ON THE ANALYSIS OF CONTINUOUS BEAMS OF NONSYMMETRICAL SECTIONS

Mohamad Taha Hassan ElKatt

Structural Engineering Department, Faculty of Engineering,
Alexandria University, Alexandria, Egypt.

ABSTRACT

The moments equations for continuous beams of nonsymmetrical sections are developed and put in their general forms. Some approximations are made on the general forms to put them in the form of three-moments equations. Numerical examples are given.

NOTATIONS

A,B,C; A^*, B^*, C^*	constants
I_1, I_2	Principal moments of Inertia of the cross-section.(about axes "1" and "2" respectively).
L_o, L_1	Spans of continuous beams.
M_1,M_2	Bending moments about axes "1" and "2" of the section respectively.
M_x, M_y	Bending moments about axes "x" and "y" of the section respect.
$\bar{M}$	Bending moments.
r,$\bar{r}$	Elastic reactions.
T	Transformation matrix.
ϕ	slope of the principal axis "1" with the horizontal.
$\bar{\phi}$, ϕ_r	Angles.
axes x,y	The horizontal and vertical axes of the cross-section.
axes 1,2	The principal axes of the cross-section.

1- INTRODUCTION

The moments equations (three, four or five, Ref [4]) for analysis of continuous beams are common and powerful methods. The equations are easy and attractive in application for beams of symmetric cross-section and which obey the rules of simple theory of bending. Recently, attention was made, Ref [3], for using a form of the above equations in the analysis of continuous beams of nonsymmetrical sections. The conditions of compatibility, see Ref [3], are satisfied for the projections of the elastic line in the vertical and horizontal planes. Two equations at each support are obtained, see Ref [3], which can be written, for the case of vertical loading, as:

$$M_{xo} L_o A_o + M_{yo} L_o B_o + 2 M_{x1} (L_o A_o + L_1 A_1) + 2M_{y1} (L_o B_o + L_1 B_1) + M_{x2}L_1A_1 + M_{y2}L_1B_1 = -6(r_{10} A_o + r_{12} A_1) \quad (1)$$

and,

$$M_{xo} L_o B_o + M_{yo} L_o C_o + 2 M_{x1} (L_o B_o + L_1 B_1) + 2M_{y1} (L_o C_o + L_1C_1) + M_{x2}L_1B_1 + M_{y2}L_1C_1 = -6(r_{10}B_o + r_{12}B_1) \quad (2)$$

in which

$$A = (\cos^2 \phi)/I_1 + (\sin^2 \phi)/I_2 \quad (3.a)$$

$$B = (\cos \phi \sin \phi)(1/I_2 - 1/I_1) \quad (3.b)$$

$$C = (\sin^2 \phi)/I_1 + (\cos^2 \phi)/I_2 \quad (3.c)$$

and,

M_x and M_y are moments about x-and y-axis of the cross-section respectively, r is the elastic reaction, ϕ is the slop of principal axis "1" with the horizontal, the suscripts 0,1,2, refer to points 0,1,2, respectively and the subscripts 01 and 12 refer to span 01 and 12 respectively, see. Figure (1).

Equations (1) and (2) should be applied at each support to satisfy the conditions of compatibility in the vertical and horizontal plane respectively, under vertical loadings. Similar equations are given for horizontal loadings, see Ref. [3].

Equations (1) and (2) above can be simpilified if the compatibility conditions, at supports, are satisfied in the principal planes themselves, as explained in the next articles

2- GENERAL FORMS OF MOMENTS EQUATIONS

Consider a continuous beam, as shown in Figure (1), with nonsymmetrical section varies from span to span. It is expedient, in this case, to relate the compatibility equations to the principal planes of one of the cross-section, which will be called, hereafter, a reference plane.

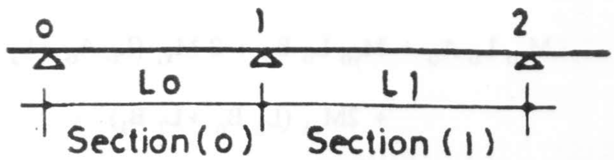

Figure 1. Continuous beam.

