MODIFIED ELASTIC PLATE METHOD FOR RAFTS ON ERRATIC SOIL

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ABSTRACT

Elastic plate method is employed to analyze flat raft on erratic soil. The mat is considered as a plate and the effect of column load is determined in the area surrounding the load, since this effect has been found to be damped out quite rapidly. A load factor is introduced in the present method to magnify the column load where softer soil underlies the zone of influence of such column. Soil non-uniformity is represented by the variation of coefficient of subgrade reaction under zones of influence for columns. By superimposing all the modified column loads within the zone of influence, the total effect of all the column loads at any point is determined. The present method is examined using finite element method in which soil is idealized as a set of isolated springs (Winkler foundation). The method of analysis proved to be powerful and relatively easy to apply without the need of large computer machine. However, the accurate determination of coefficients of subgrade reaction under each column's influence zone is essential for successful application of the present method.

INTRODUCTION

Raft is usually a large concrete slab and often rests directly on a soil that has low bearing capacity. If supporting soil is erratic or non-uniform, raft design becomes more difficult and only can be accomplished by finite element method which necessitate the use of a powerful computer. To date, no simple method of design of rafts on erratic soil is available.

A comprehensive literature review about raft analysis and design is available elsewhere (Abdrabbo, 1987). The main conclusion which may be arrived at from this review is that traditional methods of analyzing raft foundation, treat the raft as a loaded plate supported on series of springs (Winkler model), or on linearly elastic medium. Most of the studies were concerning with the effect of the relative stiffness of a raft resting on isotropic soil. However, Hooper (1975 and 1983) solved the problem of a circular raft on anisotropic medium. Hooper study concerned homogeneous soil only. Theory of plates on elastic foundations (Hetenyi, 1946) has been recommended by ACI Committee 436 (ACI, 1966) as a basis for design of flat rafts over homogeneous soil (Winterkorn and Fang, 1975).

The main objective of present study is to develop a simple, but reliable, method of analysis which can deal with flat rafts supported over erratic or non-uniform soil. It is our belief that such simple method is badly needed for more realistic design and better understanding of flat rafts founded on general soil.

METHOD OF ANALYSIS

Based on the following assumptions, the effect of non-uniformity of soil is included in theory of plates on elastic foundation (Hetenyi, 1946) :

1- The coefficient k_i of subgrade reaction under zone of influence of column (i) is to be established to represent the average elastic properties of the soil under this zone.

2- Zone of influence for column (i) is treated as a plate on elastic foundation, and therefore Hetenyi's solution can be applied within zone of influence (i).

3- The radius of effective stiffness L is calculated as (Winterkorn, 1975) :

$$L_i = \sqrt[4]{\frac{D}{k_i}} \qquad (1)$$

where D = flexural rigidity of the mat,

$$D = \frac{E\,t^3}{12(1-\mu^2)} \qquad (2)$$

E = modules of elasticity of concrete, and
μ = Poisson's ratio of the concrete.

Radius of influence of individual column load is approximately 4L. Based on Hetenyi's function for deflection Z_4, Figure (1), the deflection w at any point of radius r from column (i) may be calculated as :

$$w = \frac{P_i\,L_i^2}{4\,D}\,Z_3\,(\frac{r}{L_i}) \qquad (3)$$

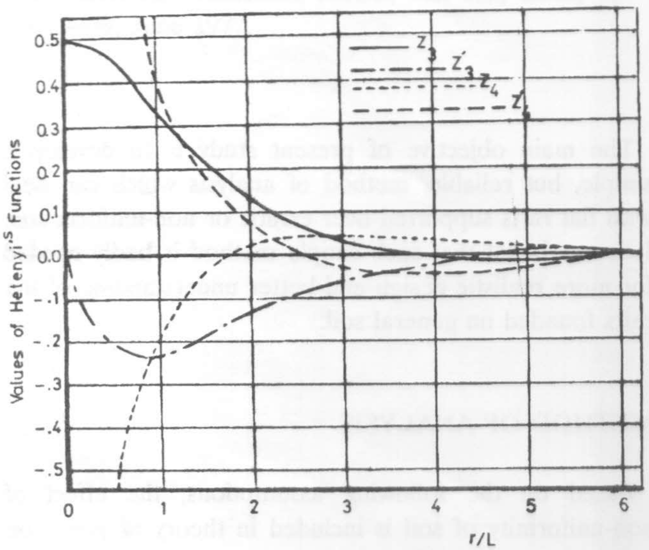

Figure 1. Function for deflection, moment and shear (hetenyi, 1946).

