EFFECT OF LINING CRACKS ON THE SEEPAGE UNDER A SIMPLE FLOOR OF A HYDRAULIC STRUCTURE IN A LINED CANAL

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ABSTRACT

The stability of a heading-up hydraulic structure will be greatly affected by the formation of upstream and/or downstream cracks in the canal lining. The boundary element method is used to analyse the uplift pressure on the floor and on the lining itself as well as the exit gradients, resulting from two cracks, of general widths and locations. A simple flat floor on a permeable layer of a limited thickness is considered. The results indicate that substantial increase in the uplift pressure and the exit gradients may develop due to narrow cracks.

Keywords

Hydraulic structure, seepage, lined canal, cracks, boundary element, uplift pressure, exit gradients.

INTRODUCTION

Lining of irrigation canals is one of the most effective means of water conservation, particularly for newly reclaimed desert areas. The design of water control and distribution structures for lined canals should be modified to include any expected different boundary conditions. The stability of such structures is greatly affected by both uplift pressure forces and the values of the hydraulic gradients at the downstream exit, resulting from seepage of water under the floor. Numerous analytical and experimental studies have been carried out to evaluate the uplift pressure distribution and the exit gradients for different boundary conditions and floor configurations, for infinite upstream (U/S) and downstream (D/S) seepage surfaces, [1-3]. For lined canals, however, these surfaces should idealy diminish and a continous impervious bed exists. Even with weep holes provided at the downstream, the possibility of clogging or malfunctioning may lead to the same situation.

For completely impervious bed, no seepage forces will act on the floor nor on the soil particles. Hence, minimum structural requirements can be adopted regarding these forces. Practically, however, strong possibility exists of limited cracks to develop in the U/S and/or the D/S lining. It is very important for the designer to have a correct evaluation for the resulting seepage forces corresponding to different probable locations and widths of such cracks.

Hathoot [4] used the theory of images and the complex velocity potentials of a line source and a line sink to derive a seepage formula for a simple floor with finite U/S and D/S pervious zones, for a finite depth of the permeable stratum. The application was limited to the calculation of the seepage discharge. Moreover, the assumption of a constant discharge per unit length for both the line source and the line sink does not represent the actual boundary conditions.

Gary and Chawla [5] and Chawla [6] derived analytical solutions using the conformal mapping technique for a floor founded on a permeable soil of infinite and finite depths, respectively, with finite pervious inlet and outlet zones and a cut-off at any general position along the floor. The analysis, however, did not cover the cases of very narrow cracks, relative to the floor length, which are considered to be the practical case. The exit gradients were calculated at one point only which may be insufficient for design purposes.

In the present study the boundary element method is used to analyse the practical problem of seepage under a simple floor constructed in a lined canal, with U/S and D/S cracks, of general widths and locations. Figure (1) shows the general layout and the main parameters of the problem. The difference of head between the water levels equls H and the length of the impervious floor EF equals B. The U/S linng, AE has a crack DE of width W_1 at a distance L_1 from the U/S edge of the floor while FJ represents the D/S lining, with a crack GI of width W_2 at a distance L_2 from the D/S edge of the floor. The permeable soil beneath the floor has a limited thickness T and is assumed to be homogenous and isotropic with constant permeability coefficient k.

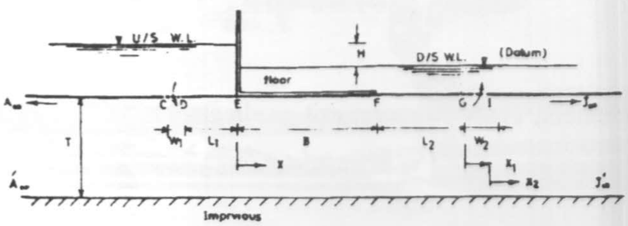

Figure 1. The physical model.

In dimensionless forms, these parameters may be rewritten as $h=H/B$, $w_1 = W_1 / B$, $w_2 = W_2 / B$, $\ell_1 = L_1 /B$, $\ell_2 = L_2/B$ and $t = T/B$.

