers considering delayed gravity response", *Water Resources Research*, Vol.10, No.2, April 1974.

[9] Power, J.P., *Construction Dewatering*, John Wiley & son Inc., USA, 1981.

[10] Somerville, S. H., *Control of Groundwater for Temporary Works*, CIRIA, London, UK, 1986.

[11] Taylor, R. L. and Brown, C. B., "Darcy flow solution with a free surface, J. of the Hydraulics Division", *ASCE*, Vol. 93, No. HY2, March 1967, pp.25-33.

[12] Zienkiewicz, O. C., Mayer, P. and Cheung, Y. K., "Solution of anisotropic seepage by finite elements", *J. of the Engineering Mechanics Division*, Vol.92, EM1, pp.111-120, 1966.

APPENDIX I

The formula of Forchheimer [2] takes the form

$$\frac{Q_p}{k\pi} = \frac{H_o^2 - (t+s)^2}{\ln\left(\frac{R_o}{r_w}\right)} \sqrt{\frac{t+0.5r_w}{t+s}} \sqrt[4]{\frac{2s+t}{t+s}} \qquad \text{(A-1)}$$

while the formula of Kozeny [5] is

$$\frac{Q_p}{k\pi} = \frac{(H_o - s)^2 - t^2}{\ln\left(\frac{R_o}{r_w}\right)} \left[1 + 7\left(\frac{r_w}{2(H_o - s)}\right)^{\frac{1}{2}} \cos\pi (H_o - s)\right] \qquad \text{(A-2)}$$

and that of Boreli [2]

$$\frac{Q_p}{k\pi} = \frac{\left[(H_o - s)^2 - t^2\right]}{\ln\left(\frac{R_o}{r_w}\right)} \left[1 + \left(0.3 + 10\frac{r_w}{H_o}\right) \sin 1.8\frac{s}{H_o}\right] \qquad \text{(A-3)}$$

where the above notations are defined earlier.