EFFECT OF MODEL DIMENSIONS ON SEEPAGE UNDER A FLOOR WITH A SINGLE SHEET PILE

Farouk A. El-Fitiany and Rabiea I. Nasr
Department of Irrigation and Hydraulics, Faculty of Engineering,
Alexandria University, Alexandria, Egypt.

ABSTRACT

Experimental models, such as sand, electric analogue and viscous flow models are extensively used to study the seepage under hydraulic structures. Effects of limiting the model lengths of the upstream and the downstream seepage surfaces and the model thickness of the pervious stratum are studied here, for a floor with an upstream or a downstream sheet pile, using the finite element method. The results indicate that exit gradients are more sensitive to such limitations than the uplift pressures on the floor.

INTRODUCTION

Experimental models for the problems of seepage under the floor of a hydraulic structure are extensively used to verify mathematically derived solutions or to overcome difficulties met with in such derivations. Sand, electric analogue and viscous flow (Hele-Show) models are the most commonly used ones [1,2,3]. It is difficult, however, to model the infinite lengths of the upstream (U/S) and the downstream (D/S) seepage faces, which is usually the case for the actual prototype problem. Similar difficulty may be also encountered in modelling the depth of the permeable stratum beneath the floor. Elganainy [4] obtained a closed-form solution for the case of a simple flat floor founded on a pervious stratum, using the conformal mapping technique. He concluded that, in general, limiting the lengths of the U/S and D/S pervious beds will result in a relatively small decrease in the uplift pressures on the U/S half of the floor and a relatively small increase of the pressures on the D/S one while causing an appreciable rise in the relative exit gradients along the D/S bed. The effects of varying the thickness of the pervious stratum were negligible. Muthukumaran and kulandaiswamy [5] investigated the case of a weir with a sheet pile at the center founded on an impervious stratum of a limited thickness, for a symmetrical lengths of the U/S and D/S seepage surfaces. The conformal mapping technique was used to obtain an analytical solution for that problem. The minimum length of the U/S and D/s surfaces required in the model to keep the difference in quantity of seepage, between model and prototype, within 1% was determined and graphically represented for various geometric dimensions.

In the present study, the more practical case of a weir with an U/S or a D/S sheet pile is investigated, using a finite element model. Unsymmetrical lengths of the seepage surfaces are also considered as well as the symmetrical ones, for an impervious stratum of a limited thickness.

THE PHYSICAL MODEL

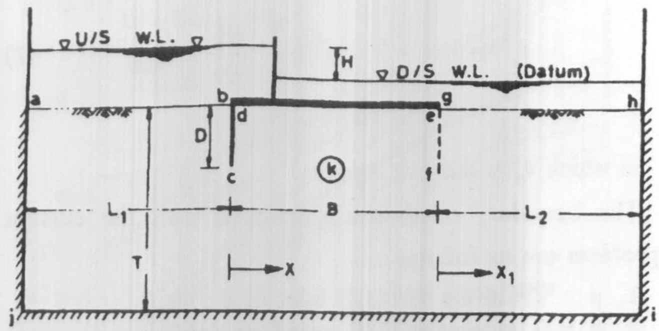


Figure 1. The physical model.

Figure (1). Shows the physical model of the problem which represents the experimental simulation of the actual *prototype conditions for which the U/S and the D/S* seepage surfaces extend to infinity. The floor, bg, is of a length B and has either an U/S sheet pile, bcd, or D/S one, efg, of a length D. An effective head, H, acts on the floor. The U/S seepage face, ab, and the D/S one, gh, have, in general, different lengths, L_1 and L_2, respectively.

The permeable stratum beneath the floor is assumed to be homogenous and isotropic, of a thickness T and a permeability coefficient k. These parameters can be expressed in dimensionless forms in terms of the floor length as : $l_1 = L_1/B$, $l_2 = L_2/B$, $d = D/B$, $t = T/B$ and $h = H/B$.

GOVERNING EQUATION AND BOUNDARY CONDITIONS

A two dimensional, steady state flow through a homogenous isotropic soil is governed by Laplac's equation:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \qquad (1)$$

in which $\phi = k\,h_1$, is the velocity potential at any point in the flow domain , $h_1 = p/\gamma + z$, is the piezometric head, p is the pressure at that point, z is its position and γ is the unit weight of water.

Equation (1) may be alternatively written in terms of the stream function ψ instead of ϕ .