Application of the Boundary Element Method

The idealization of the problem is shown in Figure (2), in which the domain Ω is bounded by a surface, S, whose normal is n, positive outward. Using a general, two dimensional cartesian system of coordinates; $(x_i, i=1,2)$, and for constant k, Laplace's equation can be written as follows, at any point (x_i) in the domain:

$$u(x)_{,ii} = 0 \qquad (1)$$

in which $u=(p/\gamma)+z$, is the potential head, (p/γ) is the pressure head and z is the position head, measured upward from the D/S water level.

The corresponding components of the flow vector v_i are given by Darcy's law as:

$$v_i = - k\, u_{,i} \qquad (2)$$

The current problem is a mixed boundary value one, with the potential head, u, or the flux, q, specified on each portion of the surfaces, S.

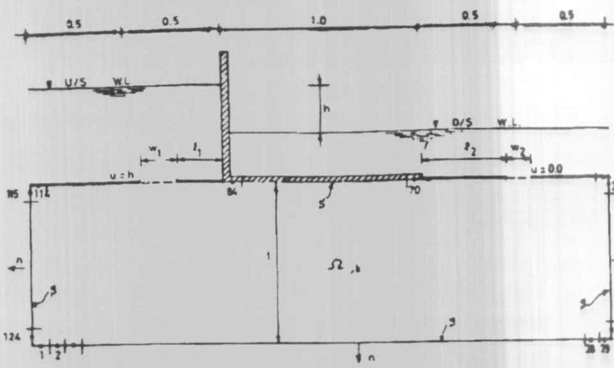

Figure 2. Idealization of the problem.

Utilizing the reciprocal integral identity corresponding to the differential equation (1) the direct boundary integral equation is written as [7,8],

$$\alpha\, u(x_o) + \int_S u\, q^* \, ds - \int_S q\, u^* \, ds = 0 \qquad (3)$$

in which,

$$q^* = \partial u^* / \partial n \qquad (4)$$

$$\alpha = \omega / 2\pi$$

ω = internal angle of the surface at point (x_o),
α = 1 at any point, x, in the domain Ω.
u^* = potential generated at a field point x_i by a unit source applied at a surcharge point x_j.

To make use of equation (3), the surface, S, of the domain is discretized to (n) linear boundary elements. The values of u and q are assumed to be constant on each element, then equation (3) is written at each nodal element. For all values of $i \neq j$, the integrals given by equation (3) were calculated using a 4-point Gauss quadrature rule, whereas for $i=j$ the values of these integrals are given by Brebbia [8] in closed form. Then equation (3) becomes;

$$(H_{ij} + \delta_{ij})\, u_j - G_{ij}\, q_j = 0 \qquad (5)$$

where

δ_{ij} = the Kronecker delta

$$G_{ij} = \int_S u^{*} \, ds \qquad (6)$$

$$H_{ij} = \int_S q^{*} \, ds \qquad (7)$$

The physical situation outlined in Figure (1) is defined by the following boundary conditions:

- The potential u along the U/S and the D/S cracks has the values u=h and u=0.0, respectively
- The flux q normal to the impervious boundaries is equal to zero

Introducing these boundary conditions into equation (5), the solution gives the following unknown boundary values:

- The flux q along the two cracks.
- The potential u along the impervious boundaries.

RESULTS AND DISCUSSION

a - Uplift pressure on the floor

Figure (3) shows the effects of the relative widths, w_1 and w_2, of the U/S and the D/S cracks, on the relative net uplift pressure (U_1/H), for $\ell_1 = \ell_2 = 0$ and t=1.

For a constant value of the relative D/S crack width w_2 equals to 0.025, the uplift pressures build up gradually, but with a decreasing rate, as the relative U/S crack width w_1 increases from 0.025 (curve IV) to 0.50 (curve VII) approaching a maximum limit for greater values of w_1. On the other hand, for a constant value of the U/S crack width w_1 equals to 0.025, the uplift pressures decrease gradually as the value of the D/S crack width w_2 increases from 0.025 (curve IV) to 0.50 (curve I), approaching also a minimum limit, irrespective of any further increase of w_2. Compared to the ordinary case of unlined canals for which $w_1 = w_2 = \infty$ (the dashed curve) it is clear that a narrow crack at the U/S side will induce a considerable uplift pressure increase on the floor if the D/S crack is very narrow. At the limiting case of $w_2=0.0$ and $w_1>0.0$, *a uniform net uplift pressure U=H acts on the floor. This* pressure diminishes (U=0) at the opposite limiting case of $w_1 = 0.0$ and $w_2 \geq 0.0$.

