OPTIMUM NUMBER OF BERTHS AT SEAPORTS

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ABSTRACT

This paper describes a methodology designed to support the decision- making process by developing seaport infrastructure to meet future demand. Seaport planner should be able to avoid inadvertent over- building and under-building. Within this methodology, the movements of ships in a port can firstly be analyzed. Ships arrivals in the port and service times at berths are approximated by theoretical models. Then, the interrelationship between number of berths, number of waiting ships, and ships delays is determined using queuing theory. Finally, the optimum number of berths that minimizes the total port costs can be decided, taking into consideration the time costs of both idle ships and idle berths. An application of the proposed methodology to Alexandria Port is also presented in the paper.

INTRODUCTION

The port transportation system includes different physical elements; e.g. berths, handling equipments, storage and traffic facilities. Although the capacity of any single element may be expressed as an absolute figure, such as the number of containers loaded per hour by a certain crane, the aggregate capacity of the whole port can not be so simply described. Each element can limit the overall port productivity.

Port productivity can be viewed from two standpoints. To ship operators, productivity implies the time needed at the port to serve ships, while at national level, port productivity can be defined as the amount of cargo transported through the port during a certain time period.

Port development is often affected by operating policies as well as by the traffic demand imposed in the port in terms of the volume of cargo expected to be accommodated, the service time at the available berths within which this volume should be handled, and the frequency of ships arrivals.

It would be possible to develop the port facilities so that its capacity is fully utilized at all times. In this manner, changes in demand have to be accommodated by forcing ships to wait (at anchorage) until ships that arrived previously had been serviced. This policy would be inefficient and uneconomic due to the delay costs of waiting ships. Conversely, developing the port so that ships are never forced to wait also represents an uneconomic use of port resources.

The ideal situation is the one in which all berths are occupied all time and no ship is ever kept waiting. This situation is impossible to achieve in practice because of the random arrivals of cargo ships and the variations in service time of ships of different sizes.

Therefore, decisions concerning port development can be made by trading- off the cost of increasing the port capacity and the costs of both waiting and service times.

The purpose of this paper is to introduce a methodology which can be used to facilitate the decision-making process of port development. The proposed methodology covers two principal areas:

(a) Investigation of the pattern of ship traffic at a seaport from the standpoint of queuing theory, and to use the findings to draw some hypotheses regarding its application to the overall operation in seaports.

(b) Determination of the optimum number of berths needed in a seaport that will minimize the total port usage costs.

The proposed methodology is then applied to the group of berths at Alexandria Port which deals mainly with general cargo vessels.

ANALYSIS OF SHIPS' MOVEMENT IN PORT

An important parameter measuring the performance of a seaport is the delays that ships experience while waiting to be processed. Two factors affect these delays: (a) the pattern of ships arrival, and (b) the berth time requirement for cargo handling.

The arrival of a cargo ship in a port is often irregular [12], and when it arrives, it may be able to move directly onto a berth or has to wait until a berth becomes empty; if all berths are occupied. The berth time needed to serve a ship is also variable, as it depends on the amount of cargo which the ship carries and the capacity of the present facilities for handling and storing cargo [2]. Figure 1 shows ship behaviour in a seaport.

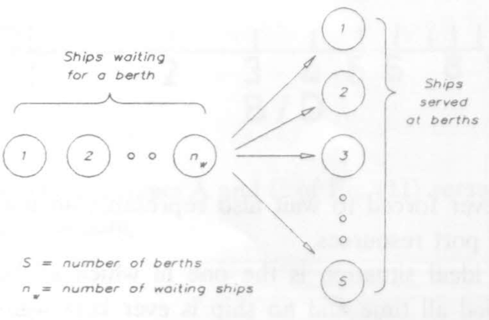

Figure 1. Seaport operation

The investigation of such random occurrences requires a complex and detailed analysis. The concept of "Queuing Theory - waiting line problem" can successfully be applied. Queuing theory is one of the most useful tools for analyzing the behaviour of waiting units (ships in this case), or for investigating the components of a multiple operation system [7]. Thus, queuing theory may be adequate for studying the ships' movement in seaports.