Solving Laplac's equation along with the appropriate boundary conditions yields the velocity potential ϕ . Hence, the pressure at any point can be directly calculated. The velocity gradient, I_e , at any point on the exit surface, gh , can be also obtained from the relation :

$$I_e = -\frac{v_e}{k} = -\frac{1}{k}\frac{\partial\phi}{\partial y}\bigg|_{exit} \qquad (2)$$

in which v_e is the exit velocity.

The boundary conditions associated with the current problem are as follows :

1. ϕ = kH along the U/S inlet face , ab ,
2. ϕ = 0 along the D/S exit face , gh ,
3. $\frac{\partial\phi}{\partial n} = 0$ along the sheet pile , bcd or efg , the horizontal floor, de, the upper surface, ij, of the impervious substratum and the artificial vertical boundaries of the model, aj and hi, where n is the normal direction to boundary.

The finite element technique has been used to get a numerical solution for equation (1) along with abovementioned boundary conditions. Figure (2). illustrates the mesh arrangement used. A summary of the derivation of the finite element equations is given in appendix (A). A computer program , FEM2D [6], was used to calculate the velocity potentials at all nodes as well as exit gradients at the D/S exit face.

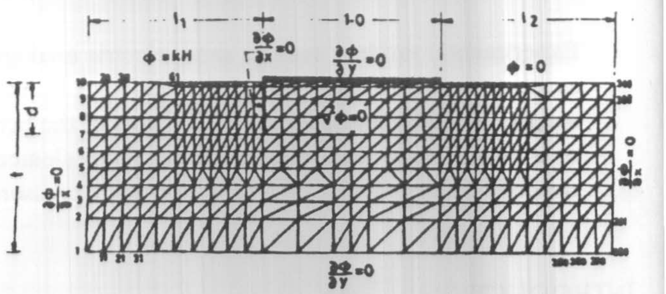


Figure 2. The finite element model.

VERIFICATION OF THE MODEL

Results obtained from the finite element model, with $l_1 = l_2 = 2.0$, have been compared with those based on the analytical, conformal mapping solution [1] , for the case of an U/S sheet pile, with infinite U/S and D/S seepage surfaces. The very good agreement between these results, as shown in Figure (3), confirms the accuracy of the model.

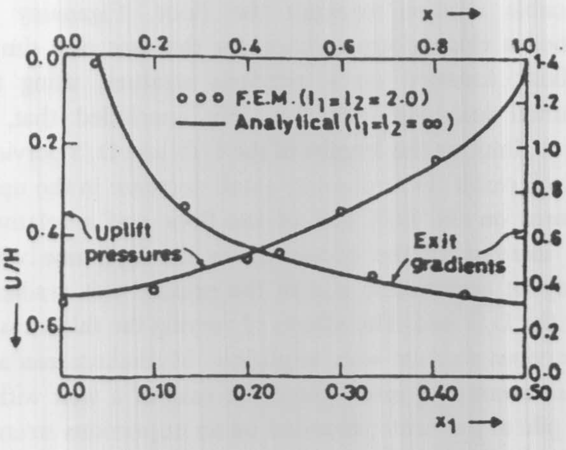


Figure 3. Comparison beetween F.E.M. and the analytical soultion (U/S sheet pile, d=o.30, t=2.0).

ANALYSIS OF THE RESULTS

The effects of limiting the lengths of the U/S and the D/S seepage surfaces and the thickness of the permeable stratum on both the uplift pressure distribution on the floor and the exit gradients behind it have been investigated.

First, the case of centrally positioned floor (case A, $l_1=l_2=1$), which is usually adopted in the experimental model, is discussed. This is followed by a discussion for the case of an eccentrically positioned floor (case B, $l_1 \neq l_2$). For both cases, two different positions for the sheet pile are considered : an U/S sheet pile and a D/S one.

CASE A: Central Floor ($l_1 = l_2 = l$)