These results are in a very good agreement with those calculated from the conformal mapping solution by Chawla [6], Figure (3).

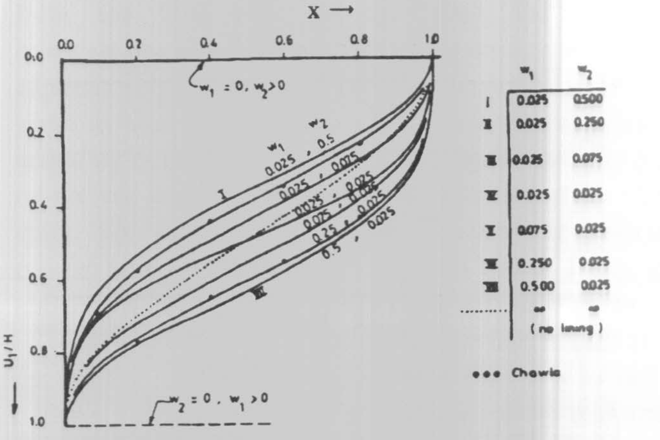

Figure 3. Effect of w_1 and w_2 on the net uplift pressures along the floor (t=1.0, $\ell_1=\ell_2=0.0$).

The effect of the relative locations ℓ_1 and ℓ_2 of the U/S and the D/S cracks are illustrated in Figure (4), for $w_1 = w_2 = 0.05$ and t = 1.0. In comparison with the case of $\ell_1 = \ell_2 = 0.0$, i.e. for cracks adjacent to the floor, significant pressure increase, particularly, on the D/S quarter of the floor, will result as the D/S crack moves away from the edge of the floor. Similar but opposite pressure changes will occur if the U/S crack moves away from the floor.

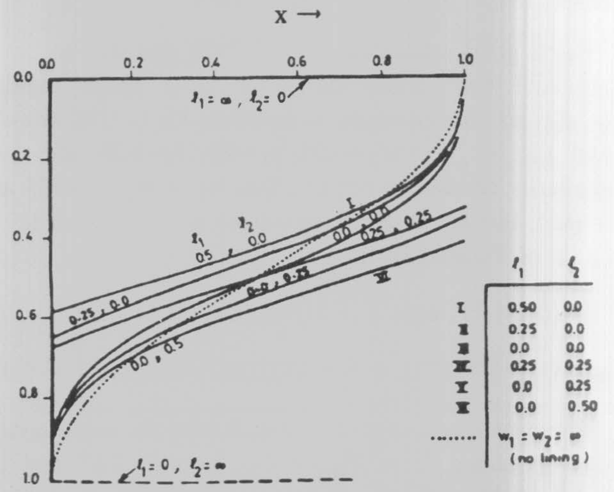

Figure 4. Effect of ℓ_1 and ℓ_2 on the net uplift pressure alonge the fllor (t =1.0, $w_1 = w_2 = 0.05$).

These results are expectable since the case of ℓ_1 and/or $\ell_2 > 0.0$ (case 2, Figure (5)) represents an U/S or a D/S lengthening of the floor. In fact, the pressure distribution for this case can be directly deduced from the simpler