Two basic elements are necessary for the application of queuing theory to a waiting line problem: an arrival function and a service function. These functions should firstly be modeled. Once the validity of these models is tested, the different characteristics of the theoretical models, which describe the actual system with the accuracy that may be realized in estimating future traffic, can then be determined.

To analyze the movement of ships in a seaport using the queuing theory, the following conditions are assumed:

- Ships arrivals and service times conform to the pattern of random occurrences.
- Ships are processed on the "first-come first-served" queue discipline.
- The queue length is unlimited, i.e. if a ship arrives and finds a long queue, it joins the waiting ships and does not leave the port.

Ship Arrival Function

Probably the two most commonly encountered arrival pattern of ships in a seaport are the random arrivals, and the scheduled arrivals with considerable delays. Thus, to predict the number of ships present in a port in a certain time period (usually a day), the arrival pattern of ships may be approximated by a Poisson function [1,10,14]. In this way, the probability P_n of the arrival of n ships in the port in a given time can be expressed as (Figure 2)

$$P_n = (k^n / n!) . e^{-\lambda}$$

where

λ = the average arrival rate of ships in a given time (one day; for example),

e = the Naperian logarithmic base, and

n! = the factorial of the number n

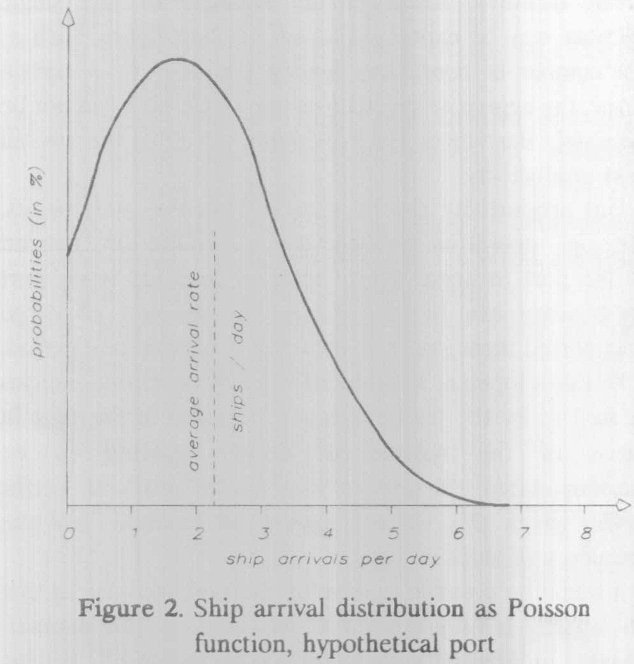

Figure 2. Ship arrival distribution as Poisson function, hypothetical port

The distribution of ships arrivals with Poisson function can be calculated, only if the average arrival rate during an entire period is known. The expected frequency F_n of n ships in port in a given time T is

$$F_n = T \cdot P_n$$

in which T = the time period port operation considered (often expressed on an annual basis as 365 days).

The Poisson distribution can also be used to focus on the time intervals "t" (i.e. the headways between successive arrivals) rather than on the number of arrivals occurring during a stated time [6]. In this case, the probability function of the arrival times will be

$$f(t) = \lambda \cdot e^{-\lambda t}$$

This equation, known as the negative exponential distribution, is expressed as a cumulative distribution function. It describes the probability of a headway "h" being greater than or equal "t":

$$P\ (h \geq t) = \int_t^{\infty} f\ (t) = e^{-\lambda t}$$

Service Time Function

The duration of ships at a berth for handling cargo may be described as an Erlangian function [5,8,9], which is usually used to present service times which are more regularly spaced in time than those represented by the Poisson distribution.

