SEEPAGE UNDERNEATH HYDRAULIC STRUCTURES ON DRAINED STRATA

Rabiea I. Nasr
Department of Irrigation and Hydraulics, Faculty of Engineering,
Alexandria University, Alexandria, Egypt.

ABSTRACT

An exact solution, using conformal mapping technique, for the problem of seepage beneath a hydraulic structure having two unequal cut-offs and founded on two pervious strata has been obtained. The lower stratum has a higher permeability than the upper one. New formulas to calculate the seepage characteristics (uplift pressures beneath the structure floor and exit gradients along the downstream bed) are derived. A computer program was written to calculate such seepage characteristics for given boundary conditions, using the derived equations. As example of calculations, the effect of the artesian head variation on the seepage characteristics is studied.

INTRODUCTION

The stability of hydraulic structures founded on permeable soil has to be insured against uplift pressures and exit gradients. Many researches were carried out, using conformal mapping technique, to study these seepage characteristics for different shapes of structures floors [2,6,7 and 8]. In such studies the structures were founded on a homogeneous isotropic pervious bed extending downward to infinity or to finite depth.

The majority of Egyptian soil is composed of a succession of layers of different permeabilities. A more pervious stratum underlying the soil on which the structure rests, is considered an actual problem for the Egyptian irrigation engineers. For such situation, different studies were also carried out, using the same technique, for the case of single flat floor [7]; a single cut-off [7]; two flat floors with a single cut-off [3], and two structures with two cut-offs [4].

In the present paper a solution has been obtained, using conformal mapping technique, for the problem of seepage beneath a single structure having two unequal cut-offs, and founded on two pervious strata. The lower stratum is of higher permeability than the upper one. A formula to calculate the uplift pressures acting along the structure floor is derived. Also, formula to calculate the values of exit gradients along the downstream bed is obtained.

For given boundary conditions, the seepage characteristics can be calculated using the derived formulas. The derived formulas are used to evaluate the affect of the seepage characteristics due to the variation of both the artesian head value and the depth of the highly pervious layer surface below the ground level.

THE PROBLEM AND THE METHOD OF SOLUTION

Figure (1-a) shows a schematic sketch of the problem. It also shows the flow domain in the Z-plane, where $Z = X + iY$. B is the total length of the hydraulic structure floor, ACEFB. The floor has two unequal cut-offs. The upstream cut-off, CDE, extends in the pervious foundation to a depth d_1. It is located at a distance b_1 from the upstream edge of the floor. The downstream cut-off, FGB, is located at the end of the floor and extends to a depth d_2. The distance between the two cut-offs equals b_2. The structure is founded on a homogeneous isotropic pervious layer of thickness T, overlying another isotropic homogeneous formation of infinite thickness. The coefficient of permeability of the lower layer is considerably larger than that of the upper one. The effective head on the floor equals $(H_1 - H_2)$. It is assumed that the head has a constant value equals H_o at the boundary of the two strata, so that $H_1 > H_o > H_2$.

The velocity potential, Φ, of the flow through the upper pervious medium is satisfied the two dimensional Laplace equation:

$$\nabla^2 \Phi = 0.0 \qquad (1)$$

In the present case $\Phi = -kH$, in which H = the head pressure, and k = the coefficient of permeability.

As shown in Figure (1-a), the subsurface contour of the floor, ACDEFGB, forms the inner boundary of the flow and therefore it can be taken as the first streamline, $\Psi = 0.0$, in which Ψ = the stream function. The upstream bed

M_1A; the downstream bed BM; and the surface of the lower pervious layer, M_1M, are all equipotential lines. Without loss of generality we shall take $\Phi_{M_1A} = -KH_1$, $\Phi_{BM} = -KH_2$, and $\Phi_{M_1M} = -kH_o$.

