A FINITE ELEMENT STUDY OF PUMPING FROM A PARTIALLY PENETRATING WELL IN AN UNCONFINED AQUIFER

Hosam El-Din Mohamed Moghazi

Irrigation and Hydraulics Department, Faculty of Engineering,
Alexandria University, Alexandria, Egypt.

ABSTRACT

Partially penetrating wells are common in areas where the aquifer is relatively thick. The water percolates upward to the well causes an additional head loss. A finite element model has been designed to investigate the effect of partial penetration of a water well in an isotropic unconfined aquifer on the mechanism of seepage close to the well screen. Close to the well the vertical components of seepage are significant and cause a seepage face to develop at the well above the level of drawdown in the well. This persists even after steady state condition have been established. The computed well yield and seepage face height are presented graphically in chart forms suitable for practical use.

NOTATIONS

{F}	global nodal force vector
h	elevation of the free surface at a radial distance r from the well axis
h_w	water depth in fully penetrating well
h_s	seepage face height measured from the impermeable bed
H_o	saturated thickness of an unconfined aquifer
k	permeability coefficient
[K]	global stiffness matrix
m	difference in water levels outside and inside the well
P	penetration depth of well to unconfined aquifer
q_o	specified amount of flux per unit length of boundary
Q	yield from fully penetrating well
Q_p	yield from partially penetrating well
Q_{seep}	flow enters well along the seepage face
r	radial distance from well axis
r_w	well radius
R_o	well radius of influence
s	distance from well bottom to the impermeable bed
s_s	water depth outside well measured from the original water table
s_w	drawdown of water in well
S_A,S_B	line boundaries on which boundary conditions types (A) and (B) are imposed respectively
t	depth of water in partially penetrating well
Φ	potential function or total potential head
{Φ}	global vector of unknown head
{Φ_p}	prescribed value of head.

INTRODUCTION

There are many instances where the depth of the aquifer is so large that full penetration is not justified from an economic point of view. In such cases the water that enters the well must percolate upward from the material situated below the bottom of the well. Thus an upward vertical movement of groundwater is produced as shown in Figure (1). The water percolates upward to the well necessarily moves a greater distance to the well than if it had percolated horizontally and thus more head is lost. The effect of this upward percolation will be reflected in the drawdown of the water level close to the well by an increase in drawdown over that which would occur had the well completely penetrates the aquifer if they are pumped at the some rate. Even they are operated at the same pumping level a partially penetrating well will discharge less than a completely penetrating well.

Various studies have been carried out by various investigators to find corrections to the drawdown at the well and the well yield due to partial penetration. Most of these studies were developed for confined aquifers such as Hsiung [7], Kozeny [1], Hantush [4,5], Franke [3], Witherspoon [16], Sternberg [14], Huisman [8] and Power [12]. Some other studies were developed analytically to study unsteady flow in unconfined aquifer such as Dagan [2], Kipp [9] and Neuman [11]. They treat the well as a line sink to determine the hydraulic properties of the aquifer. According to the author knowledge, few studies were developed to investigate the effect of partial penetration of a water well in an unconfined aquifer on the mechanism of seepage at the steady state condition. Forchheimer [1] and Kozeny [6] developed formulae to determine the yield from a partially penetrating well. They neglected the seepage face at the well. The formula of Forchheimer takes the form

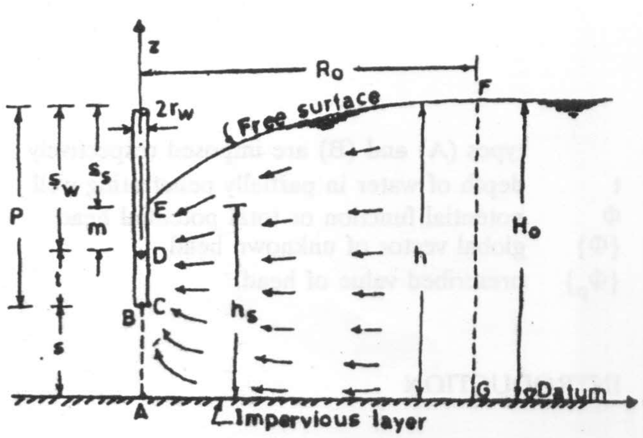

Figure 1. Definition sketch of the numerical model.

$$\frac{Q_p}{k\pi} = \frac{H_o^2 - (t+s)^2}{\ln\left(\frac{R_o}{r_w}\right)} \sqrt{\frac{t+0.5r_w}{t+s}} \sqrt[4]{\frac{2s+t}{t+s}} \quad (1)$$

and that of Kozeny

$$\frac{Q_p}{k\pi} = \frac{(H_o-s)^2 - t^2}{\ln\left(\frac{R_o}{r_w}\right)} \left[1 + 7\left(\frac{r_w}{2(H_o-s)}\right)^{\frac{1}{2}} \cos\pi \frac{(H_o-s)}{2H_o}\right] \quad (2)$$

where

Q_p well yield from a partially penetrating well
k permeability coefficient
H_o saturated thickness of the aquifer
t depth of water in the well
s distance from well bottom to the impermeable bed
R_o well radius of influence
r_w well radius.