But theory of plates on elastic foundations is based on assuming soil as isotropic, homogeneous material. Thus, to apply such theory on rafts over non-uniform soil, one should replace the characteristics of non-uniform soil by a uniform one, and at the same time keep the response of raft over the original soil under the given column loads. By considering the deflection w of Eq.(3) as the main response of the raft, and considering that the fictitious uniform soil has coefficient k_o for subgrade reaction, where;

$$k_i = n_i\,k_o \qquad (4)$$

then

$$w = \frac{P_i}{4}\left[\frac{1}{D}\sqrt{\frac{D}{k_i}}\right] Z_3$$

$$w = \frac{P_i}{4}\sqrt{\frac{1}{n_i\,k_o\,D}}\;Z_3$$

$$w = \frac{P_i'\,L_o^2}{4\,D}\,Z_3 \qquad (5)$$

Eq. (5) is analogous to Eq. (3), with the following modified values;

$$\text{Modified Column Load,} \quad P_i' = \frac{P_i}{\sqrt{n_i}} \qquad (6)$$

$$\text{and} \quad L_o = \sqrt[4]{\frac{D}{k_o}}$$

With Eq. (6), a new set of modified column loads is arised for a uniform fictitious soil. Then the ACI Committee 436 recommendations for design of mat foundation can be applied using there modified column loads. Once the deflection w is computed based on the modified column loads on a uniform soil is equivalent to the deflection w of the true column loads on erratic soil, it is assumed that the bending moments and shearing forces in the raft will follow the same coarse and, therefore, Hetenyi's functions may be used to compute them. However, a direct approach may be used to compute bending moments and shearing forces from the deflection w and raft flexural rigidity.

Using Hetenyi's function for rectangular coordinates :

$$M_x = -\frac{P_i'}{4}\left[\left(Z_4 - \frac{(1-\mu)\,Z_3'}{(\frac{r}{L_i})}\right)\cos^2\phi + \left(\mu\,Z_4 + \frac{(1-\mu)\,Z_3'}{(\frac{r}{L_i})}\right)\sin^2\phi\right] \qquad (7)$$

$$M_y = -\frac{P_i'}{4}\left[\left(Z_4 - \frac{(1-\mu)\,Z_3'}{(\frac{r}{L_i})}\right)\sin^2\phi + \left(\mu\, Z_4 + \frac{(1-\mu)\,Z_3'}{(\frac{r}{L_i})}\right)\cos^2\phi\right] \tag{8}$$

$$Q = -\frac{P_i'}{4\,L_i}\,Z_4' \tag{9}$$

where r = distance of the point under investigation from column load along radius, (Z_3', Z_4, Z_4') = functions for moment and shear (see Figure 1) and Φ = angle as defined in Figure (2).

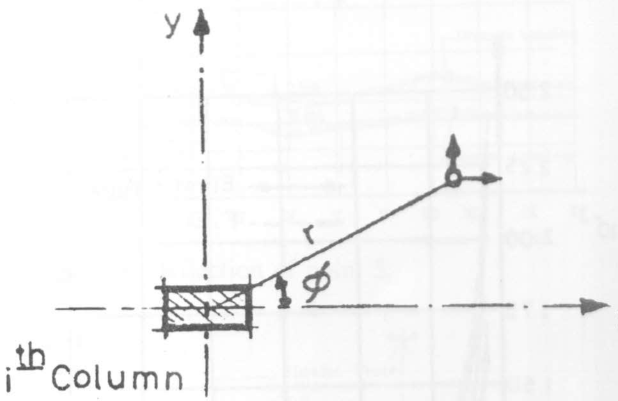

Figure 2. Angle ϕ.

FINITE-ELEMENT METHOD

With a rectangular plate and three general displacements at each node (deflection and two rotations) 12 unknowns of the displacement function are necessary. This function is of the form

$$w = a_1 + a_2x + a_3y + a_4x^2 + a_5xy + a_6y^2 + a_7x^3 + a_8x^2y + a_9xy^2 + a_{10}y^3 + a_{11}x^4 + a_{12}y^4 \tag{10}$$

The subgrade reactions of each element of the grid are prescribed to account for soil non-uniformity. In Figure (3) the dark elements rest on a softer soil.