There are purely theoretical curves (Erlang-functions), each of which is based on the assumption that the service time is split into two or more operating phases following one another, and that the ship does not leave the berth until all phases are completed. "k" is the number of "Erlangian Phases" of ships service time distribution at a berth. Each function has a negative exponential distribution. As "k" increases, the total service times become more uniform, until finally with k = ∞ all service times are identical. In the general case the total service time probability P_0 is given by (Figure 3)

$$P_0 = e^{-kb} \cdot \sum_{n=0}^{k-1} (kb) \cdot n/n!$$

where

b = Average berth service time (in days)

k = Erlangian number (k = 1, 2, 3,....., ∞)

n = Counter

for k = 1, $P_0 = e^{-b}$

for k = 2, $P_0 = e^{-2b(1 + 2b)}$

for k = 3, $P_0 = e^{-3b(1 + 3b + 9b^2/2)}$

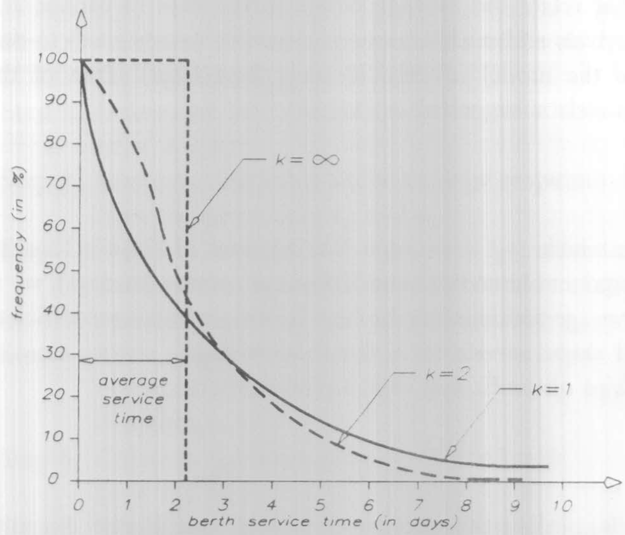

Figure 3. Service time distribution as Erlangian function, hypothetical port

Through the choice of k, a service time function may be described as anything from the purely random exponential type (k=1) to the completely regular constant service time type (k=∞). The value of k should be selected and tested to provide the best fit to the observed data.

Queuing Phenomenon

Based on the queuing theory, useful results pertaining to the amount of delays to be expected and the length of queues (here; number of ships waiting for berth) may be calculated with an appropriate model [3]. However, as the nature of the problem is defined, in this paper, as multi-channels (berths), with exponential arrivals (Poisson), and multiple exponential service (Erlang), no feasible mathematical solution is possible. The theoretical models available in the literature for multi-channel systems, are intractable for other than exponential distribution of arrivals and multiple exponential service time distribution. For investigating queuing situations of multi-channel systems, models are accessible only for the following two cases:

Case I: Exponential distributions for both arrivals and service times

Case II: Exponential arrivals and a constant service time (k = l)

Some approximate formulas have recently been proposed that relate the average delays in the case of exponential arrivals and multi-exponential service (study case) to that of the model of case II (described later). One of the models is suggested in [11]:

$$w_k = w_1 (1 + v^2)/2$$

in which, w_k = average waiting time of ships in case of exponential arrivals and Erlangian service (in days), w_1 = average waiting time in case II, v = correction coefficient of ships' service time distribution. If the service time of ships obeys Erlang with k phases, then

$$v = 1 / k^{0.5}$$

Regarding the queuing model of case II, the essential parameters are derived as follows:

λ = Average arrival rate in ships/day (Poisson-distribution)

μ = Average service rate in ships/day (Erlang-distribution; $k = \infty$)
= 1 / average berth service time
= 1 / b