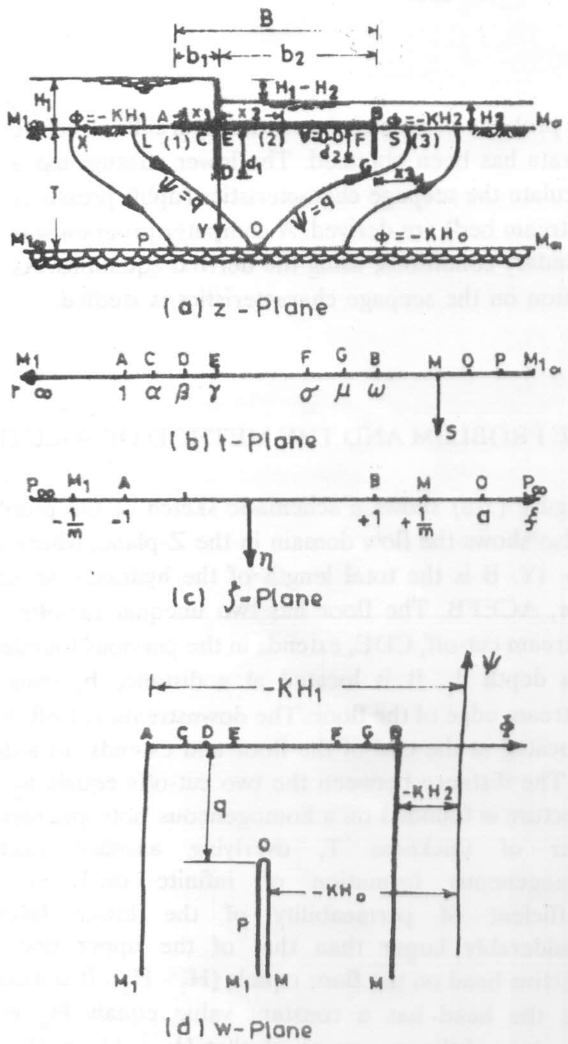

Figure 1. Transformation layout.

As shown in Figure (1-a). Part of the seepage flow, q, coming from the upstream side, LA, is drained through the upstream part of the downstream bed, BS. The seepage flow, coming from the remaining length of the upstream bed, M_1L, is drained into the lower pervious strata through the length M_10. Its remaining length, OM, works as an inlet face. The seepage flow coming from the length, OM, is drained through the length, SM, of the downstream bed.

The complex potential is presented by $W = \Phi + i\Psi$. For this particular case, conditions in the W-plane are shown in Figure (1-d). A complete solution of the problem is obtained if the relation $W = F(Z)$ is derived. To obtain the mapping from the Z-plane to the W-plane, they have been transformed onto the lower half of the same semi-infinite t-plane using the schwarz-christoffel transformation. The transformation of the W-plane onto the t-plane has been obtained through an auxiliary ζ-plane with the help of bilinear transformation.

The following relations are thus obtained:

$$Z = F_1(t) \tag{2}$$

$$W = F_2(\zeta) \tag{3}$$

and

$$\zeta = F_3(t) \tag{4}$$

Combining Eqs. 2,3, and 4

$$Z = F_1(t) = F_1 F_2^{-1} F_3^{-1}(W) \tag{5}$$

and

$$W = F_2 F_3 F_1^{-1}(Z) \tag{6}$$

in which $Z = X+iY$; $t = r+is$; $\zeta = \xi + i\eta i$ and $W = \Phi + i\Psi$

THEORETICAL SOLUTION

First operation $Z = F_1(t)$

In this operation the profile of the hydraulic structure in the Z-plane is transformed onto the real axis of the t-plane. On the t-plane points M, A and M_1 are placed at 0, +1, $\pm\infty$, respectively. Points C, D, E, F, G and B lie at α, β, γ, σ, μ and ω, respectively.

The Schwarz-Christoffel transformation giving the mapping described is [6]:

$$\frac{dZ}{dt} = N_1 \frac{(t-\beta)(t-\mu)}{t\sqrt{(t-\alpha)(t-\gamma)(t-\sigma)(t-\omega)}} \tag{7}$$

in which N_1 is a complex constant

Integration of Eq. 7 in portion CA from α to t gives [5]:

$$\frac{X_1}{N_1 g_1} = (\alpha-\gamma)[\Pi(\theta_1,n_1,r_1) - \frac{\mu\beta}{\alpha\gamma}\Pi(\theta_1,n_2,r_1)] + C_1F(\theta_1,r_1) \tag{8}$$

in which $\prod(\theta_1, n_1, r_1)$ and $\prod(\theta_1, n_2, r_1)$ are elliptic integrals of the third kind; $F(\theta_1, r_1)$ is an elliptic integral of the first kind; and

$$\theta_1 = \sin^{-1}\sqrt{\frac{(\gamma - \omega)(t - \alpha)}{(\alpha - \omega)(t - \gamma)}} \tag{9}$$

$$n_1 = \frac{(\alpha - \omega)}{(\gamma - \omega)} \tag{10}$$

$$n_2 = \frac{\gamma(\alpha - \omega)}{\alpha(\gamma - \omega)} \tag{11}$$

$$r_1 = \sqrt{\frac{(\gamma - \sigma)(\alpha - \omega)}{(\alpha - \sigma)(\gamma - \omega)}} \tag{12}$$

$$C_1 = (\gamma - \beta - \mu + \frac{\beta\mu}{\gamma}) \tag{13}$$

$$g_1 = \frac{2}{\sqrt{(\alpha - \sigma)(\gamma - \omega)}}; \tag{14}$$

t is the coordinate of any point (1) lies on CA, $(1 \geq t > \alpha)$, and X_1 is the horizontal distance for this point measured from point C.

