QUANTITATIVE ROBUST CONTROL OF TEMPERATURE AND HUMIDITY IN HOT-AND-DRY CLIMATES

A.E. Attia and S.F. Rezeka
Department of Mechanical Engineering, Faculty of Engineering,
Alexandria University, Alexandria, Egypt.

ABSTRACT

In dry and hot weather, both temperature and humidity control should be adopted in buildings, industrial plants, and passenger cars. Because of the unsatisfactory performance of the available classical control system, a quantitative servo controller is synthesized in this paper. The proposed feedback controller is constructed by a state observer related to the plant and utilizing the measured room temperature and humidity as an input. The robustness properties of such a system to model error are discussed. The feasibility of both the synthesized and the conventional control systems are investigated in the presence of uncertainty in both the room internal loads and the outside load. The introduced controller is found capable of adjusting the room temperature and humidity at the design values for all time.

NOMENCLATURE

A,B,C	Matrices defined in equations (8), (9), and (11).
c_p	Constant pressure specific heat, $kJ.kg^{-1}K^{-1}$
c_v	Constant volume specific heat, $kJ.kg^{-1}k^{-1}$
D	Weighing matrix
G	Constant observer gain matrix
g	Known function associated with the nominal system.
H	Specific humidity, kg_{vapour}/ kg_{air}
I	Identity matrix
K	Constant controller gain matrix
k_1,k_2	Controller gains
m	Mass rate, $kg.s^{-1}$
N	Vector defined in equation (13)
Q	Precooler capacity, $kJ.kg^{-1}$
q	Latent heat, $kJ.kg^{-1}$
R	Closed-loop characteristic matrix
RLH	Room latent heat, $kJ.kg^{-1}$
RSH	Room sensible heat $kJ.kg^{-1}$
sup	Supremum
T	Temperature, °C
t	Time, s
U	Input vector
V	Volume, m^3
v	Reference input
W	Washer capacity, kg_v/kg
X	State vector
x	Ratio between supplied air and mass of room air, s^{-1}
y	Output vector
Z	Combined state vector
β_i,β_o	Radius of input, and output sphere respectively
Δ	Variation
ϕ	Relative humidity
λ	Eigen value
ρ_R	Density of room air, $kg.m^{-3}$
ρ_o,ρ_1,ρ_2	Norms defined in equation (14)
τ	Time constant, s.
ζ	Norm defined in equation (14)

SUBSCRIPTS

D	Designed
R	Room
S	Supply

SUPERSCRIPTS

-	Average
^	Observer related

INTRODUCTION

Many industrial centers have been recently developed in tropical regions because of relatively cheap labor and lower taxes. In these regions (such as Arizona, New Mexico and some Third World countries) hot-and-dry climate mandates the control of both temperature and humidity in large spaces such as buildings, industrial and chemical plants, and car painting shops. Such a control is also required for small spaces such as passenger compartments and truck cabs. Figure (1) shows the control arrangement for hot and dry weather. The essential components are the precooler and the air washer. The signal from the room temperature sensor is used to modulate the cooling coil, while the signal of the room specific humidity (or dew point) sensor modulates the damper to adjust the portion of air which passes through the air washer. The applied conventional control systems use output feedback and their design has a qualitative nature [1-5]. In such a design, specific bounds are imposed on the desired response for a specified command input and steady- state accuracy. There is no guarantee that a system designed for these specifications will perform well in the presence of plant uncertainties or external disturbances.

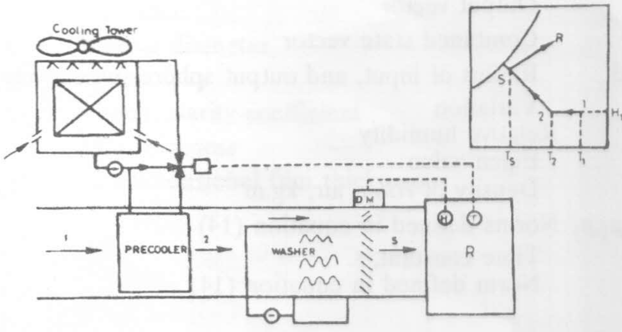

Figure 1. Air conditioning system for hot and dry weather and its psychrometry.

One of the principal applications of feedback in modern control systems is to reduce quantitatively the deviations that may arise in desired system responses due to the uncertainty either in the plant parameters or the external inputs [6-14]. Also the state feedback construction using state estimators or observers [8] ensures a servo problem and increases the robustness of the synthesized controller. Horowitz [7] approached this feedback problem from a frequency domain point of view and has developed "Quantitative Feedback Theory". Usoro [9] studied a similar problem as "a set - theoretic control synthesis". Barmish et al. [10] addressed the stability problems in the presence of unknown, but bounded, uncertainties. Schmitendorf and Barmish [11] studied the linear problem where uncertainties in the model were explicitly identified. They considered asymptotic tracking and the approach was based on the constructive use of Lyapunov theory. Jayasuriya et al. [12, 13 , 14] used the classic problem of asymptotic tracking to motivate the notion of tracking in the sense of input and output sphere.

The object of this paper is to provide quantitative servo design of the controlled air conditioning system in hot dry weather, in the presence of uncertain inputs and disturbances. The feasibility of using such a control system is demonstrated. Its robustness properties are evaluated in terms of the extent to which the design requires a perfect model.

MATHEMATICAL MODEL

Since the air conditioning system for hot and dry weather consists of sensible precooler and adiabatic humidifier (washer), its psychrometric diagram is represented as shown in Figure (1). The precooler is fed with air at point 1 which can be considered either the state of all fresh outside air or the state due to the mixing between recirculated and fresh outside air. The system equations per unit mass of the supplied air (m_s) can be written as:

$$Q = c_p \left[T_1 - T_2 \right] \tag{1}$$

$$W = \left[H_s - H_1 \right] \tag{2}$$

and

$$Wq = c_p \left[T_2 - T_s \right] \tag{3}$$

The room internal loads per unit mass are the room sensible heat, RSH and the room latent heat, RLH. Taking into consideration the room volume V_R, then the internal loads are described as

$$RSH = c_p \left[T_R - T_s\right] + \frac{V_R \rho_R}{m_s} c_v \frac{dT_R}{dt} \tag{4}$$

$$RLH = q[H_R - H_s] + \frac{V_R \rho_R}{m_s} q \frac{dH_R}{dt} \tag{5}$$

Substituting equations (1), (2) and (3) into equations (4) and (5), the system equations can be derived as