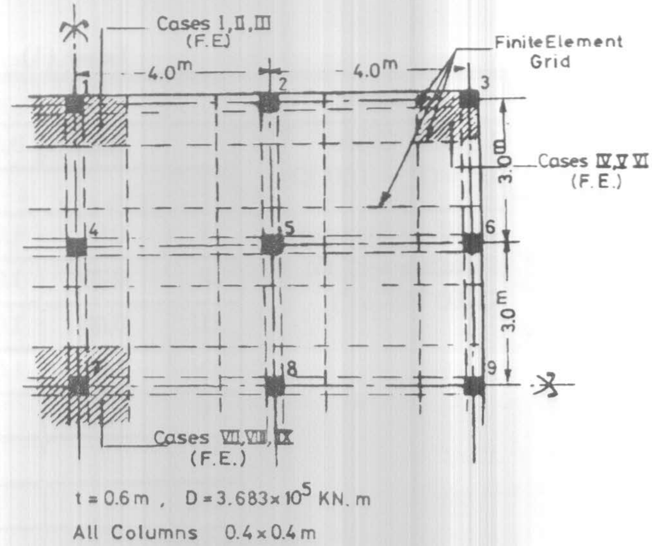

Figure 3. Studied Raft.

RESULTS

A raft having a thickness of 0.6 m and a flexural rigidity of 3.683 x 10^5 KN.m is considered in the analysis. The raft dimensions and columns arrangement as well as finite element grid are shown in Figure (3). The studied cases are presented in Table (1). These cases include unequal column loading (case A) and equal column loading (case B). For each case there are ten cases of subgrade reactions, nine of these are for erratic soil and the tenth is for a uniform soil. In Table (1) the values of P_o and k_o are 1000 KN and 25 x 10^4 KN/m^3 respectively. For the finite element solution k is prescribed according to Table (1) as shown in Figure (3).

Figures (4) and (5) illustrate the variation of deflection Δ under certain points of the studied raft for the cases of study of Table (1). As expected, the deflection increases when soft soil spots locate under studied points as may be observed from Figure (4). Also, it is easy to observe from the same figure, that the present elastic plate results nearly match those obtained using the finite element method. Figure (5) shows the deflection of point 5 for the ten cases for equal and unequal cases of loading. Again, a very good match is observed from the figure for the two methods. However, it seems that the present method is slightly stiffer than finite element method, as the deflections obtained by present method are less by about three to five percent compared to finite element results. Figure (5) shows also the minor effect of the soft soil spot on the deflection of the raft away from these spots.

Table (1). Studied Cases

Column No.		1	2	3	4	5	6	7	8	9
Loading Cases $xP_o = (P_o = 1000\ KN)$	A	0.67	0.67	0.41	1.00	1.00	0.67	1.00	1.00	0.67
	B	0.744								
Coeff. of Subgrade Reaction x k_o (k_o = 25000 $KN\backslash m^3$)	I	0.04	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	II	0.20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	III	0.40	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	IV	1.00	1.00	0.04	1.00	1.00	1.00	1.00	1.00	1.00
	V			0.20						
	VI			0.40						
	VII							0.04		
	VIII							0.20		
	IX							0.40		
	X	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