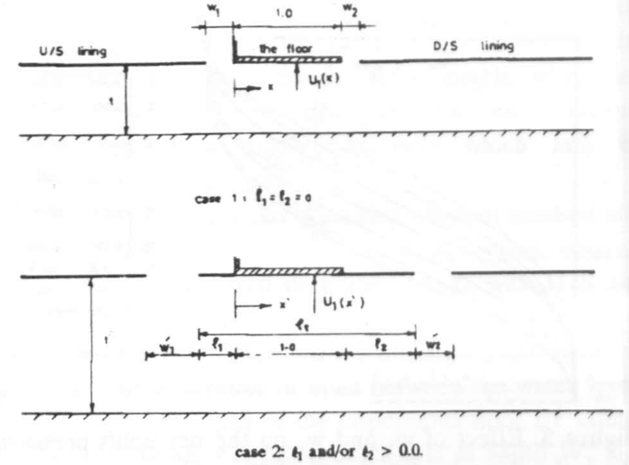

Figure 5. Cracks adjacent to the floor (case 1) or away from it (case 2).

case of $\ell_1 = \ell_2 = 0.0$ (case 1, Figure (5)), if they are geometrically similar i.e. $w_1 = w_1/\ell_t$, $w_2 = w'_2/\ell_t$ and $t = t'/\ell_t$, where $\ell_t = 1.0 + \ell_1 + \ell_2$. Then, the relative pressure at any point x′, for case 2, is given by:

$$U_1(x')/H', \text{ case } 2 = U_1(x)/H, \text{ case } (1) \quad (8)$$

where

$$x = (x' + \ell_1)/\ell_t$$

Fairly good estimates of $U_1(x')/H'$ can still be obtained with some deviation from those similarity conditions. For example, to calculate $U_1(x')/H'$ at x=0.25 and 1.0, for $w'_1 = w'_2 = 0.05$, $\ell_1 = 0.0$, $\ell_2 = 0.50$ and $t' = 1.0$, similarity conditions require that $w_1 = w_2 = 0.033$ and t = 0.67. However, using the values $w_1 = w_2 = 0.025$ and t=1.0 of Figure (3), then

$$U_1(0.25)/H', \text{ case } 2 = U_1(0.167)/H, \text{ case } 1 = 0.66$$

and $U_1(1.0)/H'$, case 2 = $U_1(0.667)/H$, case 1 = 0.43

These results compare very well with the more accurate ones obtained from Figure (4), namely 0.67 and 0.41, respectively.

Figure (6) shows the effect of the relative thickness t of the pervious layer on the relative net uplift pressure (U_1/H). As the relative thickness decreases from 1.0 to 0.25 the pressure distribution on the floor approaches the straight line distribution which represents the limiting case of t=0.0. Similar trends have been reported for infinite U/S and D/S pervious surfaces ($w_1 = w_2 = \infty$) [1].

For different combinations of w_1 and w_2, with t = 1.0 and $\ell_1 = \ell_2 = 0$, the net uplift pressures on the D/S lining, U_2, increase from 0.0 at point I, Figure (1), to a limited maximum $U_{2,max}$ after some distance, $x_2 = X_2/B < 1.0$, away from that point, Figure (7).

The relative maximum pressure ($U_{2,max}$ / H) significantly increases as the D/S crack becomes narrower and/or the U/S crack becomes wider.

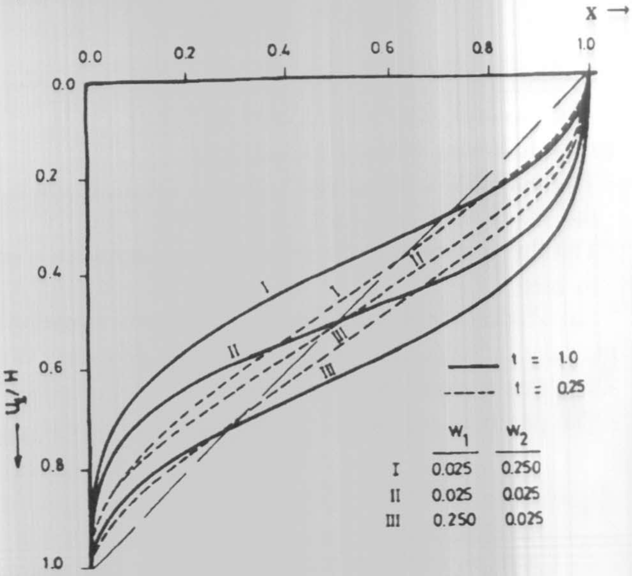

Figure 6. Effect of the relative thickness, t of the pervious layer on the net uplift pressures along the floor ($\ell_1 = \ell_2 = 0.0$).