a - Uplift Pressures

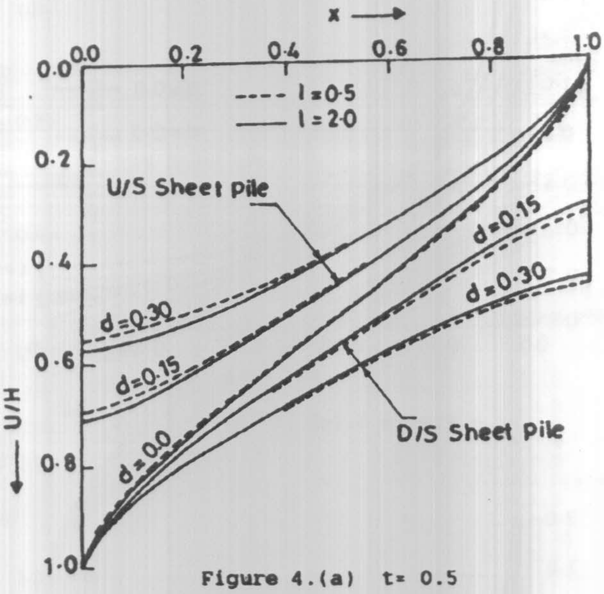
Figures (4-a,b) show the effects of limiting the relative length, l, of the U/S and the D/S seepage surfaces on the uplift pressures acting on the floor, for a relative thickness, t, of the permeable stratum equals to 0.5 and 2.0 , respectively. For both positions of the sheet pile, three different values of the relative depth, d, of the sheet pile have been considered, namely d = 0.0 , 0.15 and 0.30 .

Considering the case of a floor with an U/S sheet pile, it is found that uplift pressures are slightly reduced as the relative length l decreases. The maximum reduction always occurs at floor points adjacent to the sheet pile. Compared to the case of l = 2.0 , which can be practically considered as infinity, the maximum reduction corresponding to l = 0.5 is about 2.4% for d = 0.15 , and rises to 3.5% for d = 0.30 , with t = 0.50 for both cases. Such a reduction becomes almost negligible for the ease d = 0.0 , i.e. a simple floor with no sheet pile. As the thickness t of the pervious stratum increases to 2.0 , the maximum relative reduction slightly drops to 2.1% , for d = 0.15 but it rises to 8.5% , for d = 0.30 .

As for the case of a floor with a D/S sheet pile, it is found that similar but opposite variations occur. A pressure increase will result from using a limited length, l, for the seepage surfaces.

b - Exit Gradients

The effects of introducing vertical impervious boundaries on the exit gradients are shown in Figures (5) and (6), for the two cases of a floor with an U/S sheet pile and with a D/S one, respectively. Referring to Figure (5-a) , it is clear that exit gradients for an outlet surface of a limited length will be less than the corresponding ones for an infinite outlet surface, down to a certain distance from the floor end, then they become greater from thereon. For example exit gradients, I_{e1}, for l = 0.5 and d = 0.0 are less than I_{e2}, for l = 2.0 and d = 0.0, down to x_1 = 0.15 , where x_1 is the relative distance from the floor end, Figure (1), then they become greater after that point. As the length, d, of the sheet pile increases, the crossing points move farther to the D/S.



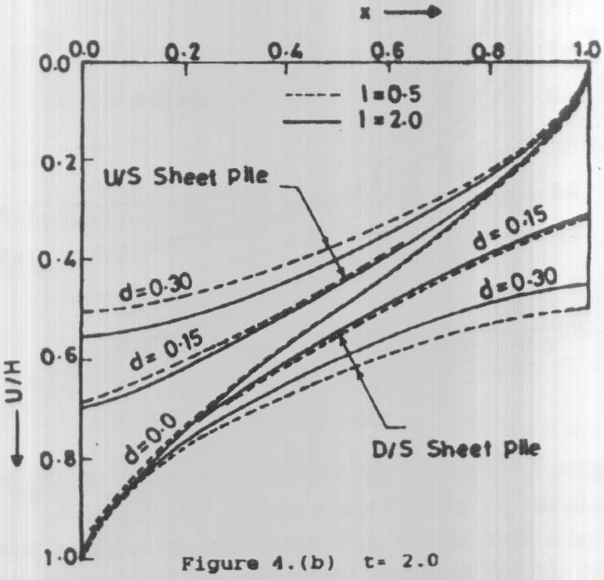


Figure 4. Effect of the vertical boundaries on the uplift pressures ($l_1=l_2=1$).

Higher exit gradients will result when the thickness, t, of the pervious stratum is increased from 0.5 to 2.0, as revealed from Figures (5-a) and (5-b). This is due to the increase of the total seepage discharge beneath the floor.

The crossing points become closer to the floor end as the thickness, t, increases.