$$\frac{dT_R}{dt} = x\frac{c_p}{c_v}[T_1 - T_R] - \frac{x}{c_v}(Q + Wq)$$

$$+ \frac{x}{c_v} RSH \tag{6}$$

$$\frac{dH_R}{dt} = x\left[W + \frac{RLH}{q}\right] - x[H_R - H_1] \tag{7}$$

where $x = m_s / V_R \rho_R$

At the design operating conditions (T_{1D}, H_{1D}, RSH_D, and RLH_D) and the design capacities of the precooler and washer (Q_D, and W_D) $\dot{T}_R$ and $\dot{H}_R$ are vanished. When the system is subjected to fluctuations in the operating conditions, the unsteady deviation in the room temperature and specific humidity can be put in the matrix form as

$$\dot{X} = AX + BU + g(X) \tag{8}$$

where

$$X = [\Delta T_R \quad \Delta H_R]^T, \; U = [\Delta Q \quad \Delta W]^T$$

$$A = \begin{bmatrix} -x\frac{c_p}{c_v} & 0 \\ 0 & -x \end{bmatrix}, \; B = \begin{bmatrix} -\frac{x}{c_v} & \frac{xq}{c_v} \\ 0 & x \end{bmatrix}$$

and

$$g(X) = \begin{bmatrix} x\frac{c_p}{c_v} & 0 & x & 0 \\ 0 & x & 0 & x \end{bmatrix} \begin{bmatrix} \Delta T_1 \\ \Delta H_1 \\ \Delta RSH \\ \frac{\Delta RLH}{q} \end{bmatrix}$$

If the room volume is small, the terms $\frac{c_v}{x}\dot{T}_R$ and $\frac{q}{x}\dot{H}_R$ in equations (4) and (5) can be neglected and the state governing equations will be

$$\dot{X} = AX + BU + g(X) \tag{9}$$

where

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} -\frac{1}{c_p} & \frac{q}{c_p} \\ 0 & 1 \end{bmatrix}, \; U = \begin{bmatrix} \dot{Q} \\ \dot{W} \end{bmatrix}$$

and

$$g(X) = \begin{bmatrix} 1 & 0 & \frac{1}{c_p} & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{T}_1 \\ \dot{H}_1 \\ R\dot{S}H \\ \frac{R\dot{L}H}{q} \end{bmatrix}$$

The system output are the room temperature and relative humidity. The relation between both the air temperature and specific humidity and the relative humidity is nonlinear. However, a linear relation may be developed about the design room temperature (T_R = 24 °C) and the design relative humidity(ϕ_R = 50%) as

$$\phi_R = 5000 H_R - 2.5 T_R + 62.5 \tag{10}$$

Therefore the system output in matrix form is

$$y = CX \tag{11}$$

where

$$y = [\Delta T_R \quad \Delta \phi_R]^T$$

and

$$C = \begin{bmatrix} 1 & 0 \\ -2.5 & 5000 \end{bmatrix}$$

It should be noted that equations (10) and (11) will be used only in the design of the feedback control and not in the plant simulation.

CLOSED - LOOP CONTROL

Although the variations in both the outside load (ΔT_1 and ΔH_1) and in the room load (ΔRSH and ΔRLH) are unknown but they are bounded. The design objective is to synthesize feedback controller which assures every output $y = y_o \pm \beta_o$ for every input with perturbation β_i, no matter what specific value of the changes in the outside load and the inside load may be. Equations (8) and (9) suggest a feedback control loop with a state observer of the form

$$\dot{\hat{X}} = A\hat{X} + BU + g(\hat{X}(t)) + GC(\hat{X} - X)$$

$$U = v(t) + K\hat{X}(t) \qquad (12)$$

Where $\hat{X}$ is the observer state, U is the control, v is the reference input, g is the known function associated with the nominal system equation; K and G are constant controller and observer gain matrices respectively. Since the state variables are the variations in the room temperature and humidity, then

$$g(\hat{x}(t)) = 0 \qquad \text{and} \qquad v(t) = 0$$

Combining the system equations with the observer equations gives

$$\dot{Z} = RZ + B_0U(t) + N(Z(t)) \qquad (13)$$

where

$$Z = [X \quad \hat{X}]^T \qquad B_0 = \begin{vmatrix} B \\ B \end{vmatrix}$$

$$R = \begin{vmatrix} A & BK \\ -GC & A+BK+GC \end{vmatrix}$$

and

$$N(Z(t)) = \begin{vmatrix} g(X(t)) \\ g(\hat{X}(t)) \end{vmatrix}$$

Upon using the tracking notion in the sense of input an output sphere [12] in designing the controller and th observer, the sufficient condition for the existence an uniqueness of solution is

$$\zeta \leq \frac{\rho_o \beta_o}{\rho_1 \beta_o + \beta_i + \rho_2} \qquad (14$$

where

$$\zeta = \max_j \int_0^\infty \left[\sum_k \mid F_{jk}(t) \mid dt \right]$$

$$[F_{jk}(t)] = \sum_{j=1}^{2n} e^{\lambda_j t} \prod_{k \neq j} \frac{D_o R D_o^{-1} - \lambda_k}{\lambda_j - \lambda_k}$$

$\lambda_1 ... \lambda_{2n}$ are the distinct eigenvalues of R

$$D_o = \text{non singular weighing matrix} = \begin{vmatrix} D & 0 \\ 0 & D \end{vmatrix}.$$

$$\rho_o = \| D_o B_o \|^{-1}$$

$$\rho_1 = \rho_o \sup \left[\frac{\| D_o[N(Z(t)) - N(Z'(t))] \|}{\| D_o(Z - Z') \|} \right]$$

$$\rho_2 = \rho_o \sup \| D_o[N(Z_o(t)) - N_o(Z_o(t))] \|$$

Z_o = Combined state of the nominal system that obtained by replacing the uncertain parameter by certain average value

$$= [\, \bar{X} \qquad \hat{\bar{X}} \,]^T$$

$$N_0(Z_0(t)) = \begin{vmatrix} g(\bar{X}(t)) \\ g(\hat{\bar{X}}(t)) \end{vmatrix}$$

The design matrices K and G must be chosen such that

1- The eigen values of the matrix R are placed in the open left hand complex plane.

2- The resulting constant ζ satisfies the inequality (14).