S = Number of berths

The ratio of the arrival rate to the service rate is usually known as the traffic intensity, and denoted by σ, thus

$$\sigma = \lambda / \mu$$

In this case, it can be noted that the average waiting time before service w_1 is given by

$$w_1 = \sigma^S((\sum_{n=1}^{S-1} (\sigma^n/n!)+\sigma^S)/(S-1)!(S-\sigma))^{-1}/(\mu(S-1)!(S-\sigma)^2)$$

From the above analysis of delays in the queue, computation can readily be made of the average length of queue, i.e. average number of ships waiting for a berth n_w. The appropriate expression is

$$n_w = \lambda . w_k$$

Thus, the average number of ships n_S present in port with S berths in a certain time period can be calculated using the following formula:

$$n_S = n_w + n_b$$

where,

n_b = average number of ships served at berths
= S x berth utilization factor
= S x (λ / μ S) = σ

Thus, it is seen that the traffic intensity σ defined in the queuing theory equals the average number of ships served at berths n_b.

ANALYSIS OF PORT CAPACITY

Minimum Capacity

The minimum number of berths S_{min} which has to be constructed in a seaport to handle a certain amount of cargo can be calculated using the following procedure:

Let Q = the total amount of cargo (in tons) handled in a port section in a time period T (for example; T = one year = 8760 hours), and R = average rate of cargo transfer between ship and berth (in tons per hour).

then, $S_{min} = Q / (R.T)$

Thus, the gross berth time available is "S_{min}.T". Now, let β equal the percentage of berth usage throughout the period T (berth utilization).

β = berth time required / berth time available, or
= $Q / (S_{min}.R.T)$

In this manner, the calculated number of berths is based on average values, regardless the random arrivals of ships and the variation in berth service times.

Optimum Capacity

If the number of berths in a port is S, the total cost spent in the port during a certain period C equals the sum of two deferent types of costs: cost related to berths and the cost related to ships present [13]. Thus, it can be expressed as (Figure 4)

$$C = c_b.T.S + c_S.T.n_S$$

in which, C = the total cost of a port with S berths during the period T, usually one year = 365 days, (in L.E.), c_b = average cost of a berth; i.e. construction and maintenance costs (L.E./day/berth), c_S = average delay cost of a waiting ship (L.E./day/ship), and n_S = average number of ships present in port.

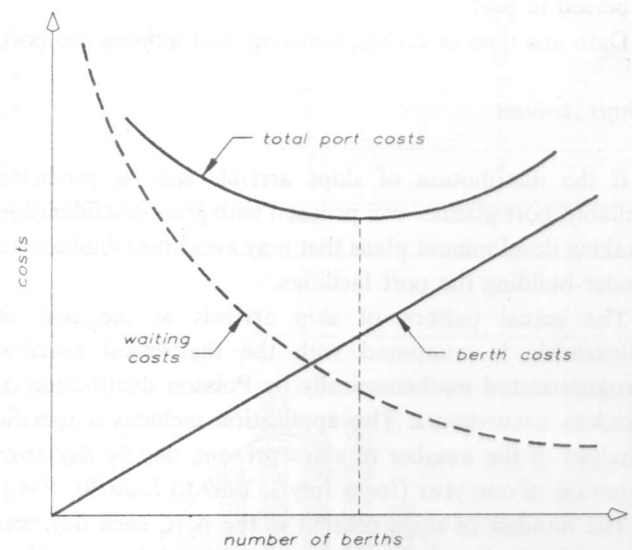

Figure 4. Total usage cost at a hypothetical port

Accordingly, if the amount of cargo that must be dealt with at a port during the period T is given as a planning target, then such number of berths S becomes the optimum that minimizes the total cost C. Therefore, C is a proper measure to examine the optimality of a port system.

Now, both sides of the above equation are divided by "C_S.T" in order to decrease the number of the parameters involved. Thus,

$$r_S = C / (c_S.T) = (c_b/c_S).S + n_S = (r_{bS}.S) + n_S$$

in which, r_S = ratio of the total annual cost for port to annual ship cost, and r_{bS} = berth-ship cost ratio.