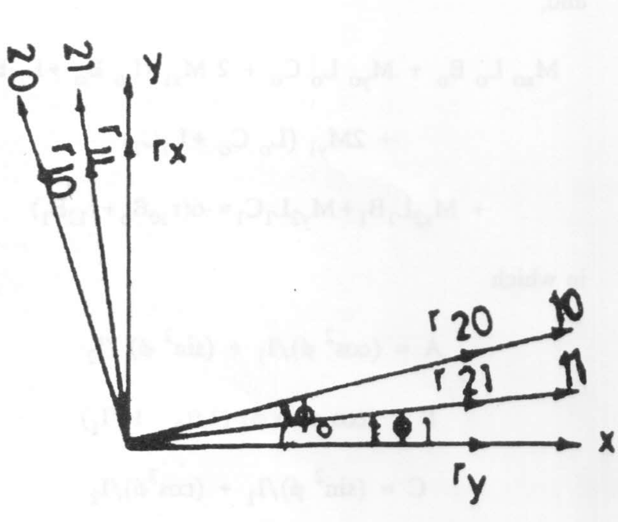

Figure 2. Positive ϕ & r.

Figure (2) shows the principal axes 10 and 20 of the cross-section of span "o", axes 11 and 21 of the cross-section of span "1", and ϕ_o and ϕ_1 are slopes of 10 and 11 with x-axis respectively. Considering 10 and 20 as the axes of plane of reference, and defining, see also Refs [1], [2],

$$\theta_{10} = - r_{10}/I_{10},\ \theta_{20} = - r_{20}/I_{20} \quad (4.a)$$

$$\theta_{11} = + r_{11}/I_{11},\ \theta_{21} = r_{21}/I_{21} \quad (4.b)$$

in which θ_{10} and θ_{20} are the elastic rotations at point "1" of span "o" about axes "1" and "2" respectively. θ_{11} and θ_{21} are same for span "1" and I_{10}, I_{20} and I_{11}, I_{21} are the principal moments of inertia of section "o" and "1" respectively. θ is positive when it acts clockwise, Refs, [1],[2].

For vertical loadings, Eqs. 4 become:

$$\theta_{10} = -(r_{xo}\cos\phi_o)/I_{10},\ \theta_{20} = -(r_{10}\sin\phi_o)/I_{20} \quad (5.a)$$

$$\theta_{11} = (r_{x1}\cos\phi_1)/I_{11},\ \theta_{21} = (r_{x1}\sin\phi_1)/I_{21} \quad (5.b)$$

in which r_{xo}, r_{x1} are the elastic reactions about x-axis (i.e. vertical reactions) at point "1" of span "0" and "1" respectively.

and for horizontal loadings:

$$\theta_{10} = (r_{yo}\sin\phi_o)/I_{10},\ \theta_{20} = -(r_{yo}\cos\phi_o)/I_{20} \quad (6.a)$$

$$\theta_{11} = -(r_y\sin\phi_1)/I_{11},\ \theta_{21} = (r_{y1}\cos\phi_1)/I_{21} \quad (6.b)$$

in which r_{yo} and r_{y1} are the elastic reactions about y-axis (i.e. horizontal reaction) of span "0" and "1" respectively.

Projecting θ_{11} and θ_{21} into plane "0", plane of reference in this derivation, then, for vertical loading

$$\begin{bmatrix} \bar{\theta}_{11} \\ \bar{\theta}_{21} \end{bmatrix} = \begin{bmatrix} \cos\bar{\Phi}_1 & -\sin\bar{\Phi}_1 \\ \sin\bar{\Phi}_1 & \cos\bar{\Phi}_1 \end{bmatrix} \begin{bmatrix} r_{x1}\ \cos\Phi_1/\ I_{11} \\ r_{x1}\ \sin\Phi_1/\ I_{21} \end{bmatrix} \quad (7.a)$$

or,

$$\bar{\theta}_1 = T.r_1 \quad (7.b)$$

in which T is the transformation matrix, Ref. [3], $\bar{\theta}_1$ is the projection of θ_1 into plane "0", and

$$\bar{\Phi}_1 = \Phi_0 - \Phi_1 \quad (7.c)$$

Noting that

$$\cos\phi_1 = \cos(\phi_0 - \bar{\Phi}_1) = \cos\Phi_0\cos\bar{\Phi}_1 + \sin\Phi_0\sin\bar{\Phi}_1 \quad (8.a)$$

$$\sin\phi_1 = \sin(\phi_0 - \bar{\Phi}_1) = \sin\phi_0\cos\bar{\Phi}_1 - \cos\Phi_0\sin\bar{\Phi}_1 \quad (8.b)$$