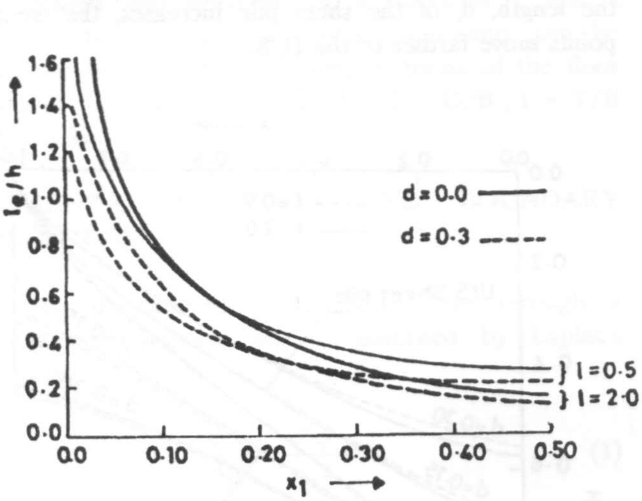


Figure 5.(a) t= 0.5

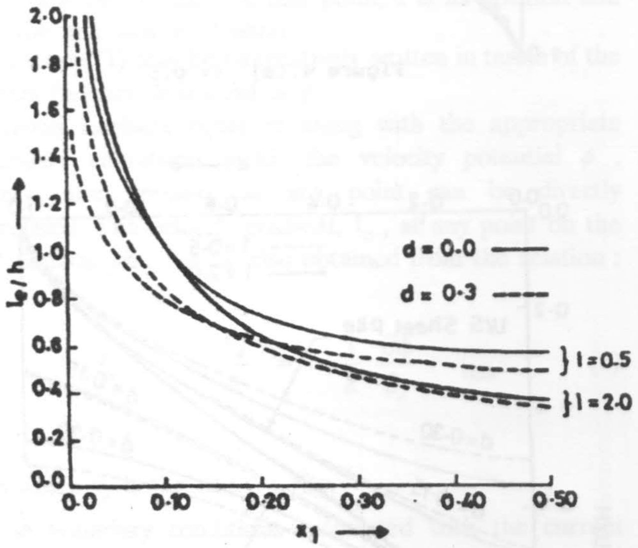


Figure 5.(b) t= 2.0

Figure 5. Effect of the vertical boundaries on the exit gradients, for an U/S sheet pile. ($l_1 = l_2 = 1$).

For a floor with a D/S sheet pile, no crossing points exist and exit gradients for l = 0.5 will be always greater than those for l = 2.0 , Figure (6). Moreover, the effects of reducing the length l are more significant for larger thicknesses of the permeable layer.

Table (1) presents some values of the ratio (I_{e1}/I_{e2}), where I_{e1} and I_{e2} are exit gradients corresponding to l = 0.5 and 2.0, respectively, at the two points x_1 = 0.025 and 0.50 .

Table 1. Values of (I_{e1}/I_{e2})

t	d	x_1 = 0.025	x_1 = 0.50
0.50	0.0	0.75	1.73
	0.3 (U/S Sh.P)	0.91	1.75
	0.3 (D/S Sh.P)	1.12	2.10
2.0	0.0	0.90	1.58
	0.3 (U/S Sh.P)	0.78	1.39
	0.3 (D/S Sh.P)	1.26	1.80

CASE B: Eccentric Floor ($l_1 \neq l_2$)

Practical considerations may necessitate using unequal lengths for the U/S and the D/S seepage surfaces in the model. These cases are discussed below, for a constant thickness of the pervious layer, t = 2.0 .

a - Uplift Pressures

Figures (7-a,b) illustrate the effects of varying the lengths l_1 and l_2 of the U/S and the D/S seepage surfaces on the uplift pressures, for sheet pile depths, d = 0.15 and 0.30, respectively. The case of $l_1 = l_2 = 2.0$ (curve 5) can be practically considered very close to the actual prototype conditions of $l_1 = l_2 = \infty$, hence other cases will be compared with it. From Figure (7), it is clear that a shorter inlet surface ($l_1 < l_2$) will reduce the uplift pressures, and vise versa, for both cases of an U/S sheet pile and a D/s one. Relative pressure head deviations ($\Delta U/U_\infty$ %) at a point just D/S of the U/S sheet pile and at the point just U/S of the D/S sheet pile are shown in table (2), for some combinations of l_1 and l_2 , for d = 0.15 and 0.30 .

Table 2. Relative Pressure Head Deviations ($\Delta U/U_\infty$ %), Close to the Sheet Pile.