Assuming that S is optimum, then the following optimization condition must be held:

$$r_S < r_{S+1}, \text{ and } r_S < r_{S-1}$$

Thus, r_S will be adopted hereafter as a measure to determine the optimum number of berths.

From the preceding information the procedure can be standardized as follows when given the data Q, R, c_b, c_S, λ, μ, k:

Step 1. Calculate the minimum number of berths from the equation $S_{min} = Q / (R.T)$

Step 2. Determine the value of traffic intensity σ ($\sigma = \lambda / \mu$)

Step 3. Compute the value of berth-ship cost ratio r_{bS} from the given data c_b and c_S

Step 4. For each number of berths, with S greater than the minimum value, estimate the number of ships present in port n_S, and predict the ratio r_S

Step 5. The number of berths which satisfies the optimization condition ($r_S < r_{S+1}$, and $r_S < r_{S-1}$) is optimum

Step 6. Compute the average berth utilization β ($\beta = \sigma / S$)

Step 7. Summarize the queuing results (average number of ships present in the port, average number of ships at berths, average number of waiting ships, average waiting time)

In order to illustrate the sequence of the calculations within this procedure, a very simple example is carried out. The aim is to determine the optimum number of berths at a hypothetical port. The following values are given as input data: Q = 1 Mill. tons, R = 1200 tons per day, c_b = L.E. 2000 per day, c_S = L.E. 8000 per day, λ = 3.19 ships per day, μ = 1.18 ships per day, and k = 2.

1. S_{min} = 1 000 000 / (1200 x 365) = 2.28 = 3 berths
2. σ = 3.19 / 1.18 = 2.7
3. r_{bS} = 2000 / 8000 = 0.25
4. Figure 5 shows the relationship between traffic intensity and r_S for a proper number of berths ($S > S_{min}$)
5. The optimum port capacity is 5 berths. In this instance, r_S = 4.09, and the annual port costs C = L.E. 11.943 Millions
6. The average berth utilization = 2.7 / 5 = 0.54

7. Queuing results:
   average number of ships present in port n_S = 2.84
   average number of ships served at berths n_b = 2.7
   average number of waiting ships n_w = 0.14
   average waiting time per ship w_k = 1.05 hours

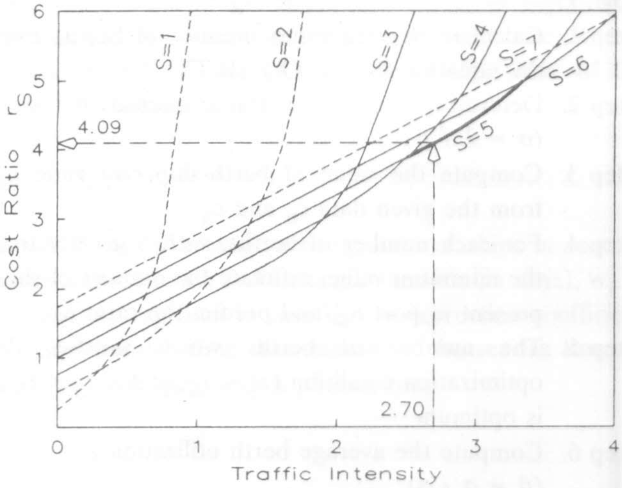

Figure 5. Determination of the optimum number of berths, illustrative example

Figure 5 also shows that a five-berths set is the optimum port capacity, in case of traffic intensity values varying between 2.50 and 3.30.

APPLICATION : ALEXANDRIA PORT

The foregoing methodology is applied to investigate the movements of ships in Alexandria Port and to predict the future capacity. The application is restricted to general cargo ships, excluding Full- container and RO/RO ships which have particular berths at the port.

Alexandria Port is the major port in Egypt. About 20.5 Million tons passed through the port in the year 1989/1990; i.e. 60 % of the total volume of the foreign trade. The amount of general cargo handled in the port in that year was 4.326 Million tons.