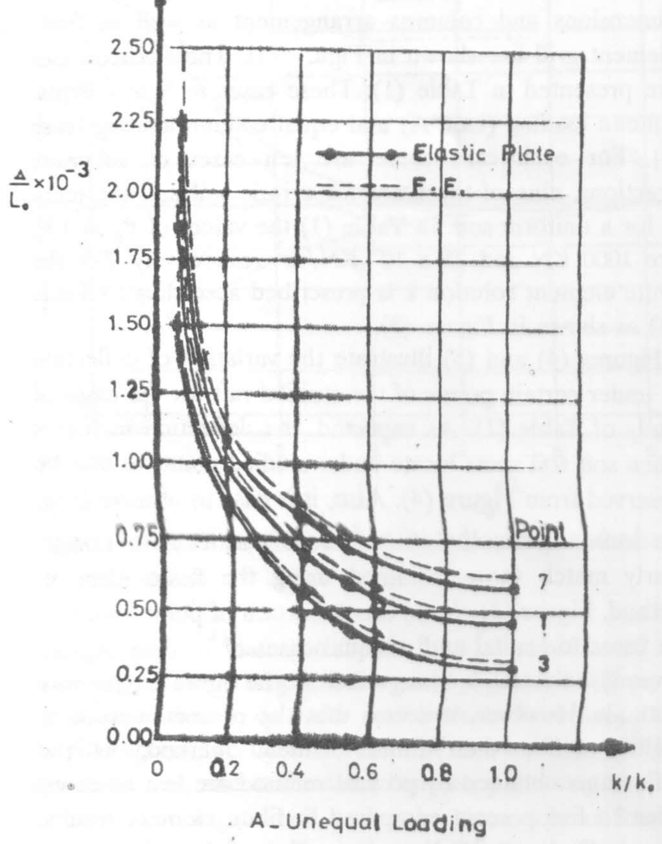

Figure 4. Deflections.

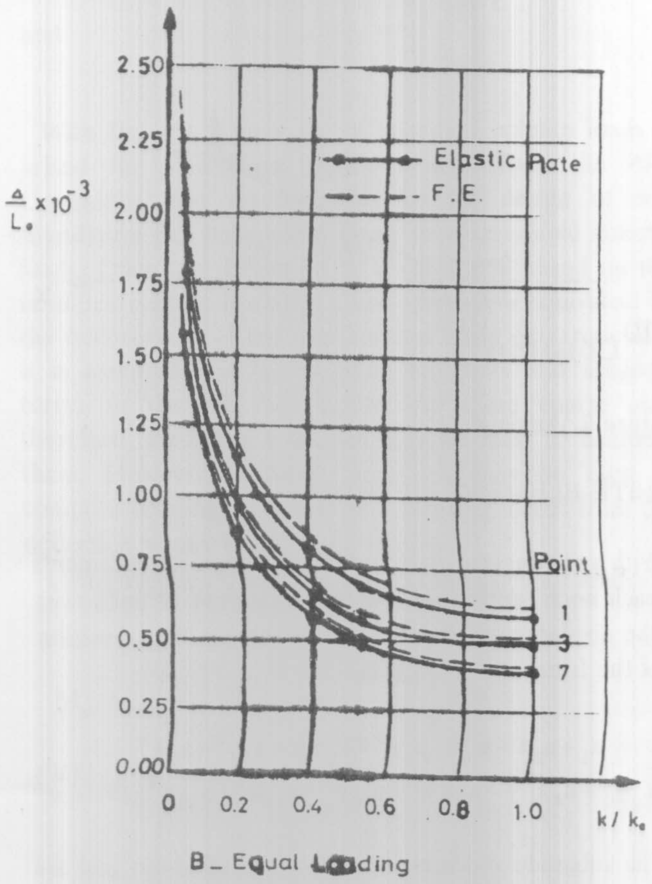

Figure 4-b. (Cont). Deflections.

The same figure shows an increase of the deflection when soft spots is an inside one (case VII) and a decrease of the deflection, when soft spot is under the corner of the raft (case IV).

The bending moments along axes of symmetry of the raft for cases of study are presented in Figures (6) and (7). Again, a very good agreement between the results of the present elastic plate and the finite element methods is observed. From the figures, one can notice that the results obtained by the elastic plate method is slightly higher than those obtained using finite element method. As may be expected, the effect of soft spots is to increase the negative moments for inner spots and to increase positive moments for outer spots, which is the case in Figures (6) and (7). Also, equal loading case increases the positive moments and consequently decreases the negative moments.

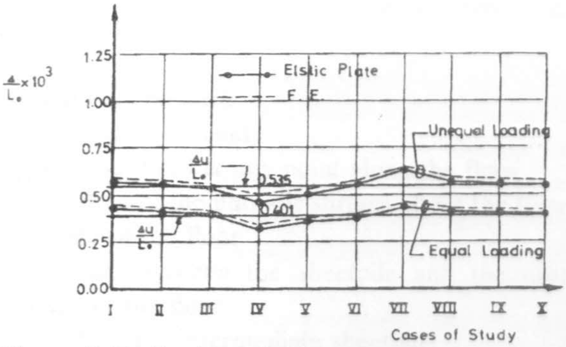

Figure 5. Deflection of point 5.