At point A, $X_1 = b_1$, and $t = 1$, therefore

$$\frac{b_1}{N_1 g_1} = (\alpha - \gamma)[\Pi(\theta_A, n_1, r_1) - \frac{\mu\beta}{\alpha\gamma}\Pi(\theta_A, n_2, r_1)] + C_1 F(\theta_A, r_1) \tag{15}$$

in which

$$\theta_A = \sin^{-1}\sqrt{\frac{(\gamma - \omega)(1 - \alpha)}{(\alpha - \omega)(1 - \gamma)}} \tag{16}$$

Integration of Eq. 7 in portion EDC from γ to t gives [5]:

$$-\frac{y_1}{N_1 g_1} = (\gamma - \sigma)[\Pi(\theta_2, n_3, r_2) - \frac{\mu\beta}{\sigma\gamma}.\Pi(\theta_2, n_4, r_2)] + C_2 F(\theta_2, r_2) \tag{17}$$

in which

$$\theta_2 = \sin^{-1}\sqrt{\frac{(\alpha - \sigma)(t - \gamma)}{(\alpha - \gamma)(t - \sigma)}} \tag{18}$$

$$n_3 = \frac{(\alpha - \gamma)}{(\alpha - \sigma)} \tag{19}$$

$$n_4 = \frac{\sigma(\alpha - \gamma)}{\gamma(\alpha - \sigma)} \tag{20}$$

$$r_2 = \sqrt{\frac{(\alpha - \gamma)(\sigma - \omega)}{(\alpha - \sigma)(\gamma - \omega)}} \tag{21}$$

$$C_2 = (\sigma - \beta - \mu + \frac{\mu\beta}{\sigma}); \tag{22}$$

t is the coordinate of any point on EDC, $(\alpha \geq t > \gamma)$, and y_1 is the vertical distance for this point measured from point E.

At point D, $y_1 = d_1$ and $t = \beta$, therefore

$$-\frac{d_1}{N_1 g_1} = (\gamma - \sigma)[\Pi(\theta_D, n_3, r_2) - \frac{\mu\beta}{\sigma\gamma}\Pi(\theta_D, n_4, r_2)] + C_2 F(\theta_D, r_2) \tag{23}$$

in which

$$\theta_D = \sin^{-1}\sqrt{\frac{(\alpha - \sigma)(\beta - \gamma)}{\alpha - \gamma)(\beta - \sigma)}} \tag{24}$$

At point C, $y = 0.0$ and $t = \alpha$, therefore

$$0 = (\gamma - \sigma)[\Pi_3 - \frac{\mu\beta}{\sigma\gamma}\Pi_4] + C_2 F_2 \tag{25}$$

where $F_2 = F(\pi/2, r_2)$ is a complete elliptic integral of the first kind and $\Pi_3 = \Pi(\pi/2, n_3, r_2)$ and $\Pi_4 = \Pi(\pi/2, n_4, r_2)$ are complete elliptic integrals of the third kind.

Integration of Eq. 7 in portion FE from t to γ gives [5]:

$$\frac{X_2}{N_1 g_1} = (\alpha - \gamma)[\frac{\mu\beta}{\alpha\gamma}\Pi(\theta_3, n_6, r_1) - \Pi(\theta_3, n_5, r_1)] + C_3.F(\theta_3, r_1) \tag{26}$$

in which

$$\theta_3 = \sin^{-1}\sqrt{\frac{(\alpha - \sigma)(\gamma - t)}{(\gamma - \sigma)(\alpha - t)}} \tag{27}$$

$$n_5 = \frac{(\gamma - \sigma)}{(\alpha - \sigma)} \tag{28}$$

$$n_6 = \frac{\alpha(\gamma - \sigma)}{\gamma(\alpha - \sigma)} \tag{29}$$

$$C_3 = (\alpha - \beta - \mu + \frac{\mu\beta}{\alpha}); \tag{30}$$

r_1 is given by Eq. 12; t is the coordinate of any point (2) lies on FE ($\gamma > t \geq \sigma$) and X_2 is the horizontal distance for this point measured from point E.

At point F, $X_2 = - b_2$ and $t = \sigma$, substituting these values into Eq. 26 therefore, $\theta_F = \pi/2$; and

$$-\frac{b_2}{N_1 g_1} = (\alpha - \gamma)[\frac{\mu\beta}{\alpha\gamma}\Pi_6 - \Pi_5] + C_3.F_1 \tag{31}$$

where $F_1 = F(\pi/2, r_1)$; $\Pi_5 = \Pi(\pi/2, n_5, r_1)$; and $\Pi_6 = \Pi(\pi/2, n_6, r_1)$.