Design values of K and G

Since the room temperature is the most important design output, the weighing matrix is chosen as

$$D = \begin{vmatrix} 1 & 0 \\ 0 & 0.5 \end{vmatrix}$$

The output sphere:

$| T_R - T_{RD} | \leq 2°C$ and

$| \phi_R - \phi_{RD} | \leq 5\%$ i.e $\beta_o = 2$

The input sphere: Fluctuations β_i in Q_D and W_D will be taken within 0.5

The design matrices K and G are chosen in the form

$$K = \begin{vmatrix} k_1 & 0 \\ 0 & k_2 \end{vmatrix} \text{ and } G = \begin{vmatrix} G_1 & G_2 \\ G_3 & G_4 \end{vmatrix}$$

When the model includes the room volume, equation (8) is applied with x = 0.002, and the resulting matrices are

$$K = \begin{vmatrix} 6 & 0 \\ 0 & -.002 \end{vmatrix} \text{ and } G = \begin{vmatrix} -50 & -0.01 \\ 200 & -0.01 \end{vmatrix}$$

with λ_j = -0.022, -0.0022,-50± 100j

When the room is small and its effects are neglected in the model, using equation (9) yields

$$K = \begin{vmatrix} 20 & 0 \\ 0 & -.35 \end{vmatrix} \text{ and } G = \begin{vmatrix} -50 & -0.01 \\ 200 & -0.01 \end{vmatrix}$$

with λ_j = -20,-0.35,-50 ± 100j

Applying the LQ theory to design a conventional controller that minimizes the output errors and the input energy, the results are

K_1 = 2.5 and K_2 = -0.001

RESULTS AND DISCUSSIONS

Three control loops performances are investigated:

I- State feedback control and the room volume is considered in the design with k_1 = 6 and K_2 = 0.002 (Figure (2-I)).

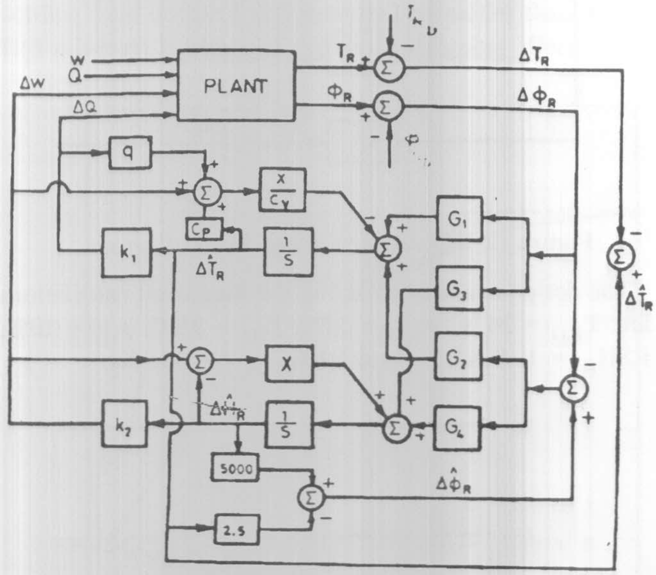

Figure (2-I).

II- State feedback control and the room volume is neglected in the state feedback reconstruction with k_1 = 20 and k_2 = - 0.35 (Figure 2-II)

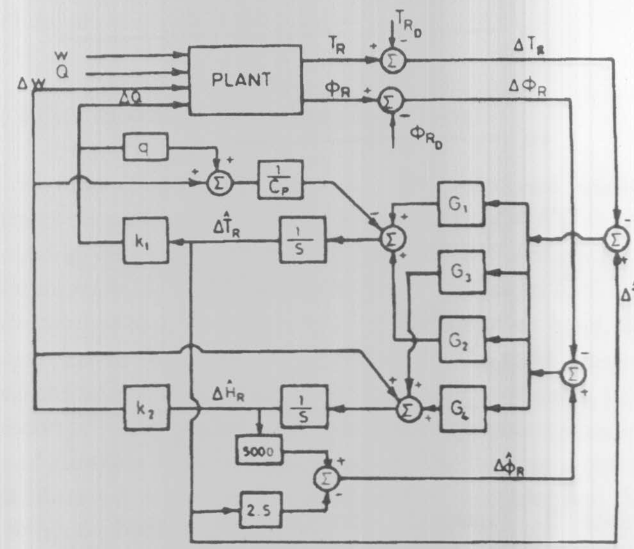

Figure (2-II).

III- Conventional control using output feedback with k_1 = 2.5 and k2 = 0.001 (Figure 2-III)

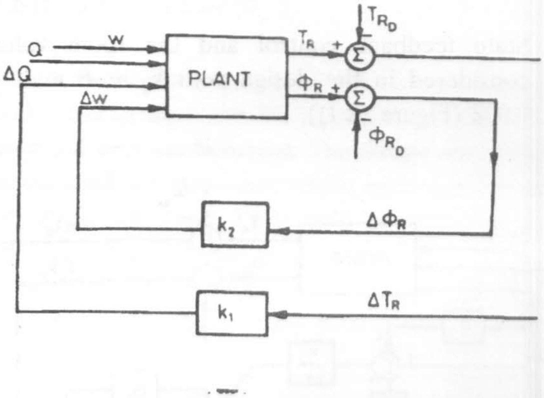

Figure (2-III). Control loops block diagram.

The design conditions used in the numerical calculations are: T_{RD} = 24 °C, ϕ_{RD} = 50%, T_{1D} = 35°C, ϕ_{1D} = 10%, RSH_D = 10.8 KJ/Kg and RLH_D = 2.7 KJ/ Kg.

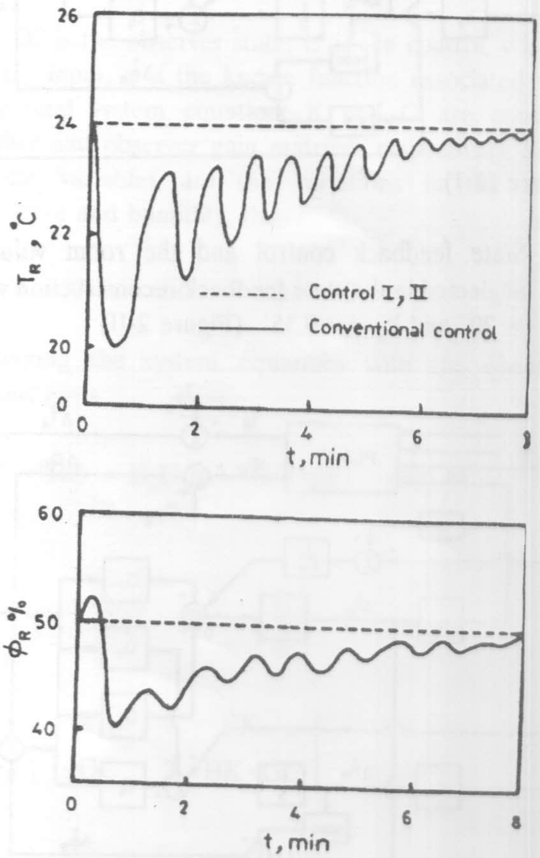

Figure 3. Transient response of large room temperature and humidity for 50% decrease in RSH.