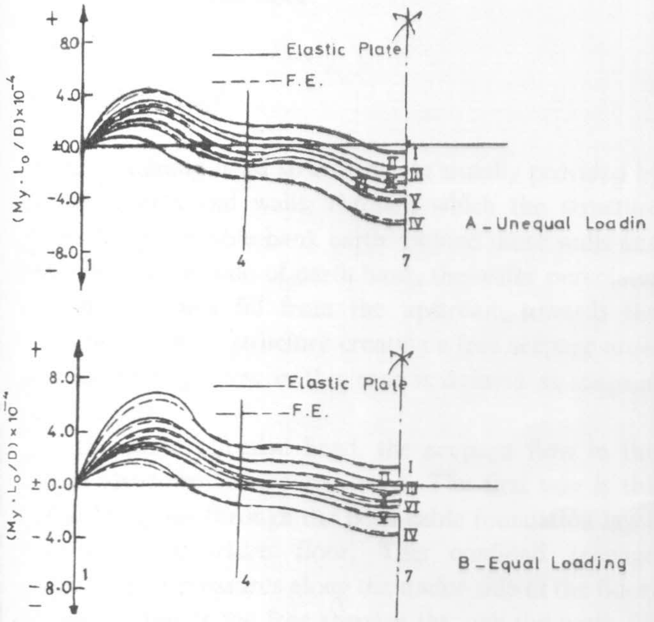

Figure 6. Bending Moment (Axis 1-7).

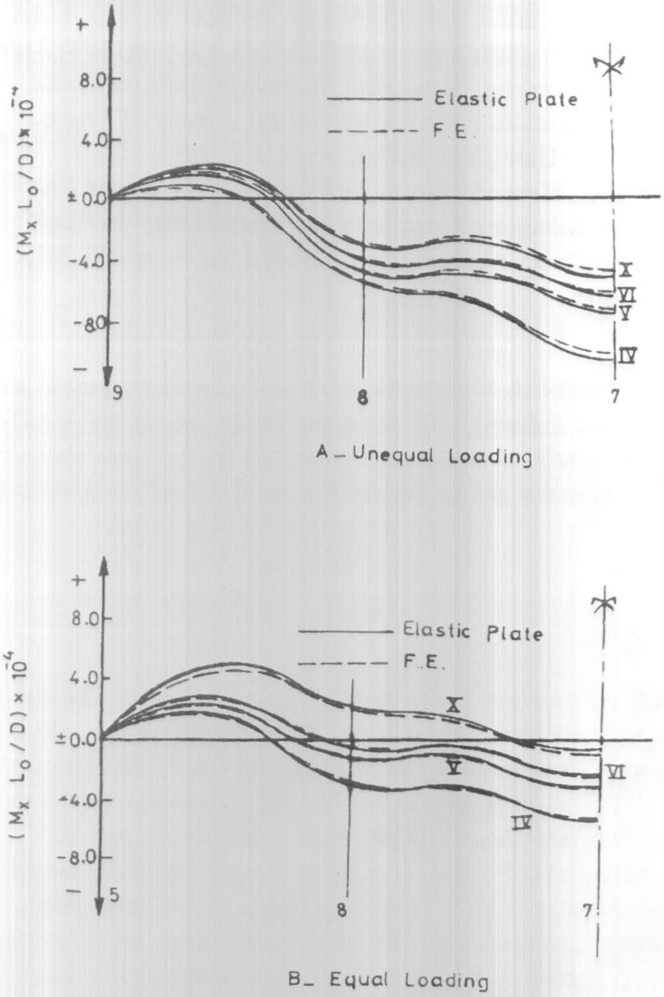

Figure 7. Bending Moment (Axis 7-9).

CONCLUSIONS

Elastic plate method with modified column loads to suit the coefficient of subgrade reaction of the soil under zones of influence of each column is introduced to analyze flat raft on erratic soil. A very good agreement between the results of the analysis using present method and finite element method on Winkler foundation is reported. Modified elastic plate method as described herein yields a slightly stiffer system compared to finite element model on Winkler foundation. The bending moments obtained using the modified elastic plate method are slightly higher than those obtained using finite element method. The effect of soft soil spots is to increase the response in the vicinity of these spots. Inner spots increase negative moments in the raft, while outer spots increase the positive moments in the raft.

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