Integration of Eq. 7 in portion BGF from t to σ gives [5]:

$$-\frac{Y_2}{N_1 g_1} = (\gamma - \sigma)[\frac{\mu\beta}{\gamma\sigma}\Pi(\theta_4, n_8, r_2) - \Pi(\theta_4, n_7, r_2)] + C_1 F(\theta_4, r_2) \tag{32}$$

in which

$$\theta_4 = \sin^{-1}\sqrt{\frac{(\gamma - \omega)(\sigma - t)}{(\sigma - \omega)(\gamma - t)}} \tag{33}$$

$$n_7 = \frac{(\sigma - \omega)}{(\gamma - \omega)} \tag{34}$$

$$n_8 = \frac{\gamma(\sigma - \omega)}{\sigma(\gamma - \omega)} \tag{35}$$

r_2 is given by Eq. 21; C_1 is given by Eq. 13; t is the coordinate of any point on BGF ($\sigma > t \geq \omega$); and y_2 is the vertical distance for this point measured from point B.

At point G, $y_2 = d_2$ and $t = \mu$, therefore

$$-\frac{d_2}{N_1 g_1} = (\gamma - \sigma)[\frac{\mu\beta}{\gamma\sigma}\Pi(\theta_G, n_8, r_2) - \Pi(\theta_G, n_7, r_2)] + C_1 F(\theta_G, r_2) \tag{36}$$

in which

$$\theta_G = \sin^{-1}\sqrt{\frac{(\gamma - \omega)(\sigma - \mu)}{(\sigma - \omega)(\gamma - \mu)}} \tag{37}$$

At point B, $y_B = 0.0$ and $t = \omega$, substituting these values into Eq. 32 therefore $\theta_B = \pi/2$; and

$$0 = (\gamma - \sigma)[\frac{\mu\beta}{\gamma\sigma}\Pi_8 - \Pi_7] + C_1 F_2 \tag{38}$$

in which $F_2 = F(\pi/2, r_2)$, $\Pi_7 = \Pi(\pi/2, n_7, r_2)$; and $\Pi_8 = \Pi(\pi/2, n_8, r_2)$.

Integration of Eq. 7 in portion MB from t to ω gives [5]:

$$\frac{X_3}{N_1 g_1} = (\sigma - \omega)[\frac{\mu\beta}{\sigma\omega}\Pi(\theta_5, n_{10}, r_1) - \Pi(\theta_5, n_9, r_1)] + C_2 F(\theta_5, r_1) \tag{39}$$

in which

$$\theta_5 = \sin^{-1}\sqrt{\frac{(\alpha - \sigma)(\omega - t)}{(\alpha - \omega)(\sigma - t)}} \tag{40}$$

$$n_9 = \frac{(\alpha - \omega)}{(\alpha - \sigma)} \tag{41}$$

$$n_{10} = \frac{\sigma(\alpha - \omega)}{\omega(\alpha - \sigma)} \tag{42}$$

; r_1 is given by Eq. 12; C_2 is given by Eq. 22; t is the coordinate of any point (3) lies on MB ($\omega > t > 0$); and X_3 is the horizontal distance for this point measured from point B.

Substituting $t = re^{i\theta}$; $dt = re^{i\theta}$, $id\theta$ into Eq. 7, as t passes around a semi-circle of small radius at point M, the corresponding change in Z-plane is iT. Thus

$$iT = N_1 \int_{\substack{0 \\ r \to 0}}^{\pi} \frac{(re^{i\theta} - \beta)(re^{i\theta} - \mu)re^{i\theta} id\theta}{re^{i\theta}\sqrt{(re^{i\theta} - \alpha)(re^{i\theta} - \gamma)(re^{i\theta} - \sigma)(re^{i\theta} - \omega)}} \tag{43}$$

from which the following relation is obtained:

$$N_1 = \frac{T}{\pi\beta\mu}\sqrt{\alpha\gamma\sigma\omega} \tag{44}$$

From Eqs. 44 and 14, the following relation is obtained:

$$\frac{T}{N_1 g_1} = \frac{\mu\beta\pi}{2}\sqrt{\frac{(\alpha - \sigma)(\gamma - \omega)}{\alpha\gamma\sigma\omega}} \tag{45}$$

$$N_2 = -\frac{k}{\pi} mm'[(H_1 - H_0) - (H_0 - H_2)] \qquad (63)$$

Substituting the value of N_2 from Eq. 63 into Eq. 58, therefore:

$$\frac{\Phi + kH_2}{k(H_1 - H_2)} = \frac{(1-2R)}{\pi}\tan^{-1}\frac{m}{m'}\sqrt{1-\zeta^2} - \frac{1}{\pi}\tan^{-1}\frac{\sqrt{1-\zeta^2}}{m'\zeta} \qquad (64)$$

where

$$R = \frac{H_0 - H_2}{H_1 - H_2} \qquad (65)$$

Third operation $\zeta = F_3(t)$

The transformation of the ζ-plane onto the t-plane is obtained from the following bilinear transformation:

$$\zeta = \frac{(1-m) - t(1+m)}{m[(1-m) + t(1+m)]} \qquad (66)$$

or

$$t = \frac{(\zeta m - 1)(m - 1)}{(\zeta m - 1)(m - 1) + 2m(\zeta + 1)} \qquad (67)$$