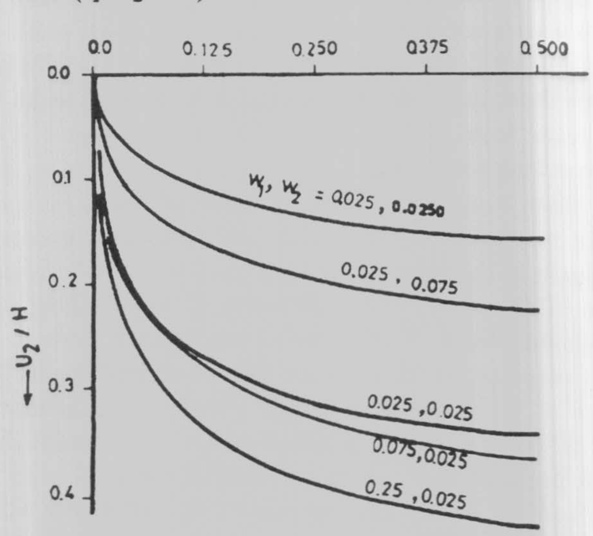

Figure 7. Effect of cracks widthes, w_1 and w_2, on the net uplift pressures on the D/S lining ($\ell_1 = \ell_2 = 0$, t=1.0).

If the cracks develop at some distance from floor (ℓ_1 and/or ℓ_2>0.0), the relative maximum pressure on the D/S lining will be smaller, Figure (8). It should be noted, however, that the part of the D/S lining adjacent to the floor will, in effect, act as an extension of the floor.

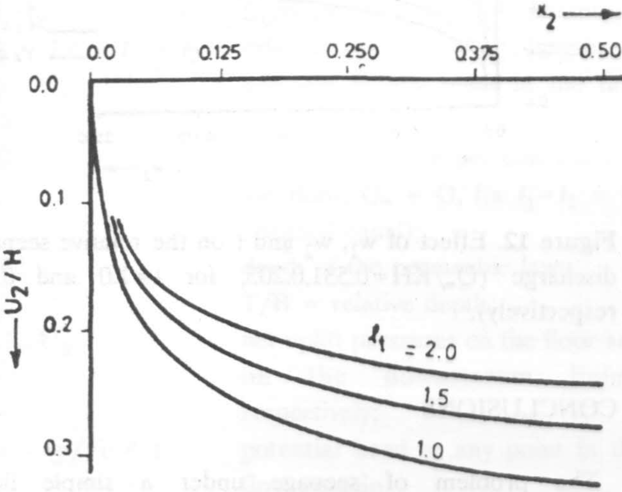

Figure 8. Effect of the total distance "ℓ_1" between the cracks on the net uplift pressures on the D/S lining. ($w_1=w_2=0.05$ $t=1.0$).

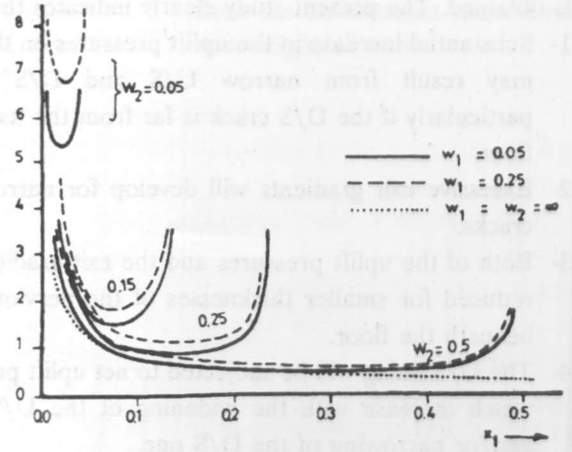

Figure 9. Effect of w_1 & w_2 on the exit gradients ($t=1.0$, $\ell_1=\ell_2=0.0$).