Curve No	l_1	l_2	d = 0.15		d = 0.30	
			U/S		U/S Sh.P	D/S Sh.P
1	0.5	2.0	-7.1	-13.3	-18.0	-12.0
4	1.5	2.0	-0.7	-3.3	-1.8	-1.1
6	2.0	1.5	0.7	1.7	0.9	2.7
9	2.0	0.5	5.7	18.3	9.9	23.0

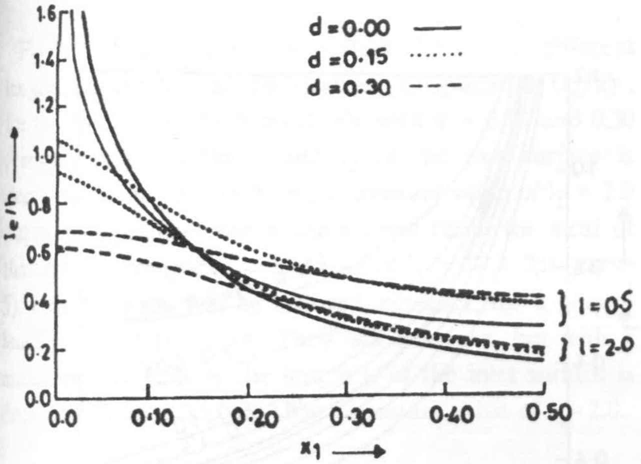


Figure 6.(a) t= 0.5

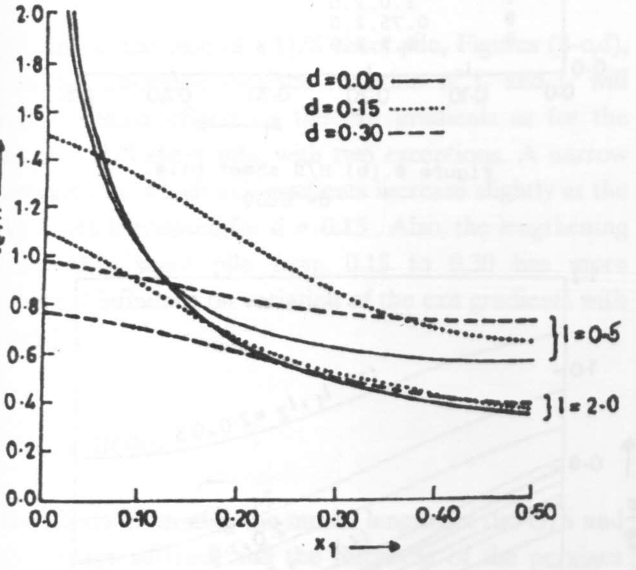


Figure 6.(b) t= 2.0

Figure 6. Effect of the vertical boundaries on the exit gradients for a D/S sheet pile, $(l_1 = l_2 = l)$.

It is noticed that increasing the length of the U/S sheet pile from 0.15 to 0.30 will increase all deviations significantly. On the other hand, similar increase of the D/S sheet pile will produce smaller increase of the positive deviations and a smaller decrease of the negative ones.