At point B, $t = \omega$, and $\zeta = +1$, therefore

$$+1 = \frac{(1-m) - \omega(1+m)}{m[(1-m) + \omega(1+m)]}$$

or

$$m = \frac{(1+\omega) - 2\sqrt{\omega}}{(1-\omega)} \qquad (68)$$

substituting the value of m from Eq. 68 into Eq. 62, therefore

$$a = \frac{1-\omega}{(1-2R)[(1+\omega) - 2\sqrt{\omega}]} \qquad (69)$$

where R is given in Eq. 65.

UPLIFT PRESSURES

The relative net uplift pressures below the floor ACDEFGB can be determined from Eq. 64 after substituting the value of $\Phi = -kH$, therefore:

$$\frac{H - H_2}{H_1 - H_2} = \frac{2R-1}{\pi}\tan^{-1}\frac{m}{m'}\sqrt{1-\zeta^2} + \frac{1}{\pi}\tan^{-1}\frac{\sqrt{1-\zeta^2}}{m'\zeta} \qquad (70)$$

in which H is the total uplift pressures below the floor, for the case of $H_0 = (H_1 + H_2)/2$, i.e., $R = 0.5$, the relative net uplift pressures along the floor can be calculate from the following relation:

$$\frac{H - H_2}{H_1 - H_2} = \frac{1}{\pi}\tan^{-1}\frac{\sqrt{1-\zeta^2}}{m'\zeta} \qquad (71)$$

The value of t which required to calculate ζ in Eq. 66, depends upon the position of the point under consideration on the floor, for point (1) which lies on the part AC at distance X_1 measured from point C Eqs. 8, 15 give

$$\frac{X_1}{b_1} = \frac{(\alpha-\gamma)[\Pi(\theta_1,n_1,r_1) - \frac{\mu\beta}{\alpha\gamma}\Pi(\theta_1,n_2,r_1)] + C_1F(\theta_1,r_1)}{(\alpha-\gamma)[\Pi(\theta_A,n_1,r_1) - \frac{\mu\beta}{\alpha\gamma}\Pi(\theta_A,n_2,r_1)] + C_1F(\theta_A,r_1)} \qquad (72)$$

in which θ_1 is given by Eq. 9, where $t = t_1$. For known value of X_1/b_1, the unknown value of t_1 can be calculated from Eq. 72 by trial and error, where $1 > t_1 > \alpha$.
For point (2), which lies on the part EF at distance X_2 measured from point E, Eqs. 26 and 31 give

$$\frac{X_2}{b_2} = \frac{(\alpha-\gamma)[\frac{\mu\beta}{\alpha\gamma}\Pi(\theta_3,n_6,r_1) - \Pi(\theta_3,n_5,r_1)] + C_3F(\theta_3,r_1)}{(\alpha-\gamma)[\frac{\mu\beta}{\alpha\gamma}\Pi_6 - \Pi_5] + C_3F_1} \qquad (73)$$

in which θ_3 is given by Eq. 27, where $t = t_2$. For known value of X_2/b_2, the unknown value of t_2 can be calculated from Eq. 73 by trial and error, where $\gamma > t_2 > \sigma$.

EXIT GRADIENTS

For any point lying on the downstream bed, the exit gradient is defined as follows:

$$I_e = \frac{1}{k}\,\mathrm{Im}\left(\frac{dw}{dZ}\right) = \frac{1}{k}\mathrm{Im}\left[\frac{dw}{d\zeta}\cdot\frac{d\zeta}{dt}\Big/\frac{dz}{dt}\right] \qquad (74)$$

Eqs. 7, 44, 57, 63 and 66, after simplifying, give the following relationship:

ANALYSIS

Figures (2) and (3) show that the calculated values of the relative uplift pressures along the floor, $(H - H_2)/(H_1 - H_2)$, and the relative exit gradients along the downstream bed $I_e/[(H_1-H_2)/B]$ are affected by the change of $(H_o-H_2)/(H_1-H_2)$. The figures indicate that the increase of $(H_o-H_2)/(H_1-H_2)$ causes an increase in both uplift pressures and exit gradients. Increasing the value of $(H_o-H_2)/(H_1-H_2)$ by equal increment causes an equal excess in relative pressure at any point between the two cut-offs, and equal excess in the relative exit gradient at any point along the downstream bed. For example, at point E in Figure (2), if the value of $(H_o - H_2)/(H_1 - H_2)$ is increased from 0.0 to 0.20 or from 0.60 to 0.80 , the relative excess in pressure is a constant and equals 0.10 $(H_1 - H_2)$.