Using the relaxation method, Boreli [1] developed a formula to determine the well yield, taking into consideration the seepage face at the well :

$$\frac{Q_p}{k\pi} = \frac{\left[(H_o-s)^2 - t^2\right]}{\ln\left(\frac{R_o}{r_w}\right)} \left[1 + (0.30 + 10\frac{r_w}{H_o} \sin \frac{1.8s}{H_o}\right] \quad (3)$$

which is reduced to the Dupuit equation for a fully penetrating well. In the same study, Boreli developed an empirical formula to determine the elevation of the free surface close to the well :

$$H_o - h = \frac{Q_p\, P^*\, \ln\left(10\,\frac{R_o}{H_o}\right)}{\pi\, k\, H_o\left[1 - 0.8\left(\frac{s}{H_o}\right)^{1.5}\right]} \quad (4)$$

where h = elevation of the free surface at a radial distance r from well axis.

$$P^* = C_x + \Delta C \quad (5)$$

$$C_x = 0.13 \ln \frac{R_o}{r} - 0.0123 \ln^2 \frac{R}{10\,r} \quad (6)$$

and

$$\Delta C = \frac{s}{h}\left[\left(\frac{1}{2.3}\ln\frac{R_o}{10r}\right)\left(1.2\frac{s}{H_o} - 0.48\right) + 0.113\ln\frac{2.4H_o}{R_o}\ln\frac{R_o}{34r}\right] \quad (7)$$

The seepage face height, h_s, at the well can be obtained by replacing r by r_w in equations (4) to (6). It should be noted that Boreli derived equations (3) to (7) based on the assumption that $R_o = 112\, r_w$ which put a serious restriction to their use. Moreover, he stated that Equation (4) is valid only for $s/H_o < 0.6$.

The main objectives of this study are to determine, numerically using the finite element method, the seepage face height and the well yield for a wide range of drawdowns in the well and a range of well penetration in the steady state condition. Attention is also paid to the quantity of flow entering the well along the seepage face. The results will be presented in non-dimensionally chart forms suitable for practical use.

ASSUMPTIONS

The idealized aquifer chosen for the study may be described as follows (1) The aquifer is composed of homogeneous and isotropic unconfined aquifer and it is of sufficient areal extent so that the effects of boundaries can be neglected. (2) Steady state conditions are established over a large area around the well. (3) The soil is fully saturated and the compressibility of both water and soil are neglected. (4) The effect of capillary flow in the zone above the phreatic surface is neglected.

THEORETICAL CONSIDERATIONS

The general governing partial differential equation for steady state flow in an isotropic and homogeneous porous continuum can be described as

$$\frac{\partial}{\partial x}\left(k\frac{\partial \phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial \phi}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial \phi}{\partial z}\right)=0 \quad (8)$$

where Φ is the potential head and k is the permeability coefficient. Since the domain and the flow are symmetrical about the well axis the cartesian coordinates in Equation (8) are transformed to cylindrical coordinates (r,z). Then the appropriate flow equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k\,r\frac{\partial \phi}{\partial r}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial \phi}{\partial z}\right)=0 \quad (9)$$

The following two boundary conditions for equation (9) are generally encountered in groundwater flow :

(A) Specified head boundary condition, where the head to be specified at a nodal point on the boundary S_A

$$\phi = \phi_P \quad (10)$$

where Φ is the potential head and Φ_p is the prescribed head.

(B) Specified flux boundary, where a specified amount of flux q_o flows into the domain per unit length of boundary S_B

$$k\frac{\partial \phi}{\partial n}+q_o=0 \quad (11)$$

where $\partial/\partial n$ is the outward pointing normal derivative to the boundary and q_o is specified discharge into the flow domain per unit length of the boundary.

By applying the Galerkin residual approach and the Green-Gauss theorem [17] to equations (10) and (12) yields a set of simultaneous equations

$$[K]\,\{\Phi\} = \{F\} \quad (12)$$

where [K] is the global stiffness matrix, $\{\Phi\}$ is the global vector of unknown head to be determined and {F} is the global nodal force vector. From Eq. (12), the final solution can be obtained after applying the boundary conditions. Details of the finite element equations are beyond the purpose of this research and found in many text books such as Zienkiewicz [17].