Effect of room internal loads fluctuations

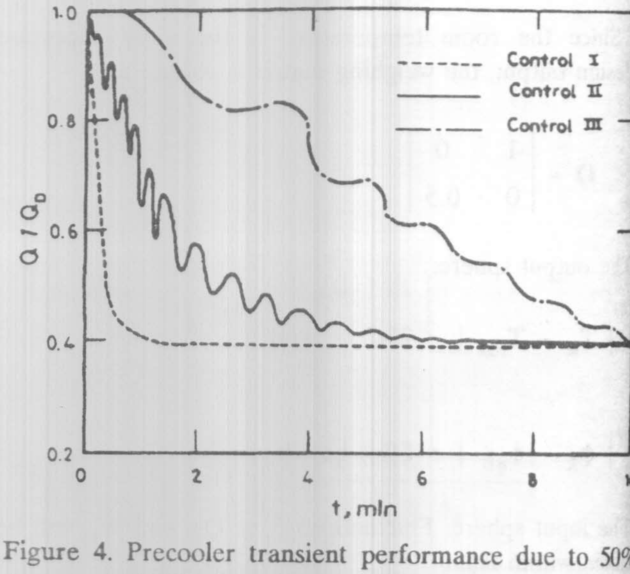

Figure 4. Precooler transient performance due to 50% decrease in RSH, for a large room.

Figure (3) shows the transient response of the room temperature and relative humidity due to a 50% decrease in the room sensible load of the design load. It can be noticed from the figure that the implementation of the observer keeps both the room temperature and relative humidity at the desired values for all time. Meanwhile, the conventional control renders an oscillatory response, the settling time is 8 minutes, and the maximum overshoots are 4°C in T_R and 10% in ϕ_R which are larger than the prespecified allowable deviation. The effects of the decrease in the room sensible heat on the control efforts are illustrated in Figures (4 and (5)). Since the precooler is used against the sensible load, 50% decrease in RSH results in 60.5% decrease in the precooler load and washer load remains the same when the steady state is reached. Both the conventional control and the control loop II result in fluctuations in both the cooling and the humidification load. The settling time upon using the control loop II is 6 minutes compared to 10.5 minutes for the system with a conventional controller. Taking into consideration the room volume in the state reconstruction (case I), the oscillations in both the cooling load and the humidification load are damped. Also the system robustness increases and the settling time is reduced to 1.5 minutes.

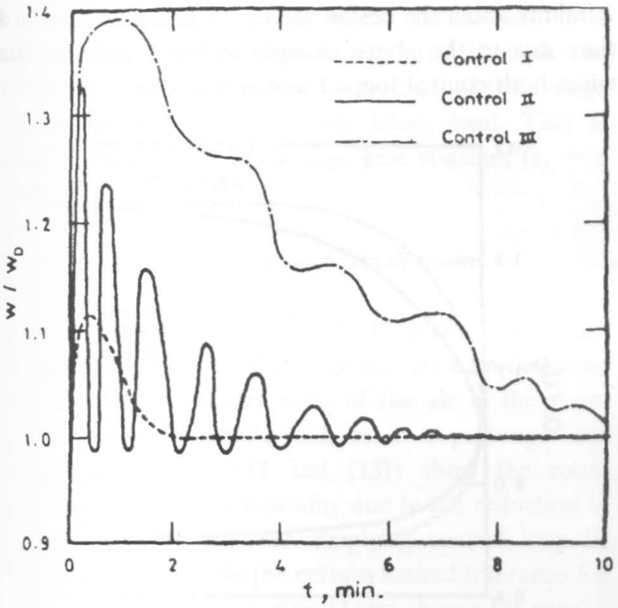

Figure 5. Air washer transient performance due to the 50% decrease in RSH for a large large room.

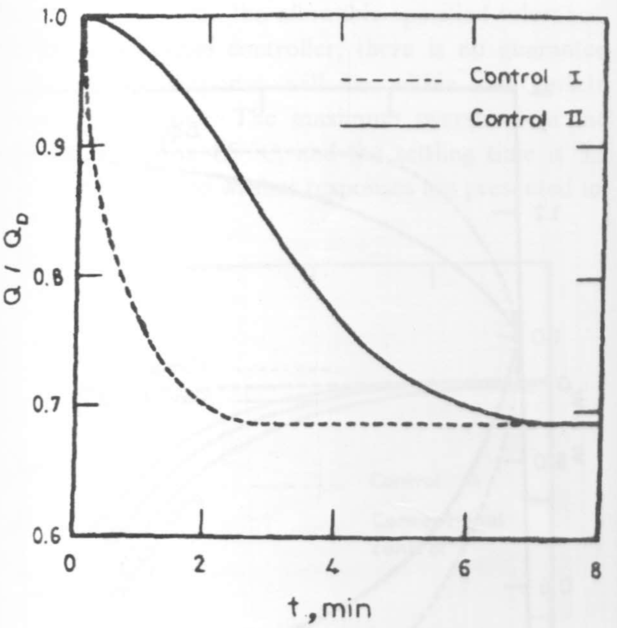

Figure 6. Precooler transient performance due to the elimination of RLH in large room.

When the room latent load is eliminated (industrial application), the room temperature and relative humidity responses are found similar to that illustrated in Figure (3). The effects on the cooling and the humidification loads are presented in Figures (6) and (7) respectively. The elimination of the room latent load increases the washer load by 20% of its design capacity. That is because in dry weather the washer is used against the outside latent load while the internal latent load works in the same direction with the humidification process. Since the humidification process is accompanied by a decrease in the sensible heat, therefore the precooler load is reduced by 30.5% as shown in Figure (6). As anticipated, the settling time in case I (2.5 minutes) is smaller than in case II(8 minutes) because of the room damping effect.