Then,

$$\bar{\theta}_{11} = r_{x1} (A^*_1 \cos \phi_0 + B^*_1 \sin \phi_0)/I_{11} \quad (9.a)$$

and,

$$\bar{\theta}_{21} = r_{x1} (C^*_1 \sin \phi_0 + B^*_1 \frac{L_{21}}{I_{11}} \cos \phi_0)/I_{21} \quad (9.b)$$

in which, the constants:

$$A^*_1 = \cos^2 \phi_1 + (\sin^2 \phi_1) I_{11} / I_{21} \quad (10.a)$$

$$B^*_1 = (\sin \phi_1 \cos \phi_1) (1 - \frac{I_{11}}{L_{21}}) \quad (10.b)$$

$$C^*_1 = \cos^2 \phi_1 + (\sin^2 \phi_1) (I_{21}/I_{11}) \quad (10.c)$$

It should be noted that for plane "0", the plane of reference, θ_{10} and θ_{20} are given by Eqs. (5a,6a).

Due to the Connecting moments at supports, the elastic reactions at support "1" of span 12 are:

$$r_{11} = M_{11} L_1 / 3I_{11} + M_{12} L_1 / 6I_{11} \quad (11.a)$$

$$r_{21} = M_{21} L_1 / 3I_{21} + M_{22} L_1 / 6I_{21} \quad (11.b)$$

in which

r_{11}, r_{21} are the elastic reactions about axes "1" and "2" for section "1" respectively.

M_{11}, M_{12} are the bending moments about axis "1" of section "1" at supports 1 and 2 respectively.

and

M_{21}, M_{22} are the bending moments about axis "2" of section "1" at supports 1 and 2 respectively.

Projecting the above reaction, Eq.11, into plane of reference gives:

$$\begin{bmatrix} \bar{r}_{11} \\ \bar{r}_{21} \end{bmatrix} = \begin{bmatrix} \cos\bar{\phi}_1 & -\sin\bar{\phi}_1 \\ \sin\bar{\phi}_1 & \cos\bar{\phi}_1 \end{bmatrix} \begin{bmatrix} r_{11} \\ r_{21} \end{bmatrix} \quad (12)$$

But,

$$M_{11} = \bar{M}_{11} \cos\bar{\phi}_1 + \bar{M}_{21} \sin\bar{\phi}_1 \quad (13.a)$$

$$M_{21} = -\bar{M}_{11} \sin\bar{\phi}_1 + \bar{M}_{21} \cos\bar{\phi}_1 \quad (13.b)$$

in which, $\bar{M}_{11}$ and $\bar{M}_{21}$ are the bending moments acting at section "1" but about directions parallel to the principal axes of reference plane, plane "0" in this case. Similar equations can be given for M_{21} and M_{22} at point 2.

Substituting Eqs. (13) into Eqs. (12) gives:

$$\bar{r}_{11} = A^*_1 (\frac{\bar{M}_{11} L_1}{3I_{11}} + \frac{\bar{M}_{12} L_1}{6I_{11}}) + B^*_1 (\frac{\bar{M}_{21} L_1}{3I_{11}} + \frac{\bar{M}_{22} L_1}{6I_{11}}) \quad (14.a)$$

and,

$$\bar{r}_{21} = C^*_1 (\frac{\bar{M}_{21} L_1}{3L_{21}} + \frac{\bar{M}_{22} L_1}{6I_{21}}) + B^*_1 (\frac{\bar{M}_{11} L_1}{3I_{11}} + \frac{\bar{M}_{21} L_1}{6I_{11}}) \quad (14.b)$$