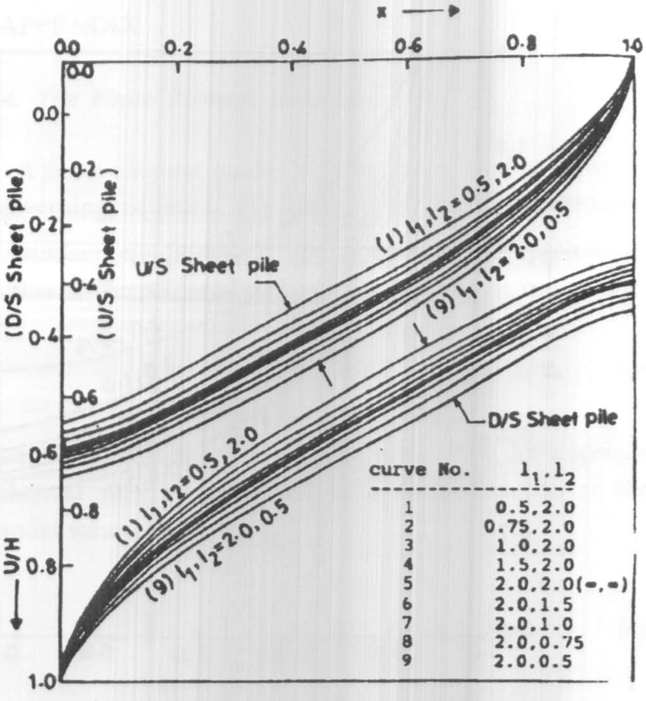


Figure 7.(a) d= 0.15

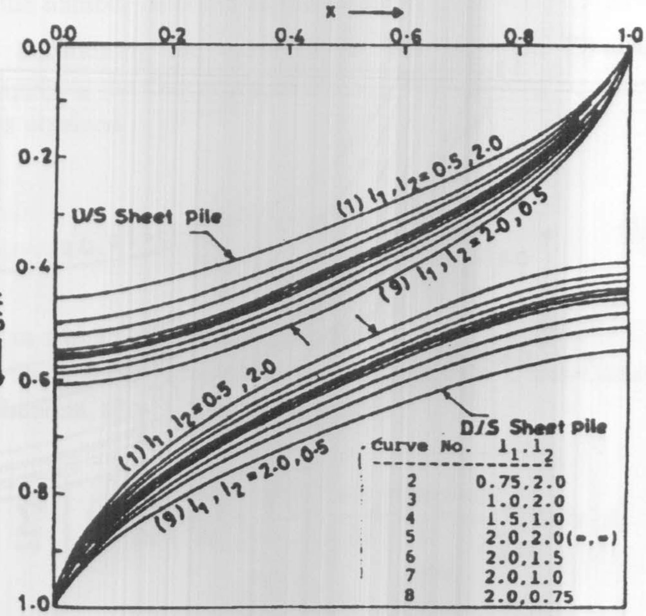


Figure 7.(b) d= 0.30

Figure 7. Effect of the vertical boundaries on the uplift pressures (case B:$l_1 \neq l_2$), for t = 2.0.