BOUNDARY CONDITIONS

Referring to the geometry of the model in Figure (1), the boundary conditions associated with the flow towards a partially penetrating well are as follows :

Water boundaries

These constitute the faces BCD and FG. The total potential head, Φ, along these faces equal to the elevation of the water face above the datum :

$$\Phi = t + s \text{ (along BCD)}, \quad \Phi = H_o \text{ (along FG)} \quad (13)$$

Phreatic surface

Along EF the total head equal the elevation above the datum and the flow across this boundary is nil :

$$\left.\begin{aligned}\phi &= z\\ \frac{\partial \phi}{\partial n} &= 0\end{aligned}\right\}\text{ (along EF)} \quad (14)$$

Seepage face

At which the pressure is atmospheric and the total head equal the elevation head :

$$\phi = z \quad \text{(along DE)} \tag{15}$$

Impervious boundaries

Along the surface AG, $\partial\Phi/\partial n = 0$. Also, along the boundary AB, and due to symmetry condition $\partial\Phi/\partial n = 0$.

MODEL DIMENSIONS AND ANALYSIS

Values of the saturated aquifer thickness, H_o, and the well radius, r_w, were chosen equal to 50.0 and 0.15 m respectively. The permeability coefficient of the soil, k, was assumed equal to 0.001 m/sec. Four cases of well penetration ratio, P/H_o, are studied (P/H_o = 1, 0.8, 0.6, 0.4) where P is the well penetration depth to the aquifer. For each value of P/H_o different values of the drawdown ratio, t/P, are studied (t/P = 0.8, 0.6, 0.4, 0.2, 0.0). The steady state well radius of influence, R_o, is estimated using the Sichart [13] empirical formula which has been recommended by many authors (Power [12] and Leonard [10]) :

$$R_o = 3000\, s_w \sqrt{K} \tag{16}$$

where

s_w = drawdown of water in well (m)

k = permeability coefficient (m/sec)

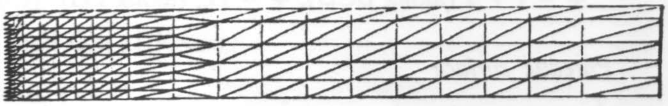

Figure 2. A finite element mesh for a partilly penetrating well No. of elements = 405. No. of nodes = 238.

Figure (2) shows a typical finite element mesh used to investigate the problem using axisymmetric triangular elements. The Taylor and Brown [15] technique was used to determine the position of the free surface, in which the vertical height of each node is adjusted until the condition of zero gauge water pressure is fulfilled.

RESULTS AND DISCUSSIONS

1. Well Yield (Q_p)

Figure (3) shows the relationship between Q_p/kH_o^2 and t/P for different penetration ratios, P/H_o. It can be seen that the well yield decreases as the penetration ratio decreases. A comparison between the results and the corresponding well yield calculated by the Forchheimer, Kozeny and Boreli equations are listed in Table (1). It can notice that the finite element results agree well with the results of Forchheimer and Kozeny rather than that obtained by the Boreli equation. This is mainly attributed to the assumption made by Boreli in deriving his formula, where the well radius of influence was assumed equal to 112 well radius and independent on the drawdown in the well. This contradicts the fact that the radius of influences increases as the increase of drawdown in the well. This put a serious limitation to the use of Boreli equation. It can also notice from Table (1) that the Forchheimer equation (Eq. 1) fails to estimate the well yield accurately at t = 0.0 when the well is fully drained. This is due to the great influence of s in the second term of the right hand side of his equation.

Table 1. Comparison between the well yield obtained by the finite element methods and others.

P/Ho	t/p	Q(FEM) m3/sec	Q(Forchheimer) m3/sec	Q(Kozeny) m3/sec	Q(Boreli) m3/sec
0.80	0.8	0.276	0.251	0.276	0.210
	0.6	0.462	0.411	0.450	0.350
	0.4	0.590	0.510	0.570	0.440
	0.0	0.687	0.080	0.640	0.500
0.60	0.8	0.177	0.175	0.167	0.124
	0.6	0.310	0.284	0.273	0.203
	0.4	0.400	0.344	0.343	0.255
	0.0	0.475	0.050	0.388	0.290
0.40	0.8	0.094	0.103	0.082	0.058
	0.6	0.171	0.170	0.135	0.095
	0.4	0.217	0.198	.169	0.119
	0.0	0.271	0.030	0.190	0.133