For Continuity at support 1, under vertical loadings, then,

$$\theta_{10} + r_{10} = \bar{\theta}_{11} + \bar{r}_{11} \quad (16.a)$$

and

$$\theta_{20} + r_{20} = \bar{\theta}_{21} + \bar{r}_{21} \quad (16.b)$$

Expanding Eqi (16.a), noting that

$$\theta_{10} = -\frac{r_{xo}}{I_{10}} \cos\phi_o \quad (5.a)$$

and,

$$r_{10} = -(\frac{\bar{M}_{10} L_o}{6I_{10}} + \frac{\bar{M}_{11} L_o}{3I_{10}}) \quad (17)$$

Then,

$$-\frac{r_{xo}}{I_{10}} \cos\phi_o - (\frac{\bar{M}_{10} L_o}{3I_{10}} + \frac{\bar{M}_{11} L_o}{3I_{10}}) = r_{x1} (A^*_1 \cos\phi_o + B^*_1 \sin\phi_o)/I_{11} + A^*_1 (\frac{\bar{M}_{11} L_1}{3I_{11}} + \frac{\bar{M}_{12} L1}{6I_{11}}) + B^*_1 (\frac{\bar{M}_{21} L_1}{3I_{11}} + \frac{\bar{M}_{22} L_1}{6I_{11}}) \quad (18)$$

Rearranging Eq. 18, gives.

$$\bar{M}_{10} L_o/I_{10} + 2\bar{M}_{11} [L_o/I_{10} + (L_1/I_{11})A_1^*]$$

$$+2\bar{M}_{21}(B^*_1 L_1/I_{11}) + \bar{M}_{12}L_1A^*_1/I_{11} + \bar{M}_{22}L_1B^*_1/I_{11}$$

$$= -6[r_{xo}\cos\phi_o/I_{10} + r_{x1}(A^*_1\cos\phi_o + B_1^*\sin\phi_o)/I_{11}] \quad (19.a)$$

Equation (19.a) is applicable only when section "o" is the plane of reference, otherwise Eq. (19.a) becomes:

$$\bar{M}_{10} L_o A^*_o/I_{10} + \bar{M}_{20} L_o B_o^*/I_{10} + 2\bar{M}_{11}(L_o A_o/I_{1o}$$

$$+ L_1 A_1^*/I_{11}) + 2\bar{M}_{21} (L_o B^*_o/I_{10} + L_1 B_1^*/I_{11})$$

$$+ \bar{M}_{12} L_1 A^*_1 / I_{11} + \bar{M}_{22} L_1 B^*_1 /I_{11}$$

$$= -6 [r_{xo} (A^*_o \cos\phi_r + B_o^* \sin\phi_r)/I_{10}$$

$$+ r_{x1} (A_1^*\cos\phi_r + B_1^*\sin\phi_r) /I_{11}] \quad (19.b)$$

in which,

ϕ_r is the slope of plane of reference.

Similarly, Eq. (16.b) can be expanded to give,

$$\bar{M}_{20} L_o/I_{20} + 2\bar{M}_{21} (L_o/I_{20} + L_1 C_1^*/I_{21})$$

$$+ 2\bar{M}_{11} L_1 B_1^*/I_{11} + \bar{M}_{12} L_1 B^*_1/I_{11}$$

$$+ \bar{M}_{22} L_1C_1^*/I_{21} = -6[r_{xo}\sin\phi_o/I_{20}$$

$$+ r_{x1}(C_1^*\sin\phi_o + B_1^* \frac{L_{21}}{I_{11}} \cos\phi_o)/I_{21}] \quad (20.a)$$

if "o" is a plane of reference, otherwise

$$\bar{M}_{20} L_o C_o^*/I_{20} + \bar{M}_{10} L_o B_o^*/I_{10}$$

$$+ 2\bar{M}_{21} (L_o C_o^*/I_{20} + L_1 C_1^*/I_{21})$$

$$+ 2\bar{M}_{11}(L_o B_o^*/I_{1o} + L_1 B_1^*/I_{11}) + \bar{M}_{12} L_1 B^*_1/I_{11}$$

$$+ \bar{M}_{22} L_1 C_1^*/I_{21}$$

$$= -6 [r_{xo} (C_o^* \sin\phi_r + \bar{B}_o \frac{L_{21}}{I_{11}} \cos\phi_r)/I_{20}$$

$$+ r_{x1} (C_1^* \sin\phi_r + B_1^* \frac{L_{21}}{I_{11}} \cos\phi_r)/I_{21}] \quad (20.b)$$