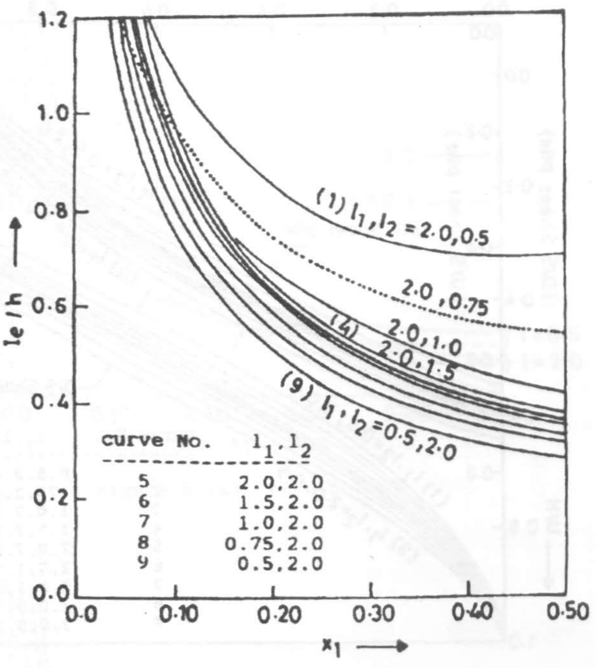


Figure 8.(a) U/S sheet pile. d= 0.15

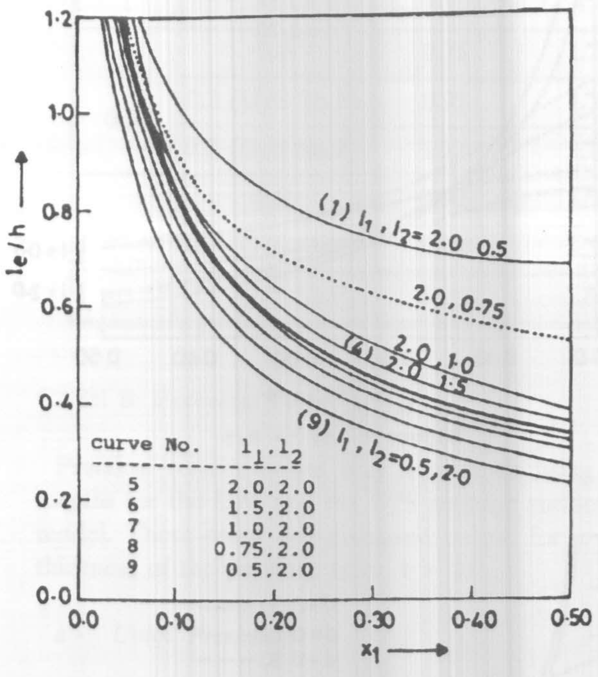


Figure 8.(b) U/S sheet pile. d= 0.30

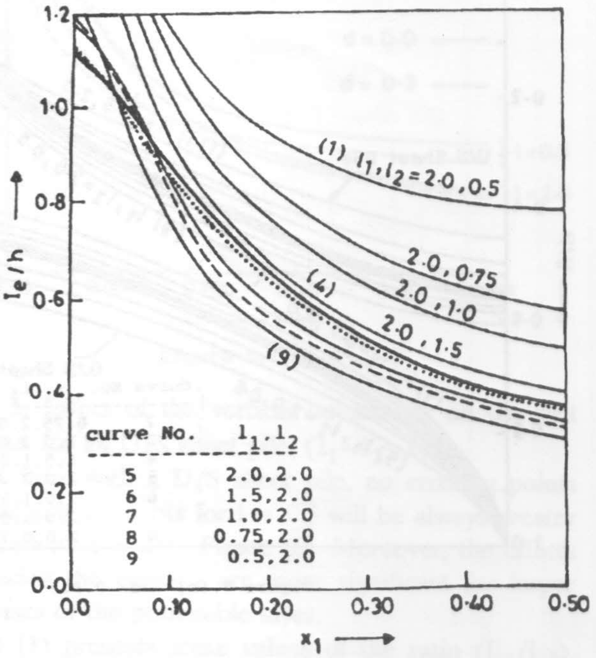


Figure 8.(c) D/S sheet pile d= 0.15

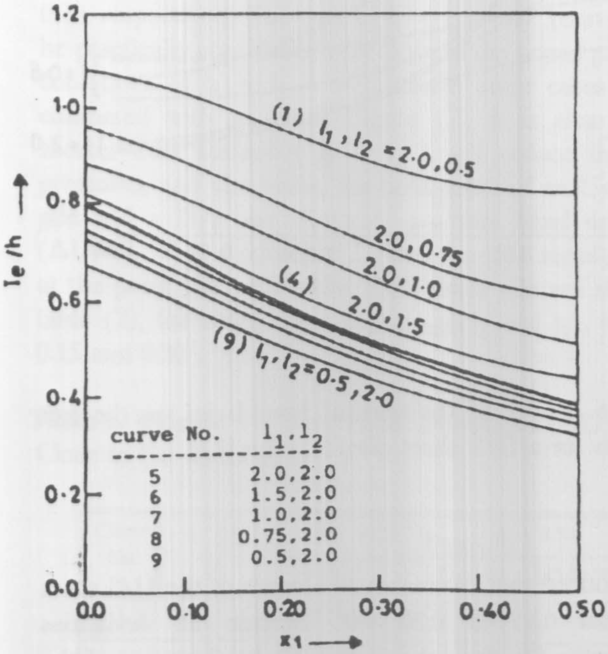


Figure 8.(d) D/S sheet pile. d= 0.30

Figure 8. Effect of the vertical boundaries on the exit gradients (case B: $l_1 \neq l_2$), for t=2.

b - Exit Gradients

Figures (8-a,b) show the effects of using different lengths, l_1 and l_2 , on the relative exit gradients (I_e/h) , for the cases of an U/S sheet pile with d = 0.15 and 0.30 , respectively. As the length l_2 of the exit surface is decreased from 2.0 to 0.5 , for a constant value of l_1 = 2.0 , appreciably higher exit gradients will result for most of that surface compared to the case of $l_1 = l_2 = 2.0$ (curve 5). Exit gradients will be reduced, however, for a limited length next to the floor. They will decrease, but with a much smaller rate, as the length l_1 of the inlet surface is decreases from 2.0 to 0.5 , for a constant value of l_2=2.0.

The lengthening of the U/S sheet pile from 0.15 to 0.30 will, as expected, slightly reduce the exit gradients but the effects of varying l_1 and l_2 are very similar.

Turning to the case of a D/S sheet pile, Figures (8-c,d), it can be concluded that the variation of l_1 and l_2 will produce similar effects on the exit gradients as for the case of a U/S sheet pile, with two exceptions. A narrow zone exists for which exit gradients increase slightly as the inlet length is reduced,for d = 0.15 . Also, the lengthening of the D/S sheet pile from 0.15 to 0.30 has more prominent influence on variation of the exit gradients with l_1 and l_2 .