Alexandria Port is constituted of an old and complicated layout with short quays and too narrow or too long piers. A large number of quays has a limited drought under 8.0 meters, and only a lower number of berths is capable to receive ships with more than 130.00 meter length. The number of berths available for general cargo in the port is 32 berths.

Data Base

The daily "log books" of the traffic department of the Alexandria Port Authority include (among others) the arrival time of each ship at the pilot vessel. In addition, detail information concerning the movement of each ship in the port is also available in the so-called "ship log sheets". Every sheet is a ship report, and it contains the following data:
- Ship name, nationality, type of cargo, and total tonnage.
- Berth occupancy, including berth changes during the period in port.
- Date and time of arrival, berthing, and quitting the port.

Ships Arrivals

If the distribution of ships arrivals can be predicted reliably, port planner can proceed with great confidence in making development plans that may avoid over-building or under-building the port facilities.

The actual pattern of ship arrivals at the port of Alexandria is compared with the theoretical function prognosticated mathematically by Poisson distribution of random occurrences. The application includes a specific analysis of the number of ships present, day by day, over a period of one year (from July 1, 1989 to June 30, 1990).

The number of ships present in the port, each day, was transcribed from the port "log books" and then summarized to obtain the number of days, that various number of ships were present during the period studied. The theoretical distribution (Poisson) is computed. Table 1 compares the predicted distribution with the actual one. The average arrival rate was 5.69 ships per day.

In Table 1, it can be seen the good agreement between actual and predicted distributions. The number of days that various numbers of ships are predicted to be present in the port is in agreement with the actual distribution on 336 days of 360 days, i.e. on 92 % of days.

To judge whether the observed frequencies of ship arrival distribution is compatible with the predicted theoretical frequencies, Chi-square is computed, and the result, X^2 = 20.0 with 10 % probability, indicates a good fit. From the statistical standpoint, probability values between 5 % and 95 % designate good fit from which it is concluded that this theoretical distribution is plausible [6]. Figure 6 demonstrates the goodness of fit between actual and predicted distributions.

Table 1. Comparison of actual versus predicted ship arrival distribution

Arrival rate ships/day	Actual number of days (A)	Predicted number of days (B)	Predicted days agreed with actual (Min.: A or B)
0	1	1	1
1	6	7	6
2	18	20	18
3	27	37	27
4	49	54	49
5	73	62	62
6	60	59	59
7	61	47	47
8	37	34	34
9	15	21	15
10	9	12	9
11	5	6	5
12	2	3	2
13	2	2	2
Total	365	365	336

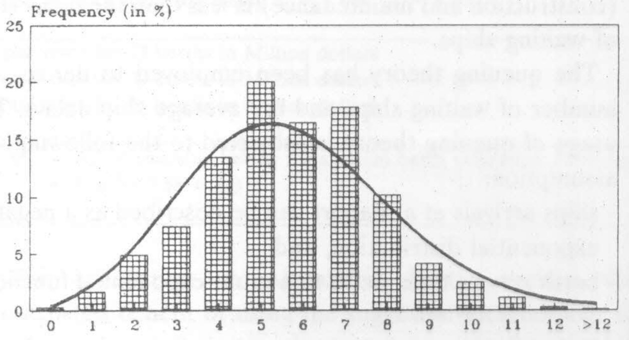

Figure 6. Frequency distribution of ships arrivals, Alexandria Port 1989/1990

Berth Service Times

Information giving the date and time of arrival at a berth and the date and time of departure from the berth were obtained from the "ship log sheets". A total of 315 observations, including those general cargo ships which were tied up at the berths between July 1, 1989 and june 30, 1990 were randomly selected to be analyzed. A class interval of 15 hours was selected for such analysis.