In similar way, the equations of compatibility, under horizontal loadings, can be obtained. But it is worthy noted that the changes will occur only on the right-hand side of Eqs. 19&20, and they will be:

$$\text{R.H.S. of Eq. (19-a)} = -6[-r_{y1}(-B_1^*\cos\phi_o + A_1^*\sin\phi_o)/I_{11}$$

$$-r_{yo}\sin\phi_o/I_{10}] \quad (21.a)$$

$$\text{R.H.S. of Eq. (19.b)} = -6[-r_{yo}(-B_o^*\cos\phi_r + A_o^*\sin\phi_r)/I_{10}$$

$$-r_{y1}(-B_1^*\cos\phi_r + A_1^*\sin\phi_r)/I_{11}] \quad (21.b)$$

$$\text{R.H.S. of Eq. (20.a)} = -6[r_{yo}\cos\phi_o/I_{20}$$

$$+ \frac{r_{y1}}{L_{21}} (C_1^*\cos\phi_o - B_1^* \frac{L_{21}}{I_{11}} \sin\phi_o)] \quad (21.c)$$

$$\text{R.H.S. of Eq. (20.b)} = -6[r_{yo}(C_o^*\cos\phi_r - B_o^* \frac{L_{20}}{I_{10}} \sin\phi_r)/I_{20}$$

$$+ r_{y1} (C_1^* \cos\phi_r - B_1^* \frac{L_{21}}{I_{11}} \sin\phi_r)/I_{21}] \quad (21.d)$$

Having obtained the values of $\bar{M}_1$ and $\bar{M}_2$ at each support, their components M_x and M_y can be calculated using the transpose of transformation matrix, T, of Eq. (7.b) and ϕ_r i.e.

$$T^t = \begin{bmatrix} \cos\phi_r & \sin\phi_r \\ -\sin\phi_r & \cos\phi_r \end{bmatrix} \quad (22)$$

3-THE THREE MOMENT EQUATION

The three moment equation for continuous beams of nonsymmetrical sections can be obtained directly if the beams have a constant section for all spans. In this case, if the conditions of compatibility are satisfied about the principal axes, three-moment equations will result, which can be written, for vertical loading, as:

$$M_{10}\,L_o + 2M_{11}\,(L_o+L_1) + M_{12}\,L_1 = -\,6r\,\cos\phi \quad (23.a)$$

and,

$$M_{20}\,L_o + 2M_{21}\,(L_o+L_1) + M_{22}\,L_1 = -\,6r\,\sin\phi \quad (23.b)$$

Equation (23) and its similar equations for horizontal loadings are exact for beams of constant sections. But, in case of beams with section varies from span to span, eqs. 19-21, some approximations should be made so that equation of three-moment can be obtained. The approximation made, in this work, is that, $\sin\bar{\phi} \approx 0$ and $\cos\bar{\phi} \approx 1$. Therefore, the constant of Eqs. (10) have the values of $A^* = C^* = 1$, and $B^* = 0$, and hence the Three-Moment equations for beams of nonsymmetrical varied sections are:

For vertical loadings:

$$\bar{M}_{10}\frac{L_o}{I_{10}} + 2\bar{M}_{11}\left(\frac{L_o}{I_{10}}+\frac{L_1}{I_{11}}\right) + \bar{M}_{12}\frac{L}{I_{11}} = -6\left(\frac{r_{xo}}{I_{10}}+\frac{r_{x1}}{I_{11}}\right)\cos\phi_r \quad (24\text{-a})$$

and,

$$\bar{M}_{20}\frac{L_o}{I_{20}} + 2\bar{M}_{21}\left(\frac{L_o}{I_{20}}+\frac{L_1}{I_{21}}\right) + \bar{M}_{22}\frac{L1}{I_{21}} = -6\left(\frac{r_{xo}}{I_{20}}+\frac{r_{x1}}{I_{21}}\right)\sin\phi_r \quad (24\text{-b})$$