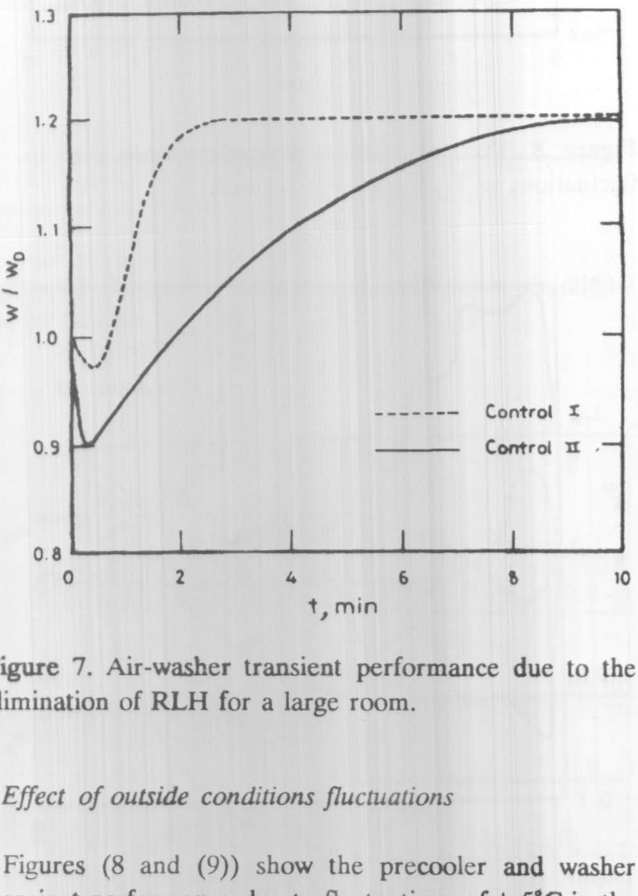

Figure 7. Air-washer transient performance due to the elimination of RLH for a large room.

Effect of outside conditions fluctuations

Figures (8 and (9)) show the precooler and washer transient performance due to fluctuations of ± 5°C in the outside air temperature. As the outside temperature increases by 5 °C, the precooler load increase by 65% and the washer load decreases by 10%. On the other hand, the decrease in the outside temperature by the same amount results in a 67% decrease in the precooler load and a 10% increase in the washer load to keep the room temperature and humidity at the designed values. The hunting problem is observed in the performance of the precooler and the air washer when control loop II is used and the settling time is 9 minutes. The settling time is reduced to 3 minutes with control loop I.

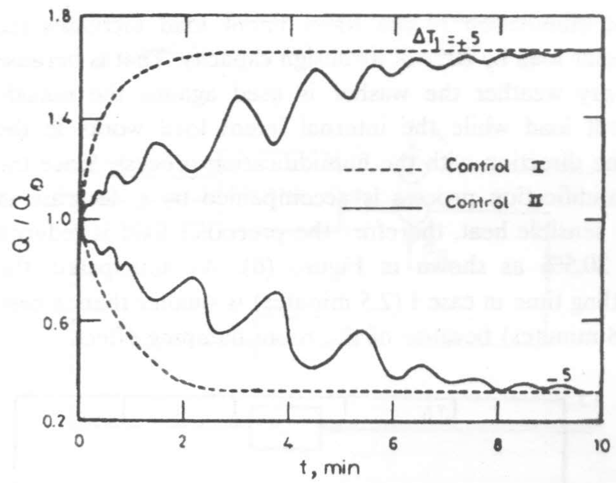

Figure 8. Precooler transient performance due to fluctuations in T_1, for a large room.

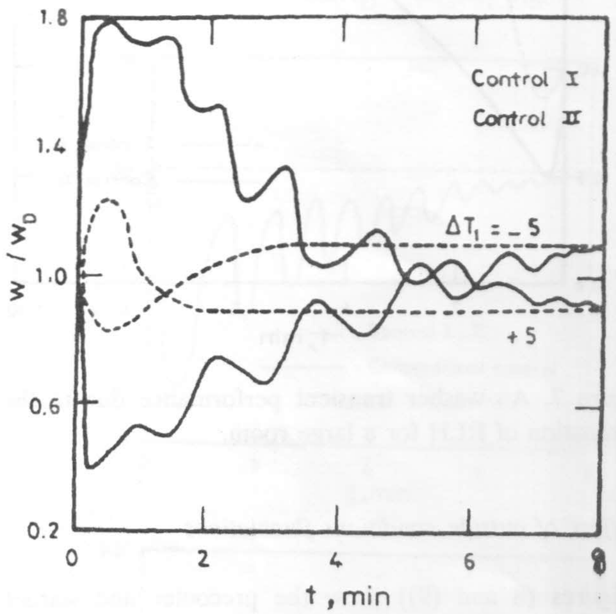

Figure 9. Air-washer transient performance due to fluctuations in T_1, for a large room.

The effects of the changes in the outside relative humidity on both the precooler and washer loads are plotted in Figures (10 and (11)). The increase in outside relative humidity is accompanied by an increase in the precooler load and a decrease in the washer load. The settling time is also decreased when the room volume is considered in the design.

The fluctuations in the room temperature and relative humidity about the design values are less than 0.005 for all time due to the above changes in the outside conditions when both control loops I and II are used.

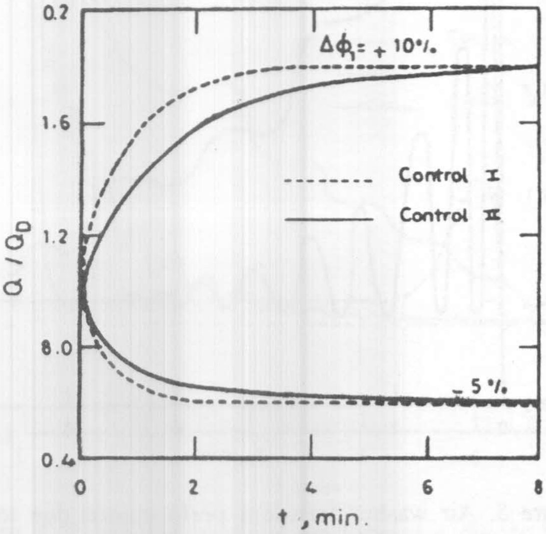

Figure 10. Precooler transient performance due to fluctuations in ϕ_1, for a large room.