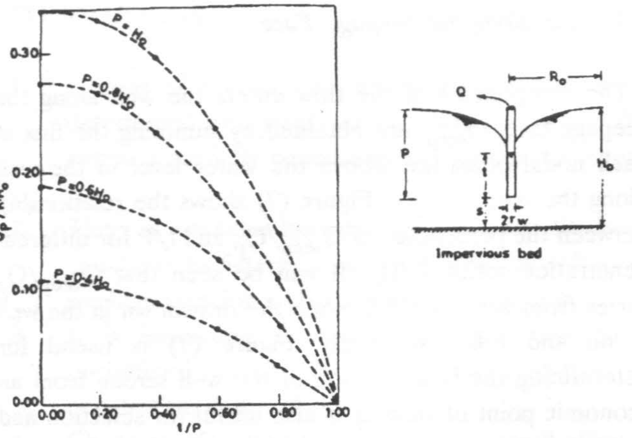

Figure 3. Well yield from a partially penetrating well (H_o = 50.00m, r_w=0.15m).

In order to show the effect of varying the well radius, r_w, on the well yield various values of r_w were examined for various penetration ratios and the results are shown in Figure (4). It can be seen that doubling r_w from 0.15 m to 0.30 m results in only 7% increase in Q_p. Thus, it can conclude that r_w does not greatly affect the well yield within the range of the studied variables. This agrees with the Dupuit [11] equation to determine the yield, Q, from a fully penetrating well :

$$Q = \frac{\pi k \left[H_o^2 - (t+s)^2\right]}{\ln\left(\frac{R_o}{r_w}\right)} \tag{17}$$

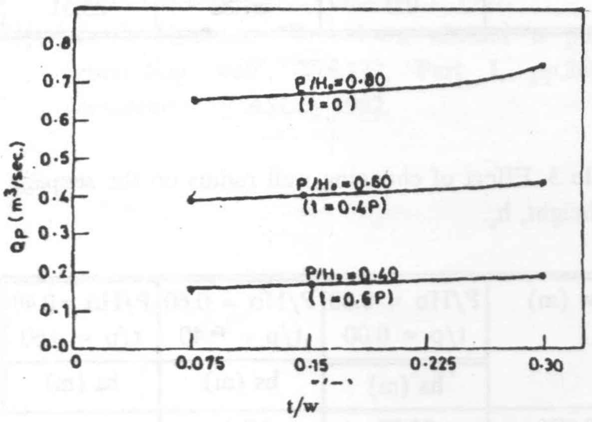

Figure 4. Variation of well yield well radius.

where r_w appears as a log function and changes in the well radius do not result in proportionate changes in Q.

2. The Seepage Face

Figure (5) shows the relationship between m/(t+s) and t/P for different penetration ratios, P/H_o, where m is the difference in water levels outside and inside the well and t+s represents the water level in the well (see Figure (1)). It can be seen that m increases as the well penetration ratio or drawdown in the well increases and reaches a maximum at t= 0.0 when the well is fully drained. However, Figure (5) gives an infinity value for m when the well is fully penetrating the aquifer and becomes completely drained (s = 0 and t = 0). Thus, the results are presented in a different way as shown in Figure (6). It shows the relationship between s_s/s_w and t/P for different values of P/H_o, where s_s and s_w represent the water depth outside and inside the well measured from the original water table respectively (see Figure (1)). It can be noticed from Figure (6) that at small drawdown ratio t/P the effect of penetration ratio P/H_o on the value of s_s/s_w is nearly negligible. Even at deep drawdown, this effect is very small. It can also notice that the drawdown outside the well s_s cannot decrease than 0.45-0.50 the distance between the original water level and the well bottom even when the well is completely drained (t = 0). This means that it is not possible to drawdown the water table in an unconfined aquifer by more than 0.45-0.50 the penetration depth of the well.

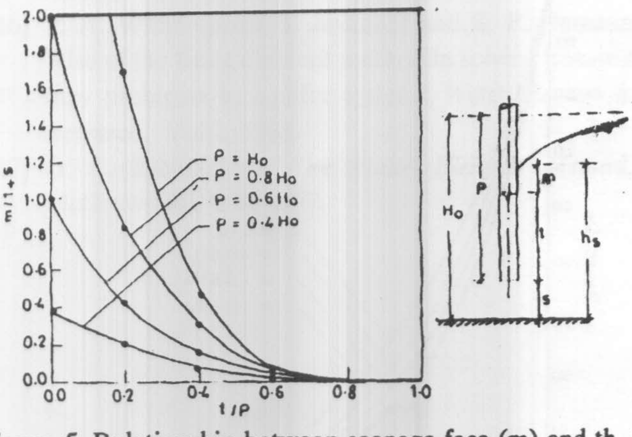

Figure 5. Relationship between seepage face (m) and th drawown in the well. (H_o = 50, r_w = 0.15m).