and, for horizontal loadings:

$$\bar{M}_{10}\frac{L_o}{I_{10}} + 2\bar{M}_{11}\left(\frac{L_o}{I_{10}}+\frac{L_1}{I_{11}}\right) + \bar{M}_{12}\frac{L1}{I_{11}} = -6\left(\frac{-r_{yo}}{I_{10}}-\frac{r_{y1}}{I_{11}}\right)\sin\phi_r \quad (25\text{-a})$$

$$\bar{M}_{20}\frac{L_o}{I_{20}} + 2\bar{M}_{21}\left(\frac{L_o}{I_{20}}+\frac{L_1}{I_{21}}\right) + \bar{M}_{22}\frac{L1}{I_{21}} = -6\left(\frac{r_{yo}}{I_{20}}+\frac{r_{y1}}{I_{21}}\right)\cos\phi_r \quad (25\text{-b})$$

Having obtained $\bar{M}_1$ and $\bar{M}_2$ at each support, their component M_x, M_y are calculated using the transformation matrix of eq. (22)

4- NUMERICAL EXAMPLES

The first example

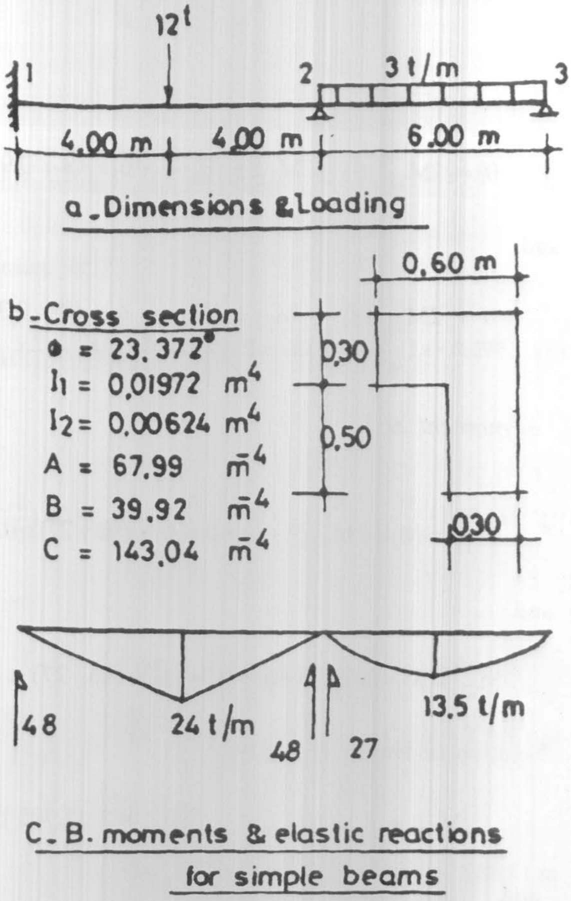

Figure 3. Continuous beam of constant section.

is a continuous beam of constant section given in Figure (3-a). Under vertical loading, the properties and section constants are given in Figure (3-b). An application of eqs. (1) & (2) results in the following equations:

$$\begin{bmatrix} 1 & 0.5872 & 0.5 & 0.2936 \\ 1 & 3.5832 & 0.5 & 1.7916 \\ 1 & 0.5872 & 3.5 & 2.055 \\ 1 & 3.5832 & 3.5 & 12.541 \end{bmatrix} \begin{bmatrix} M_{x1} \\ M_{y1} \\ M_{x2} \\ M_{y2} \end{bmatrix} = \begin{bmatrix} -18 \\ -18 \\ -56.25 \\ -56.25 \end{bmatrix}$$

The solution of the above equations gives:

M_{x1} = - 11.625 tim, M_{x2} =- 12.75 tim

$$M_{y1} = M_{y2} = 0.$$

while the application of Eqs. (23) gives:

3- moment at 1

$$0 + 2M_{11}\ (8) + M_{12} \times (8) = -6 \times 48 \times 0.918 \quad \text{(a)}$$

and,

$$0 + 2M_{21}\ (8) + M_{22} \times (8) = -6 \times 48 \times 0.397 \quad \text{(b)}$$