Figure (4) shows that the relative net uplift pressures along the floor have a slightly affect by the variation of T/B. The pressures decreased when the ratio T/B is increased. The figure indicates that the rate of reduction in pressures is decreased when the ratio T/B is increased. For example, at point E, if the ratio T/B increases from 0.65 to 1.25 the reduction in head is 0.02 $(H_1 - H_2)$, while the same value of reduction can be occurs when the ratio T/B increases from 2.5 to 5.0. Fig. 5 show that the relative exit gradients along the downstream bed are also affected by the variation of T/B. The relative exit gradients are decreased when the value of T/B is increased. If the value of T/B is more than 2.50 its effect on exit gradients is dissipated.

CONCLUSIONS

An exact solution has been obtained, using conformal mapping technique for the problem of seepage beneath a hydraulic structure founded on two successive pervious strata. The lower stratum is of higher permeability than the upper one. The structure floor has two unequal cut-offs. The downstream cut-off is located at the end of the floor, while the upstream one can be changed at any place along the floor. The depths of the two cut-offs can be varied.

Formulas (70) and (75) are obtained to calculate the relative net uplift pressures along the structure floor and the relative exit gradients along the downstream bed, respectively.

A computer program was written to use the derived formulas for computation of the calculated seepage characteristics. A sample of calculations is presented in the form of curves. For constant structure floor dimensions, the effect of both relative artesian head, $(H_o - H_2)/(H_0 - H_2)$, and the relative depth of the upper pervious layer, T/B, are studied. For a constant value of T/B, the increase of the artesian head $(H_o - H_2)$ value cause a significant increase in both uplift pressures along the floor and downstream exit gradients. Also, for a constant value of $(H_o - H_2)/(H_1 - H_2)$, the uplift pressures and exit gradients are also affected by the variation of the relative depth T/B. If the value of T/B is more than 2.5 its effect will be dissipated.

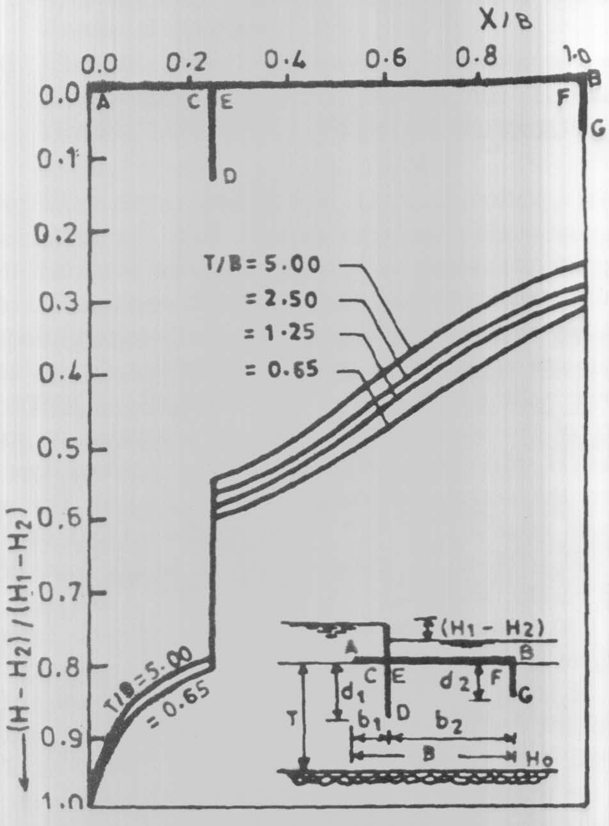

Figure 4. Effect of T/B on the relative uplift pressure along the floor
($b_1/B = 0.25, d_1/B = 0.2, d_2/B = 0.10$ and $T/B = 0.70$)

$$\frac{I_e}{(H_1-H_2)/B} = \frac{B}{T}\,\frac{G_3(2R-1)m'\beta\mu}{2(\beta-t)(\mu-t)}\sqrt{\frac{(\alpha-t)(\gamma-t)(\sigma-t)(\omega-t)}{\alpha\gamma\sigma\omega(G_2^2-m^2G_1^2)}} \qquad (75)$$

in which $G_1 = 1 - m + t\,(1+m)$ (76)

$G_2 = 1 - m - t\,(1 + m)$ (77)

$G_3 = (1 - m)\,(1 - am) - t\,(1 + m)\,(1 + am)$ (78)