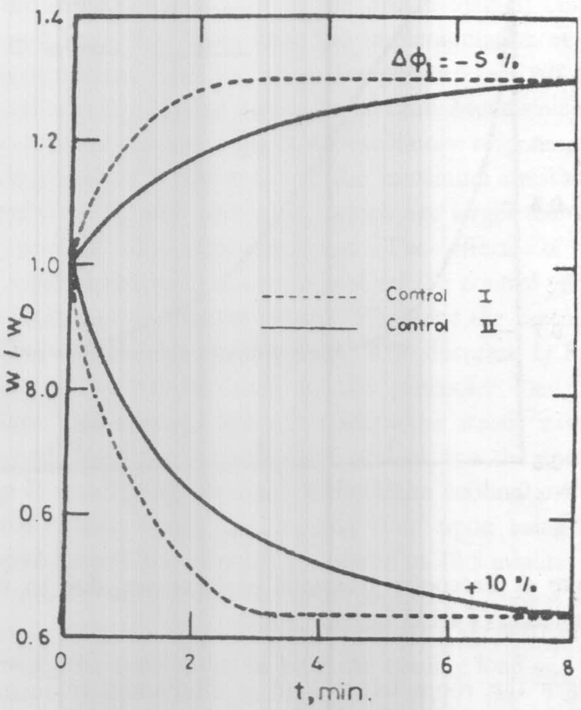

Figure 11. Air-washer transient performance due to fluctuations in ϕ_1, for large room.

It should be mentioned that when control loop II is used, no oscillatory behavior is exhibited in both the precooler and washer response as a result of the change in either the internal or outside latent load. That is because the latent load has a large time constant ($\tau_L \approx q = 2500$).

Control loops performance with small room

Both the control loops II and III performances are investigated when the mass of supplied air is in the same order of magnitude as the mass of the air in the room (i.e., small room volume), such as in a passenger car compartment. Figures (12 and (13)) show the room temperature and relative humidity due to the reduction in the room sensible heat load. Applying control loop II, both responses are within the certain limited tolerance for all time. Since the weight matrix D was chosen for precise tracking of the room temperature and partially tracking of room humidity, the fluctuation in the room temperature about the design value is less than 0.003 °C. Meanwhile the maximum deviation in the room relative humidity is 4.7% which is less than the allowable specified tolerance. With the conventional controller, there is no guarantee that the output response will be within the certain tolerance for all time. The maximum overshoot in the room temperature is 4.5 °C, and the settling time is 3.5 sec. The precooler and washer responses are presented in

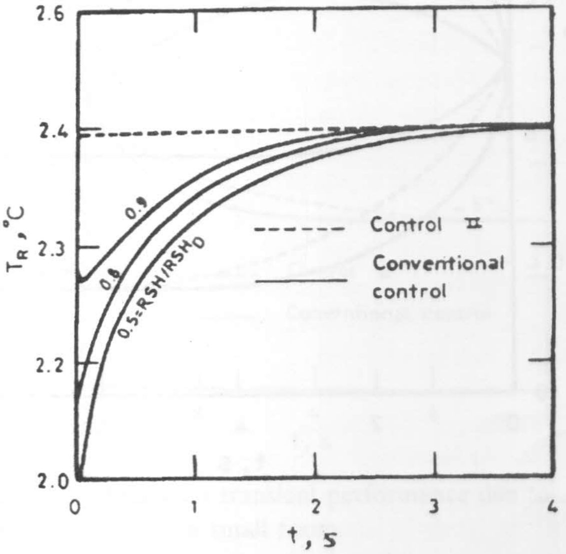

Figure 12. Transient response of small room temperature due to changes in RSH.

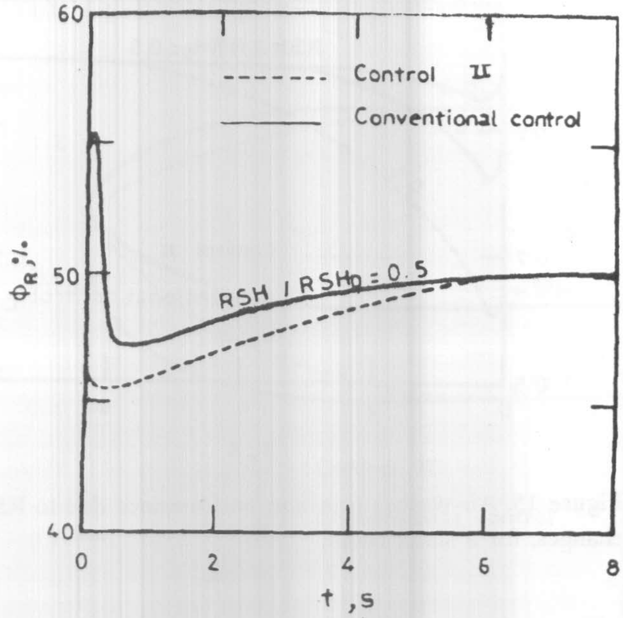

Figure 13. Transient response of small room humidity due to changes in RSH.

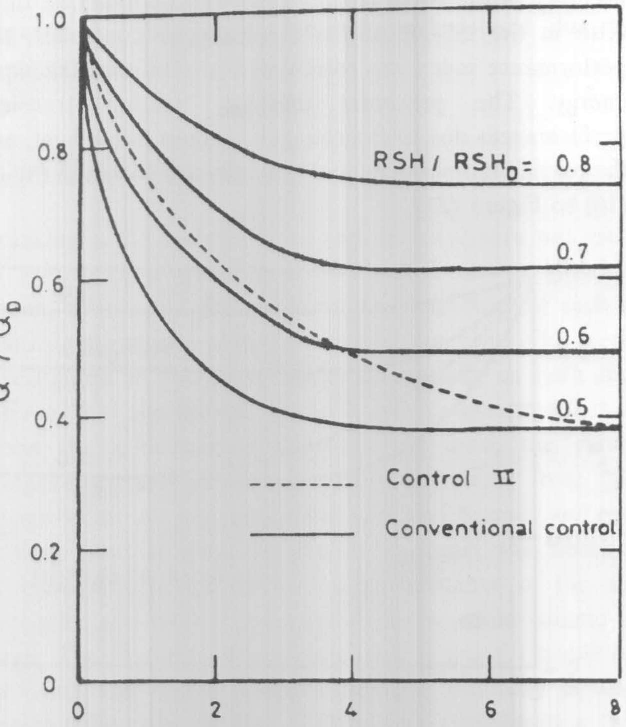

Figure 14. Precooler transient performance due to RSH changes for a small room.