3- moment at 2

$$M_{11} \times (8) + 2M_{12}(8+6) + 0 = -6 \times (48+27) \times 0.918 \quad \text{(c)}$$

and,

$$M_{21} \times (8) + 2M_{22}(8+6) + 0 = -6 \times (48+27) \times 0.397 \quad \text{(d)}$$

Solution of Eqs. a & c gives:

$$M_{11} = -10.672 \text{ t.m}, \quad M_{12} = -11.705 \text{ t.m}$$

and,

Solution of Eqs. b&d gives:

$$M_{21} = -4.615 \text{ t.m}, \quad M_{22} = -5.062 \text{ t.m}$$

Then, using Eq. (22),

$$\begin{bmatrix} M_{x1} \\ M_{y1} \\ M_{x2} \\ M_{y2} \end{bmatrix} = \begin{bmatrix} 0.918 & 0.397 & 0 & \\ -0.397 & 0.918 & & \\ 0 & 0 & 0.918 & 0.397 \\ 0 & 0 & -0.397 & 0.918 \end{bmatrix} \begin{bmatrix} 10.672 \\ -4.615 \\ -11.705 \\ -5.062 \end{bmatrix} = \begin{bmatrix} -11.639 \\ 0 \\ -12.755 \\ 0 \end{bmatrix}$$

A second example

is the beam shown in Fig.3.a. with section shown in Figure (3-b) for span 1-2 and for span 2-3 another section which is shown in Figure (4).

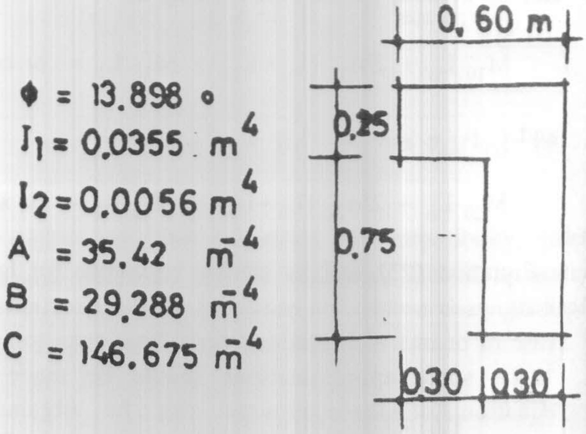

Figure 4. Cross section of span 2-3.

Equations 1 and 2 give:

$$\begin{bmatrix} 1 & 0.5872 & 0.5 & 0.2936 \\ 1 & 3.5832 & 0.5 & 1.7916 \\ 1 & 0.5872 & 2.7814 & 1.8204 \\ 1 & 3.5832 & 3.1005 & 12.677 \end{bmatrix} \begin{bmatrix} M_{x1} \\ M_{y1} \\ M_{x2} \\ M_{y2} \end{bmatrix} = \begin{bmatrix} -18 \\ -18 \\ -46.5494 \\ -50.8568 \end{bmatrix}$$

From which:

$$M_{x1} = -11.755 \text{ t.m} \quad M_{y1} = 0.0175 \text{ t.m}$$

$$M_{x2} = -12.491 \text{ t.m} \quad M_{y2} = -0.035 \text{ t.m}$$

Applying Eqs. (24) considering the relative Inertia, then

3- moment at 1

$$0 + 2\bar{M}_{11} * \left(\frac{8}{1}\right) + \bar{M}_{12} * \left(\frac{8}{1}\right) = -6 * \frac{48}{1} * 0.918 \quad \text{(e)}$$

and,

$$0 + 2\bar{M}_{21} * \left(\frac{8}{1}\right) + \bar{M}_{22} * \left(\frac{8}{1}\right) = -6 * \frac{48}{1} * 0.397 \quad \text{(f)}$$