b- Exit gradients

Theoretically, the exit gradients, $G_E=V_E/k$, where V_E is the velocity at the exit, become infinity at both ends of the cracks, if the thickness of the lining is neglected and no filter is provided. Figure (9) illustrates the variation of (G_E/h) with the relative distance, $x_1=X_1/B$, measured from the U/S edge of the crack, for different combinations of w_1 and w_2. The cracks are assumed to be adjacent to the floor and the relative thickness of the previous layer is taken as $t=1.0$. The value of the relative width, w_2, of the D/S crack has a very significant influence on the exit gradients in comparison with that of the U/S crack. For example, the average relative exit gradient (G_E/h) at $x_1=0.025$ decreases from 6.1 to 2.5, a drop of about 60%, as w_2 is increased from 0.05 to 0.25, for a constant $w_1=0.05$. If w_1 is increased from 0.05 to 0.25, and for $w_2=0.05$, the average exit gradient rises only by 18%.

Figure (10) shows that the greater the relative thickness t of the pervious layer the higher the values of (G_E/h) will be. Such an increase becomes less prominent for wider D/S cracks. The results obtained for wide D/S cracks are in good agreement with those calculated by Chawla's formula [6], where less agreement is noticed for narrow D/S cracks.

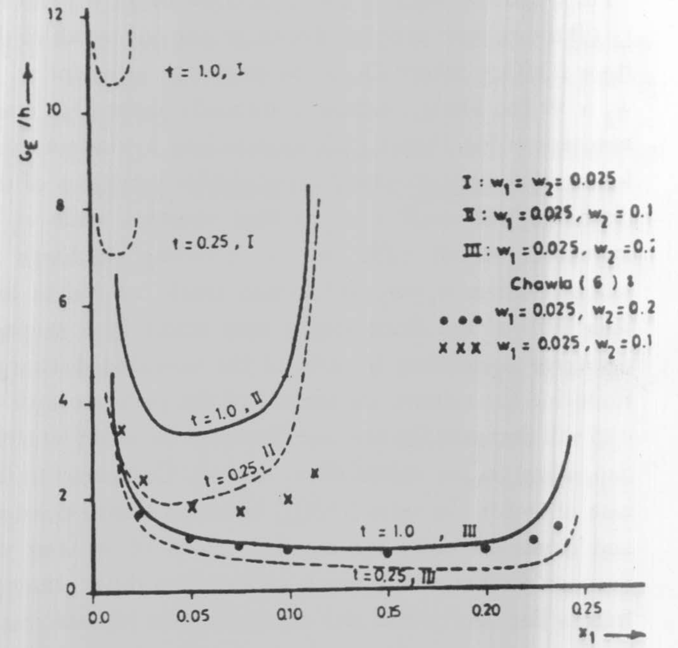

Figure 10. Effect of the reltive thickness, t, on the exit gradients (ℓ_1 = ℓ_2 = 0).

If the U/S crack and/or the D/S one occurs at some distance from the floor, the total relative length ℓ_t of the impervious surface between them will increase resulting in a gradual reduction of the average exit gradient, as shown in Figure (11). Almost the same percentage reduction is obtained for different crack widths, for the same increase of ℓ_t.

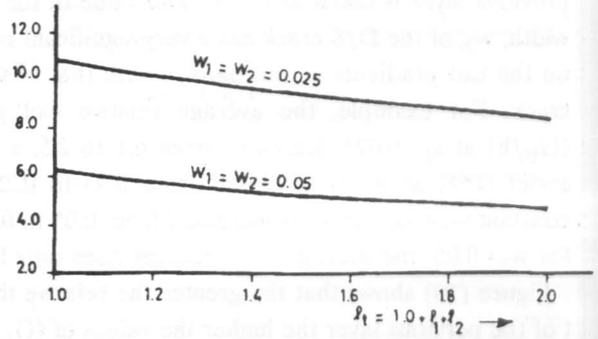

Figure 11. Effect of the total distance between the two cracks on the exit gradients. (t=1.0).