Search for a suitable model for the distribution of the durations at berths led to an Erlangian distribution having k = 3. The mean time spend at a berth was found 5.58 days for the 315 observations. The standard deviation of the distribution was computed and found to be ± 1.43 days. Figure 7 presents the frequency and the cumulative distributions of the observed data and compares the values of the cumulative distribution with those of the Erlangian function having k = 3. A Chi-square test was also performed to test the goodness of fit between the observed frequency distribution and the postulated Erlangian function, and a value X^2 = 14.87 for 42 % probability was found. Comparison with other Erlangian functions (k = 1, k = 2, and k = 4) indicates that k = 3 is the best choice for this distribution function. Figure 7 shows the observed data points and a plot of the selected function.

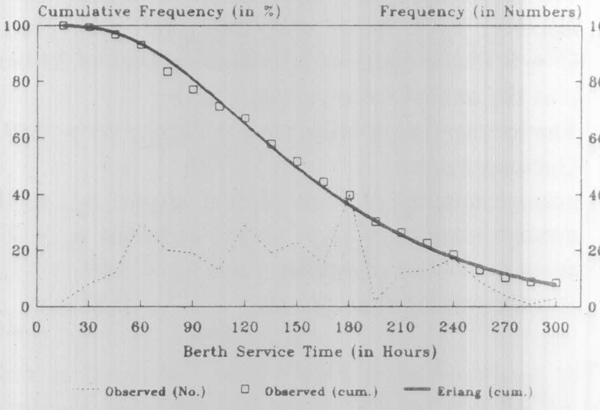

Figure 7. Frequency distribution of berth service time, Alexandria Port 1989/1990

Optimum Number of Berths

To establish the optimum number of berths needed for general cargo handling at Alexandria Port in the year 2005, applying the proposed procedure, the following input data are used:

- Due to the further development of the egyptian ports, particularly the Dekheila Port, the annual general cargo tonnage to be handled at the berths of Alexandria Port will be only about 4.00 Million tons at the target year (tonnage in year 1989/1990 = 4.326 Million tons)[4].
- The average arrival rate k of general cargo ships will be 5.26 ships/day, assuming that the average ship load equals 2084 tons (the present value).
- The average rate of cargo handling at a general cargo berth R = 373.5 tons per day (the existing rate).

- The average cost of a berth C_b = L.E. 2000 per day (approximately \$ 600 per day), based on the development program of the Alexandria Port [4].

  The average delay cost of a general cargo ship C_S = \$ 6000 per day

The calculations are carried out as follows:

1. S_{min} = 4 000 000 / (373.5 x 365) = 29.34
   = 30 berths
2. μ = 1 / 5.58 = 0.18 ships/day
   σ = 5.26 / 0.18 = 29.35
3. r_{bS} = 600 / 6000 = 0.10
4. Figure 8 shows the relationships between traffic intensity and the cost ratio r_S for a proper number of berths (from S= 29 to S = 34)
5. The optimum port capacity is 33 berths. In this instance, r_S = 34.34, and the total port costs C = \$ 75.200 Millions (the development of 33 berths plus the annual maintenance costs)
6. The average berth utilization = 29.35 / 33 = 0.89
7. Queuing results:
   average number of ships present in port n_S = 31.04
   average number of ships served at berths n_b = 29.35
   average number of waiting ships n_w = 1.69
   average waiting time per ship w_k = 0.32 days

The relationships in Figure 8 are prepared as design curves derived to determine the optimum number of berths for Alexandria Port by changing the traffic intensity and/or the cost ratio r_{bS}. The optimum number of berths corresponding to a suitable cost ratio r_{bS} values (from 0.10 to 0.30) is noted in this figure. It can also be seen that a 33-berths set is the optimum port capacity in case of traffic intensity values varying between 27.58 and 29.60.

Table 2 shows the calculation of the costs of idle berths and idle ships for 33 berths in view of the expected frequency (number of days per year). It also presents the combined costs (vacant berths and ships) in case of port size 32, 33 and 34 berths. The cost comparison indicates that the total port cost is least when there are 33 berths. This conclusion confirms the result previously obtained by applying the proposed methodology.