For a point (3) lying on the downstream bed at distance X_3 measured from point B, Eqs. 39, and 36, give:

$$\frac{X_3}{d_2} = \frac{(\sigma-\omega)[\frac{\mu\beta}{\sigma\omega}\Pi(\theta_5,n_{10},r_1) - \Pi(\theta_5,n_9,r_1)] + C_2F(\theta_5,r_1)}{(\gamma-\sigma)[\Pi(\theta_6,n_7,r_2) - \frac{\mu\beta}{\gamma\sigma}\Pi(\theta_G,n_8,r_2)] - C_1F(\theta_G,r_2)} \qquad (79)$$

in which θ_5 is given by Eq. 40, where $t = t_3$. For known value of X_3/d_2, the unknown value of t_3 can be calculated from Eq. 79 by trial and error, where $\omega > t_3 > 0$.

COMPUTATIONS AND RESULTS

The equations derived herein have been used for computation of seepage characteristics for different boundary conditions. A computer program is written to calculate these characteristics for different combinations of variables involved. In this paper, the structure floor dimensions are assumed constant for all calculations as follows: $b_1/B = 0.25$, $d_1/B = 0.20$, and $d_2/B = 0.10$. The effect of the artesian head which is depending on its values (H_o-H_2) and the depth of the upper pervious layer, T, on the seepage characteristics are studied. For a constant value of $T/B = 0.70$, the relative artesian head $(H_0 - H_2)/(H_1-H_2)$ is changed from 0.0 to 1.0. Some of these results, which are $(H_o-H_2)/(H_1-H_2) = 0.0$, 0.20, 0.40, 0.50, 0.60, and 0.80 have been presented in the form of curves in Figures (2) and (3). For a constant value of $(H_0 - H_2)/(H_1-H_2) = 0.60$, four specific doubled values of the relative depth, T/B, are taken into consideration which are 0.65, 1.25, 2.5 and 5.0, respectively. For each value of T/B the relative uplift pressures along the floor and the relative exit gradients are also calculated. These results are also presented in Figures (4) and (5).

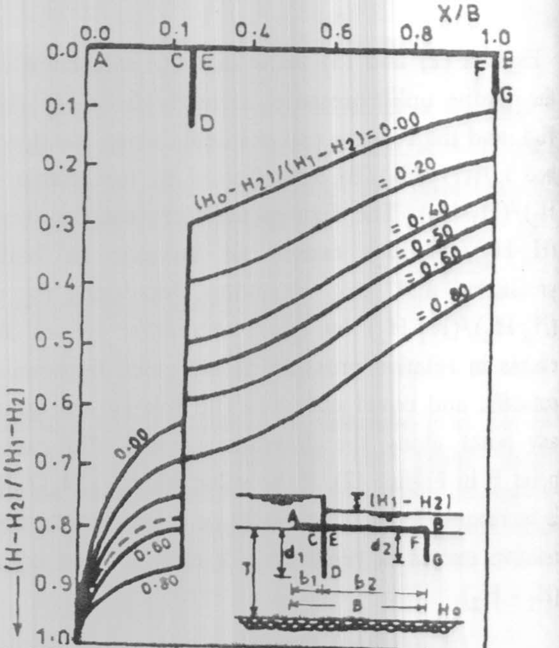

Figure 2. Effect of $(H_o-H_2)/(H_1-H_2)$ on the relative uplift pressures along the floor

($b_1/B=0.25, d_1/B=0.20, d_2/B=0.10$ and $T/B=0.70$)

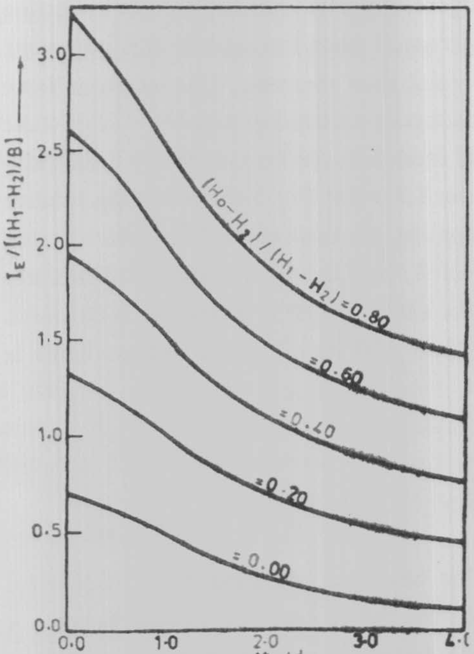

Figure 3. Effect of $(H_o-H_2)/(H_1-H_2)$ on the relative exit gradients downstream the floor

($b_1/B=0.25, d_1/B=0.2, d_2/B=0.10$ and $T/B=0.70$)