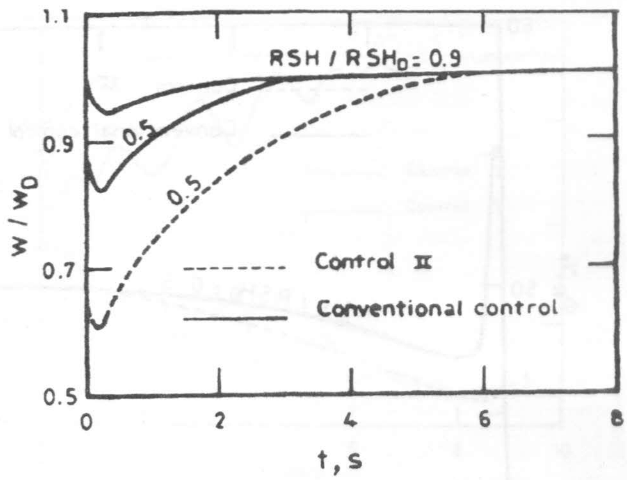

Figure 15. Air-washer transient performance due to RSH changes, for a small room.

Figures (14 and (15)). One may notice that the control efforts (cooling and humidification) are higher upon using the control loop II than that associated with the conventional controller. The larger control efforts arise mainly due to the high demand placed to improve the accuracy of the room temperature and humidity all time. Also in the design of the conventional controller, the performance index was formulated to minimize the input energy. The precooler and air washer transient performances due to the changes in room latent heat, and the outside temperature and humidity are shown in Figure (16) to Figure (21).

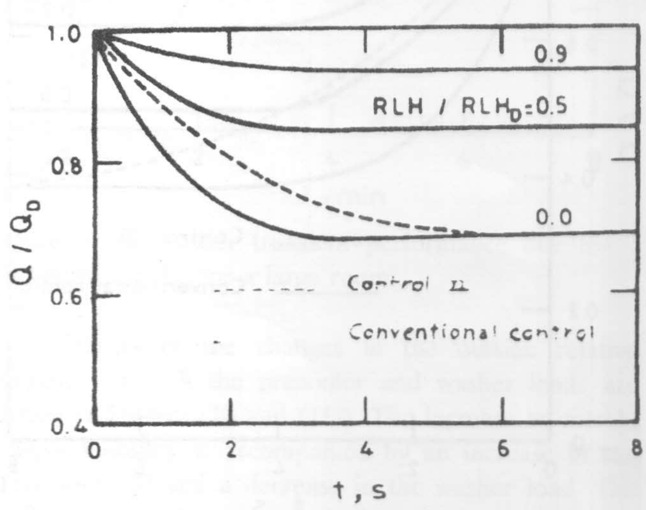

Figure 16. Precooler transient performance due to RLH changes, for a small room.

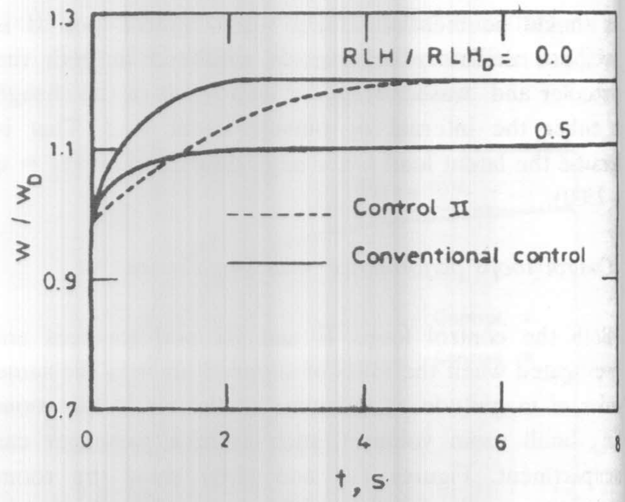

Figure 17. Air-washer transient performance due to RLH changes, for a small room.

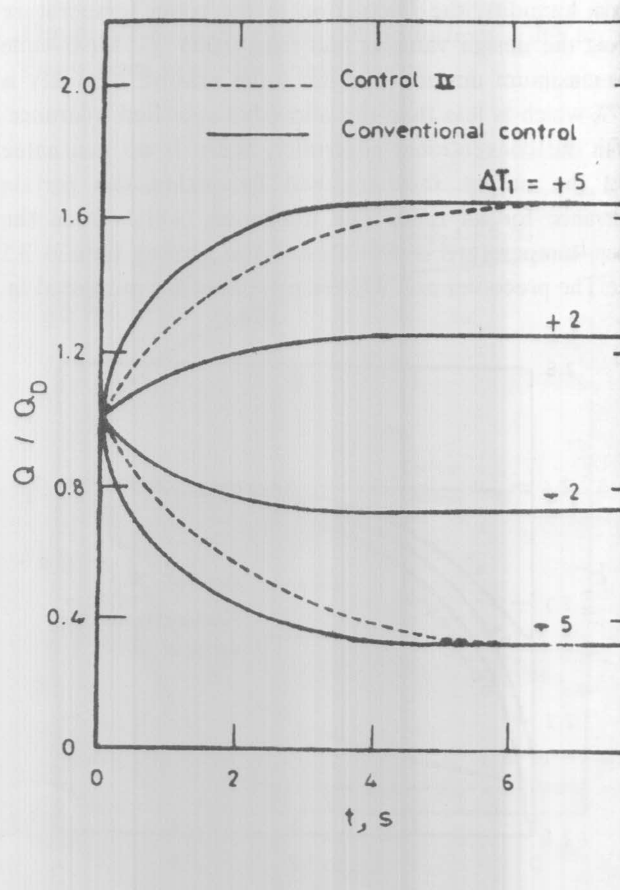

Figure 18. Precooler transient performance due to changes in T_1, for a small room.

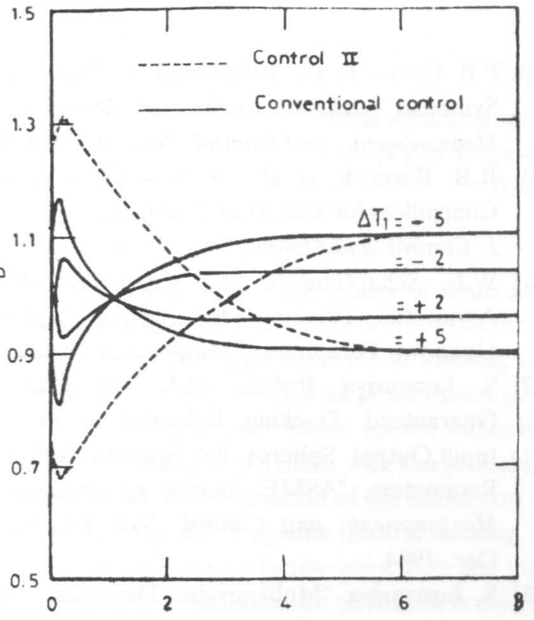

Figure 19. Air-washer transient performance due to changes in T_1, for a small room.