CONCLUSIONS

The paper presents a methodology proposed to predict the optimum number of berths required in a seaport to meet the future traffic volumes. The methodology is based on the hypothesis that the number of berths can be increased as long as the marginal cost of berths (construction and maintenance) is less than the delay costs of waiting ships.

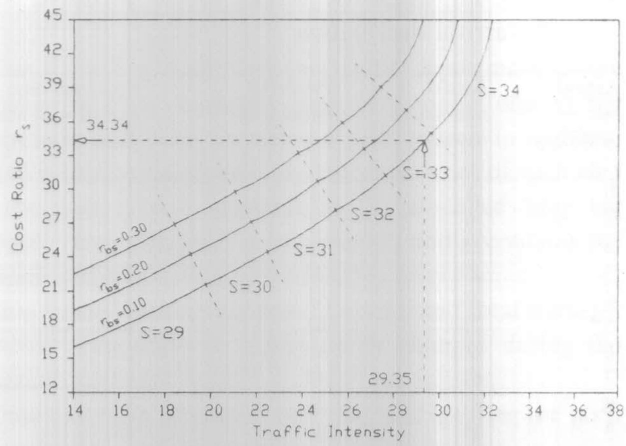

Figure 8. Determination of optimum number of berths, Alexandria Port 2005

The queuing theory has been employed to derive the number of waiting ships and the average ship delays. The usage of queuing theory is subjected to the following two assumption:

- ships arrivals at a seaport can be described as a negative exponential distribution, and
- berth service time yields to a multi-exponential function.

The employment of the queuing theory to study the movements of general cargo ships at Alexandria port was profitable. The observed pattern of ships arrivals appears to agree with Poisson's law of random distribution. In addition, the berth service times for 315 ships were found to conform most closely to an Erlangian distribution with k = 3. The usage of an approximate model of queuing theory led to acceptable results. The criterion for acceptance of this model was the reasonable agreement achieved between the computed and observed values of average waiting time and average number of waiting ships in queues at berths.

Thus, there are no doubt that ships arrive at Alexandria port in accordance with a random pattern and that the future distribution of such arrivals can be predicted to a degree of accuracy that compares favorably with the accuracy that may be realized in estimating future traffic.

Table 2. Cost calculation of both idle ships and idle berths in case of 33 berths, and the comparison of the resulting value with those for 32 and 34 berths

Arrival rate (ships/day) λ	Predicted frequency (in days) F	Berth utilization $\beta < 1.0$	Required number of berths σ	Over-building: Number of berths (33-σ)	Over-building: Berths-Days F x (33-σ)	Under-building: Number of ships λ - X*	Under-building: Ships-Days F x (λ-X)
0	2	0.00	0	33	66		
1	10	0.17	6	27	270		
2	26	0.34	12	21	546		
3	45	0.51	17	16	864		
4	60	0.68	23	10	600		
5	64	0.85	28	5	320		
6	56	1.00	33	0	0	0	0
7	42					1	42
8	28					2	56
9	16					3	48
10	9					4	36
11	4					5	20
12	2					6	12
13	1					7	7
Total	365				2666		215
Costs in Million dollars (using c_b = \$ 600, c_S = \$ 6000)					1.60		1.29
Total costs for 33 berths in Million dollars						2.89	
Total costs for 32 berths in Million dollars						3.66	
Total costs for 34 berths in Million dollars						2.96	

* X = No. of available berths x maximum berth utilization / average service time
= 33 x 1 / 5.58 = 6.0

The application to Alexandria Port verifies the anticipated benefit of using the suggested methodology to evaluate the port size in the best interests of both ship operators and the port authority. The evaluation is settled on the premise that maximum port efficiency results when the total port cost is minimum, i.e. the cost of vacant berths over a substantial period plus the time cost of ships waiting for a berth during the same period.

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