CONCLUSIONS

The effects of limiting the model lengths of the U/S and D/S seepage surfaces and the thickness of the pervious stratum have been investigated, using the finite element technique, for a floor with an U/S sheet pile or a D/S one. Compared to the prototype conditions of infinite U/S and D/S seepage surfaces, it is found that if equal lengths are adopted for the U/S and the D/s seepage surfaces, the resulting deviations in the uplift pressures will be relatively small. Exit gradient deviations are, however, significant and increase for larger thicknesses of the pervious stratum. If, on the other hand, different lengths are used, these deviations will be greater and the exit face length will have an appreciable influence on the exit gradients. It can be stated that using equal inlet and exit lengths 1.5 to 2.0 times the floor length is sufficient to keep the resulting deviations within 3% .

APPENDIX

A. The Finite Element Solution

A finite element model [6,7] has been used to solve the governing equation (1) along with the aforementioned boundary conditions. If $\bar{\Phi}$ represents an approximate value of the potential ϕ , equation (1) may be rewritten as :

$$\nabla^2 \bar{\Phi} = R \tag{3}$$

in which R is the residual. The value of $\bar{\Phi}$ for a specific element may be expressed as a linear function of the nodal values ϕ_n :

$$\bar{\Phi} = \sum_{n=1}^{I} N_n \, \phi_n \text{ , } (n=1,2,..,I) \tag{4}$$

in which N_n are linear interpolation functions and I is the number of nodes in the element.

Substituting the value of $\bar{\Phi}$ into equation (3) and applying the Galerkin's conditions, the following equation is obtained :

$$\sum_{n=1}^{I} \int_A \nabla^2 \phi_n \, dA = 0 \tag{5}$$

in which A is the flow domain.

Integrating by parts and making use of the Green-Gauss theorem, then equation (5) yields :

$$\sum_{n=1}^{I} \left\{ \int_{A^e} \left[\frac{\partial N_m}{\partial x}\left(\frac{\partial N_n}{\partial x} + \frac{\partial N_n}{\partial y} \right) + \frac{\partial N_m}{\partial y}\left(\frac{\partial N_n}{\partial x} + \frac{\partial N_n}{\partial y} \right) \right] dx\,dy \right\} \phi_n$$

$$- \int_s N_m \, q_n \, ds = 0 \text{ ,} (m=1,2,...I) \tag{6}$$

in which A^e is the element domain, S is the boundary of the domain and q_n is the normal flux across the element boundary.

Equation (6) can be also written as follows :

$$\sum k^e_{nm}\phi^e_n = F^e_n \tag{7}$$

where

$$k^e_{nm} = \int_{A^e}\left[\frac{\partial N_m}{\partial x}\left(\frac{\partial N_n}{\partial x}+\frac{\partial N_n}{\partial y}\right)+\frac{\partial N_m}{\partial y}\left(\frac{\partial N_n}{\partial x}+\frac{\partial N_n}{\partial y}\right)\right]dx\,dy \tag{8}$$

and

$$F^e_n = \int N_m\, q_n\, ds \tag{9}$$

For the present case, k^e is symmetric, i.e. $k^e_{nm} = k^e_{mn}$.

Equation (7) represents the finite element form of equation (1), for the element n of the mesh. Considering all elements, a set of simultaneous linear equations is obtained which can be written as :

$$[k]\,\{\Phi\} = \{F\} \tag{10}$$

where [k] is the global stiffness matrix, $\{\Phi\}$ is the global vector of the unknown potentials to be determined, and {F} is the global nodal force vector.

The final solution is obtained form equation (10) after the application of the boundary conditions.

APPENDIX B.

REFERENCES

[1] M.E., Harr, *Ground Water and seepage*, Mc Graw-Hill Book Co. Inc., New York, N.Y., 1962 .

[2] S., Leliavsky, *Design of Dams for Percolation and Erosion*, Chapmen and Hall Ltd., London 1965.

[3] D.K., Tood, *Ground Water Hydrology*, John Willy & Sons, Inc. New York and London, 1959.

[4] M.A., Elganainy Seepage Beneath Flat Floors on Bounded Pervious strata, *The Bulletin of the Faculty of Eng.*, Alexandria Univ., Vol. XXIV, pp. 587-604, 1986.

[5] Muthukumaran S. and Kulandaiswamy, End Effects in Models for Seepage Below Weirs, *Journal of the Hydraulic Division, ASCE*, Vol. 98 No. 3, July 1972, pp 541-552.

[6] J.N., Reddy, *An Introduction to the Finite Element Method*, Mc Graw-Hill Book Company, New York, 1987.

[7] J.J., Connor, and C.A., Brebbia, *Finite Element Techniques for Fluid Flow*, Newnes-Butterworths, London, 1976.