3- moment at 2

$$\bar{M}_{11}(\frac{8}{1}) + 2\bar{M}_{12}(\frac{8}{1}+\frac{6}{1.8}) + 0 = -6(\frac{48}{1}+\frac{27}{1.8}) * 0.918 \qquad (g)$$

and,

$$\bar{M}_{21}(\frac{8}{1}) + 2\bar{M}_{22}(\frac{8}{1}+\frac{6}{1.042}) + 0 = -6(\frac{48}{1}+\frac{27}{1.042}) * 0.397 \qquad (h)$$

which give:

$$M_{11} = -10.77 \text{ t.m}, \; M_{12} = -11.508 \text{ t.m}$$

$$M_{21} = -4.618 \text{ t.m}, \; M_{22} = -5.055 \text{ t.m}$$

and, Using eq. (22)

$$\begin{bmatrix} M_{x1} \\ M_{y1} \\ M_{x2} \\ M_{y2} \end{bmatrix} = \begin{bmatrix} 0.918 & 0.397 & 0 & 0 \\ -0.397 & 0.918 & 0 & 0 \\ 0 & 0 & 0.918 & 0.397 \\ \text{zero} & 0 & -0.397 & 0.918 \end{bmatrix} \begin{bmatrix} -10.77 \\ -4.618 \\ -11.508 \\ -5.055 \end{bmatrix} = \begin{bmatrix} -11.72 \\ +0.036 \\ -12.57 \\ -0.0718 \end{bmatrix}$$

Consider a continuous beam shown in Fig.5 Under the horizontal loadings shown. Sections properties of the beam are given in Fig.3.b and Fig.4. The solution given, using equations of Ref. [3] is:

$$M_{x1} = -0.017 \text{ t.m}, \; M_{y1} = -3.87 \text{ t.m}$$

$$M_{x2} = +0.034 \text{ t.m}, \; M_{y2} = -4.261 \text{ t.m}$$

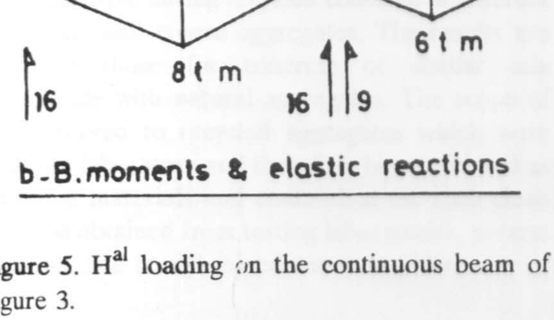

Figure 5. H^{al} loading on the continuous beam of Figure 3.

Applying Eqs. (25), gives:

3- moment at 1

$$0 + 2\bar{M}_{11}(\frac{8}{1}) + \bar{M}_{12}(\frac{8}{1}) = -6x\frac{16}{1} * -0.397 \qquad (i)$$

and,

$$0 + 2\bar{M}_{21}(\frac{8}{1}) + \bar{M}_{22}(\frac{8}{1}) = -6x\frac{16}{1} * 0.918 \qquad (j)$$

3- moment at 2

$$\bar{M}_{11}(\frac{8}{1}) + 2\bar{M}_{12}(\frac{8}{1}+\frac{6}{1.8}) + 0 = -6(\frac{16}{1}+\frac{9}{1.8}) * -0.397 \qquad (k)$$

and,

$$\bar{M}_{21}(\frac{8}{1}) + 2\bar{M}_{22}(\frac{8}{1}+\frac{6}{1.042}) + 0 = -6(\frac{16}{1}+\frac{9}{1.042}) * 0.918 \qquad (l)$$

which give:

$$M_{11} = 1.553 \text{ t.m}, M_{21} = -3.56 \text{ t.m}$$

$$M_{12} = +1.659 \text{ t.m}, \; M_{22} = -3.896 \text{ t.m}$$

and, Using Eq. (22), then

$$M_{x1} = +0.12 \text{ t.m}, \; M_{y1} = -3.885 \text{ t.m}$$

$$M_{x2} = -0.024 \text{ t.m}, \; M_{y2} = -4.235 \text{ t.m}$$

CONCLUSIONS

The Three Moment Equations, eqs. (23), can be applied directly for continuous beams of constant nonsymmetrical sections. For beams of varied sections, a good approximate solution can be obtained using the Three Moment Equations, given in Eqs. (24)& (25). The results are acceptable.

APPENDIX -I- REFERENCES

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