c- Seepage discharge

For a specific value of the U/S crack width w_1, and for t=1.0 the relative seepage discharge per unit width of the floor (Q/Q_∞), where Q_∞ is the discharge value for $w_1 = w_2 = \infty$ (no lining), increases gradually as the D/S crack becomes wider, Figure (12), approaching a constant value for $w_2>0.5$, approximately. Very similar variations of the discharge will result if w_2 is kept constant while w_1 is increased. About 92% of the no-lining discharge is attained when $w_1=w_2=0.5$. Small cracks of widths less than 0.01 of the floor length may result in a seepage discharge amounting to 40% of the no-lining discharge. Reducing the relative thickness t of the pervious layer to 0.25 will decrease the seepage discharge by about 40-60%, depending on the values of w_1 and w_2. Compared to the case of t=1.0, the ratio (Q/Q_∞) becomes almost constant and closer to unity for smaller values of w_1 and w_2. Seepage discharges are much greater than those obtained from a line source-line sink representation [4], whereas a good agreement has been obtained with values calculated by the following formula, which is directly derived from equation (18) of Chawla's solution [6] by considering point F at w' and ζ planes:

$$q/(kH) = K'/K \qquad (9)$$

where K and K' are complete elliptic integrals of first kind [6].

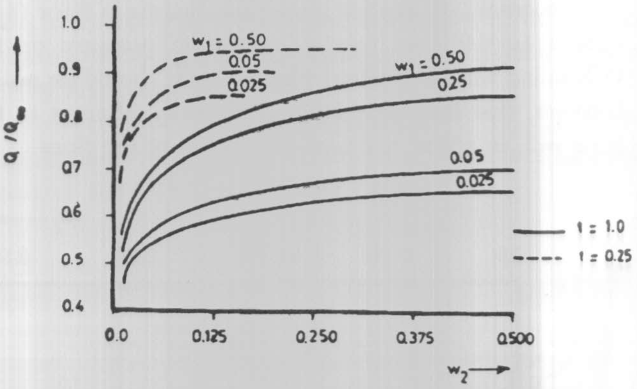

Figure 12. Effect of w_1, w_2 and t on the relative seepage discharge (Q_∞/KH=0.531,0.205, for t=1.0 and 0.25, respectively).

CONCLUSIONS

The problem of seepage under a simple floor constructed in a lined canal due to limited upstream and downstream cracks has been numerically analyses using the boundary element method. Very good agreement with the analytical, conformal mapping solution by Chawla [6] is obtained. The present study clearly indicates that:

1- Substantial increase in the uplift pressures on the floor may result from narrow U/S and D/S cracks, particularly if the D/S crack is far from the toe of the floor.
2- Exeessive exit gradients will develop for narrow D/S cracks.
3- Both of the uplift pressures and the exit gradients are reduced for smaller thicknesses of the pervious layer beneath the floor.
4- The D/S lining will be subjected to net uplift pressures which increase with the widening of the U/S crack and/or narrowing of the D/S one.
5- Cracks of widths as small as 0.01 of the floor length may produce a seepage discharge amounting to 40% of that for unlined canal.

Notations

The following symbols are used in this paper:

B	impervious floor length;
G_E	exit gradient;

H	effective head;
h	H/B = relative effective head;
k	coefficient of permeability;
L_1, L_2	distances between floor edges and upstream and downstream cracks, respectively;
ℓ_1, ℓ_2	L_1/B and L_2/B, relative distances;
$\ell_t = 1.0 + \ell_1 + \ell_2$	relative total impervious length;
p	pressure at any point in the flow domain;
Q	seepage discharge per unit width of the floor; $Q_\infty = Q$, for $\ell_1 = \ell_2 = \infty$ (unlined canal);
T	depth of the permeable layer;
t	T/B = relative depth;
U_1, U_2	net uplift pressures on the floor and on the downstream lining, respectively;
$u = (p/\gamma) + z$	potential head at any point in the flow domain;
W_1, W_2	widths of the upstream and downstream cracks, respectively;
w_1, w_1	W_1/B and W_2/B, relative widths
z	position head, measured upward from the downstream water level;
γ	unit weight of water.

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