In order to show the effect of the Boreli assumption in the derivation of his equation (Eq.4) on the value of the seepage face height, h_s, a comparison between the finite element results and the corresponding values using Eq. (4)

is listed in Table (2). It can point out that values of h_s calculated by the Boreli equation increase as the increase of the drawdown in the well. This makes a contradiction to the fact that elevation of the free surface decreases with the increase of the drawdown in the well and indicates the serious limitation of the use of the Boreli equation.

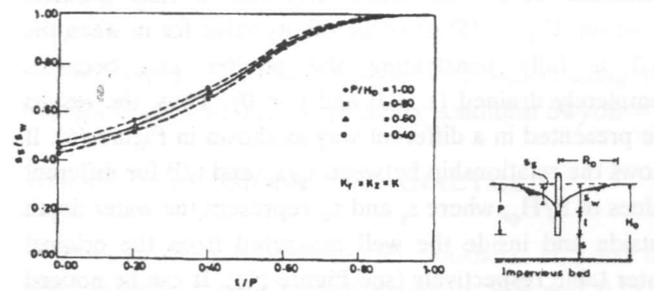

Figure 6. Relationship between the drawdowns outside and inside the well (H_o = 50.00m, r_w = 0.15m)

The effect of r_w on the seepage face height is studied by changing the well radius and the corresponding seepage face heights are obtained as listed in Table (3). It can point out that very slight differences are noticed in the seepage face heights due to the change of the well radius within the range of the studied variables.

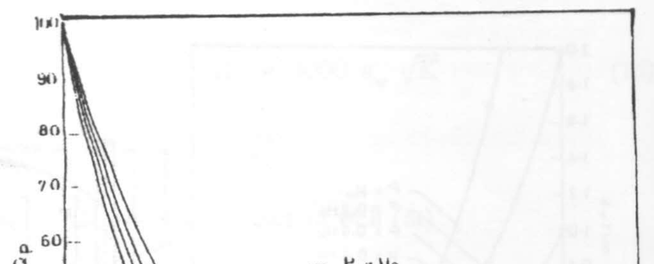

Figure 7. Relationship between the flux alonge the seepage face and well yield (H_o = 50 m, r_w = 0.15 m).

3. *Flux along the Seepage Face*

The components of the flow enters the well along the seepage face, Q_{seep}, are obtained by summing the flux at each nodal point lays above the water level in the well along the seepage face. Figure (7) shows the relationship between the percentage of Q_{seep}/Q_p and t/P for different penetration ratios P/H_o. It can be seen that Q_{seep}/Q_p varies from zero to 100% when the drawdown in the well is nil and full respectively. Figure (7) is useful for determining the best position of the well screen from an economic point of view. It is also useful for selection and design of the pumping equipment.

Table 2. Comparison between the Boreli equation and the finite element method to determine the seepage face height.

P/Ho	t/p	hs (Boreli) (m)	hs (FEM) (m)
0.6	0.8	48.60	44.17
	0.6	49.46	39.36
	0.4	50.93	38.30
0.8	0.8	47.94	42.11
	0.6	48.29	37.41
	0.4	49.34	35.12
1.0	0.8	43.24	40.30
	0.4	34.30	29.40
	0.0	31.82	25.51

Table 3. Effect of changing well radius on the seepage face height, h_s.

rw (m)	P/Ho = 0.80 t/p = 0.00	P/Ho = 0.60 t/p = 0.40	P/Ho = 0.40 t/p = 0.60
	hs (m)	hs (m)	hs (m)
0.075	32.70	37.72	43.10
0.150	32.60	37.70	43.09
0.300	30.30	37.55	42.88

CONCLUSIONS

1 Groups of non-dimensional charts are obtained to determined well yield, seepage face height and flow enters a partially penetrating well along the seepage face.

2 There is a serious limitation to the use of the Boreli equations to determine the well yield as well as the seepage face for a partially penetrating well.

3 It is impossible to lower the water level in an unconfined aquifer by more than 0.45-0.50 the penetration depth of the well.

4 Changing of well radius yields a very small effect on the well yield and the seepage face height within the range of the studied variables.

5 The results of the investigation demonstrate how the finite element method can be used to prepare design charts suitable for use in water well design studies.

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