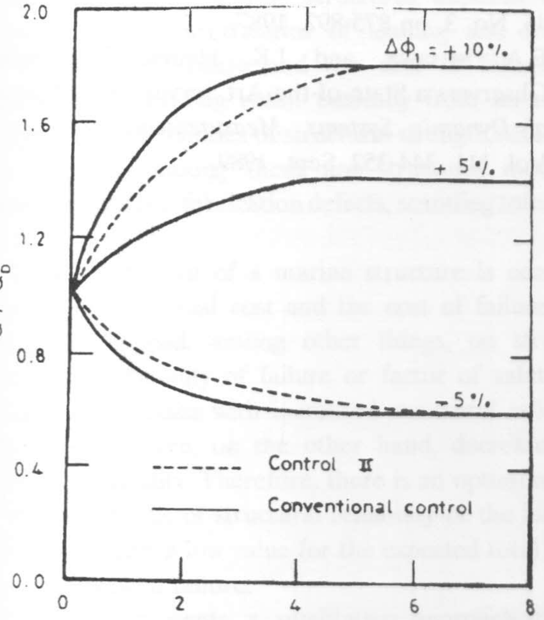

Figure 20. Precooler transient performance due to changes in ϕ_1, for a small room.

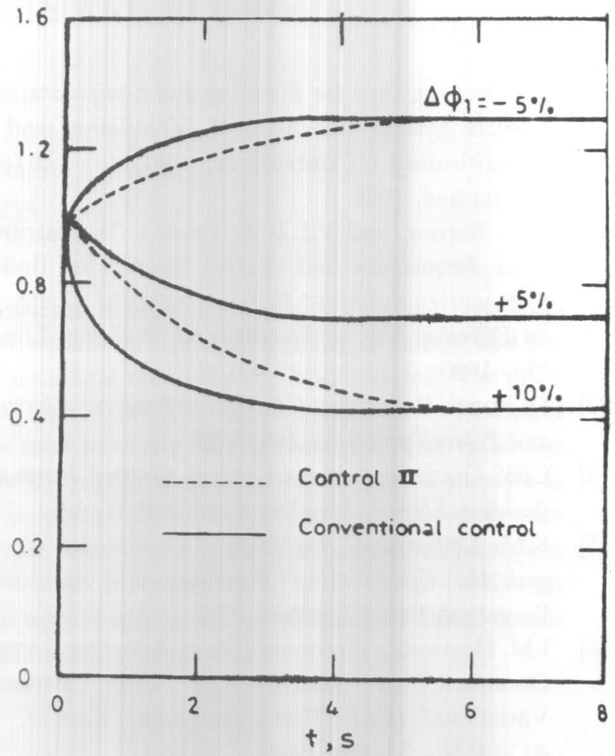

Figure 21. Air-washer transient performance due to changes in ϕ_1, for a small room.

CONCLUSIONS

Tracking in the sense of input and output sphere was used to synthesize a quantitative controlled air-conditioning system in hot and dry weather. The feasibility of such a control system and also the conventional controller were investigated in the presence of uncertainty in both the internal and the outside loads. It was demonstrated that when the conventional controller is used, the room temperature and relative humidity deviations from the design value are not within the specified bounds and the settling time is long. In the mean time, the hunting problem is experienced in the performance of the air precooler and washer specially when the room volume is large. The synthesized control circuit is found capable of adjusting the room temperature and humidity at the design values for all time. Taking into consideration the room size in the model used in the observer, the oscillations are eliminated and the robustness is increased.

REFERENCES

[1] Anonymous, *Data for Planning and Equipment*, Stafa Control System International, Ventilation and Air Conditioning Controllers, 90516 GB/81,8, Switzerland, 1981.

[2] G.C. Barney, and V.L.B de Freitas, "A Distributed Data Acquisition and control System for Building Engineering Services," *IEE International Conference on Effective Use of Electricity in Building*, London, May 1980.

[3] H. Bondi, "Intelligent Controls in Energy," *Electronic and Power*, pp 87-93, Jan. 1980.

[4] CIBS Guide, Automatic Control, *Charted Institute of Building Service*, London, Section B-11, 1981.

[5] K.M. Letherman, *Automatic Controls for Heating and Air, Conditioning, Principles and Application*, Pergamon Press, London, 1981

[6] I.M. Horowitz, "Optimum Linear Adaptive Design of Dominant,Type System with Large Parameter Variations," *IEEE Trans. Automatic Control*, Vol. AC-14, No. 3, June 1969.

[7] I.M. Horowitz, "Synthesis of Feedback System with Nonlinear Time-Varying Uncertain Plants to Satisfy Quantitative Performance Specification," *IEEE Proceedings*, Jan,1976.

[8] G. Zames, "Feedback and Optimal Sensitivity: Model Reference Transformations, Multiplicative Seminorms, and Approximate Inverse," *IEEE Trans. Automatic Control*, Vol. AC-26, No. 2, Apr. 1981

[9] P.B. Usoro, et al., "Ellipsoidal Set-Theoretic Control Synthesis," *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 104, No. 4, 1982.

[10] R.B. Barmish, et al., "A New Class of Stabilizing Controllers for Uncertain Dynamical Systems," *SIAM J. Control and Optim.*, Vol. 21, No 2, 1983.

[11] W.E. Schmitendorf, and R.B. Barmish, "Robust Asymtotic Tracking for Linear Systems with Unknown Parameters," *Automatica*, August, 1986.

[12] S. Jayasuriya, Rabins, M.I., and R.D. Barnard, Guaranteed Tracking Behavior in the Sense of Input,Output Spheres for Systems with Uncertain Parameters, "ASME *Journal of Dynamic Systems Measurement, and Control*, Vol. 106, pp 273-279, Dec. 1984.

[13] S. Jayasuriya, "Multivariable Disturbance Rejection Tracking Controllers for Systems with Slow and Fast Modes," *ASME Journal of Dynamic System, Measurement, and Control*, Vol. 109, pp363-369, Dec. 1987.

[14] S. Jayasuriya, "On Robust Tracking for a class of Uncertain Linear Systems," *Int. J. of Control*, Vol. 45, No. 3, pp 875-892, 1987.

[15] E.A. Misawa, and J.K. Hedrick, "Nonlinear Observers a State-of-the-Art Survey," *ASME Journal of Dynamic Systems, Measurement*, and Control, Vol. 111, 344-352, Sept. 1989.