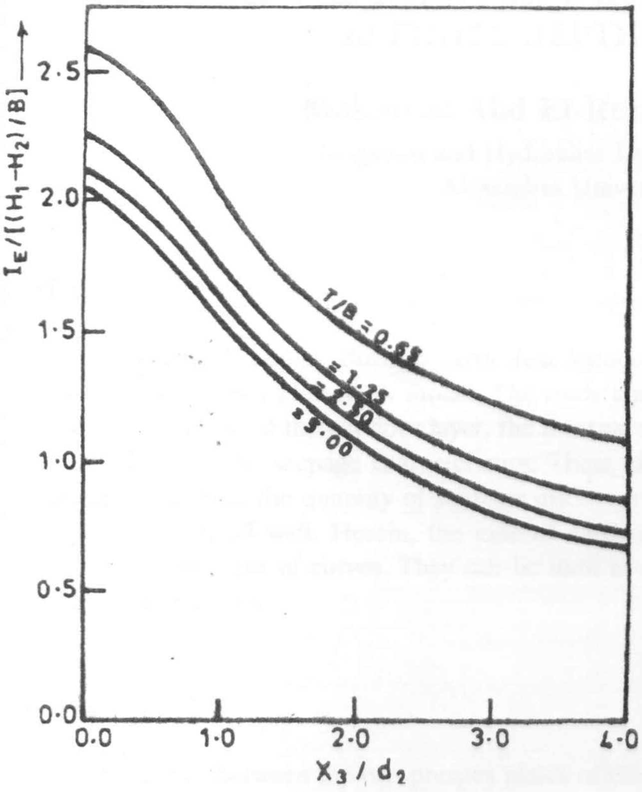

Figure 5. Effect of T/B on the relative exit gradient downstream, the floot the ($b_1/B=0.25, d_1/B=0.20, d_2/B=0.10$ and $T/B=0.70$)

NOTATIONS

a	transformation parameter,
b_1	the length of floor between the upstream cut-off and the upstream edge;
b_2	the length of floor between the two cut-offs;
B	the total length of the floor;
C_1, C_2, C_3	constants;
d_1	the depth of the upstream cut-off;
d_2	the depth of the downstream cut-off;
$F(\theta, r)$	incomplete elliptic integral of first kind;
$F(\pi/2, r)$	Complete elliptic integral of the first kind.
H	the head at any point;
H_0	the head at the boundary of the two pervious strata,
H_1	the head acting along the upstream bed;
H_2	the head acting along the downstream bed;
I_e	exit gradient
I_m	imaginary part of complex function;
K	coefficient of permeability;
m, m=	transformation parameters;
N_1, N_2	complex constants;
R	relative artesian head, $(H_0-H_2)/(H_1-H_2)$;
X_1, X_2, X_3	horizontal distances;
y_1, y_2	vertical depths;
$\alpha, \beta, \gamma, \sigma, \mu, \omega$	transformation parameters;
$\Pi(\theta, n, r)$	elliptic integral of third kind;
$\Pi(\pi/2, n, r)$	Complete elliptic integrals of the third kind;
Φ	velocity potential;
ψ	stream function.

REFERENCES

[1] Abramowitz M., and Stegun I.A., *Handbook of Mathematical Functions with Formulae, Graphs, and Mathematical Tables*, Appl. Math. Series 55, 9th Printing, 1970 US Dept. of Commerce, National Bureau of Standards.

[2] Chawala A.S., "Stability of Structures with Intermediate Filters", Journal of the Hydraulic Division, *ASCE*, Vol. 101, No. HY2, Proc. Paper 11139, February, 1975, pp. 223-241.

[3] Elganainy M.A., "Flow Underneath a pair of Structures With Intermediate Filters On a Drained Stratum, *Applied Mathematical Modelling*, Butterworth, England, Dec. 1986.

[4] Elganainy M.A., Hathoot H., and Nasr R., " Seepage Underneath Structures With Intermediate Filters On Drained Strata", *The Bulletin of the Faculty of Engineering, Alexandria University, Alexandria, Egypt*, Vol. XXV, 1986.

[5] Gradeshtaien E.S., and Rejek E.M., *Tables of Integrals, Sums, Series, and Derivatives*, Gosudarestvennoe Izd., Fiz. -Math. Liter., Moscow, 1963.

[6] Harr M.E., *Ground Water and Seepage*, McGraw-Hill Book Co., Inc., New York, 1962.

[7] Kochina P., *Theory of Ground Water Movement*, Translated by Roger De Wiest, Princeton University Press, Princeton, New Jersey, 1962.

[8] Kumar A., Sing B., and Chawla A.S., "Design of Structures With Intermediate Filters", Journal of the Hydraulic Division, *ASCE*, Vol. 112, No. HY3, Proc. Paper 20436, March 1986, pp 206-219.