DESIGN OF DOWNSTREAM FILTER OF A DAM

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ABSTRACT

A study on the proper length of a filter installed downstream and a dam with an end sheet pile is presented in this paper. The finite element method is applied and a numerical model which covers a wide and practical range of variables is designed. The results are provided in non-dimensional graphs which may be used for design purposes. In addition a general design equation is presented to suit cases which cannot be evaluated directly from the graphs. Finally, two numerical examples are given to illustrate the procedure to be followed in designing the length of a downstream filter.

NOTATIONS

A	constant contained in Eq. 11;
A^e	element domain;
A'	flow domain;
B	length of dam floor;
C	constant contained in Eq. 11;
D	depth of permeable soil;
{F}	global nodal force vector;
I	number of nodes in element;
[K]	global stiffness matrix of a domain;
K^e_{nm}	element stiffness matrix;
k	hydraulic conductivity of soil;
L	length of filter;
N_m	linear interpolation functions;
N_n	linear interpolation functions;
q_n	normal flux across the boundary;
q_t	total seepage beneath a dam;
R	residual;
S	length of sheet pile;
s	boundary of flow domain;
ϕ	velocity potential;
$\{\phi\}$	global vector unknown velocity potential;
ϕ_n	nodal values of velocity potential;
$\overline{\phi}$	approximate value of velocity potential; and
ψ	stream function

INTRODUCTION

The problem of confined flow through homogeneous soil beneath a concrete dam has been solved by considering the case of a dam without any filter and taking into account an infinite downstream discharge face [6,7,9]. A graphical solution for the above mentioned problem [1] according to Forchheimer's trial and error method and an approximate solution for the problem [5], accounting for the existence of a downstream filter, are also available. The case of a concrete dam with a filter installed partially underneath the floor and extending in the downstream direction was investigated by Hathoot [4]. Pavlovsky [9] and Muskat [6] solved, independently, the problem of a dam with single sheet pile. They also provided graphical solutions for the case of symmetrically placed pilings [3].

In this paper the objective is to investigate the proper length of a downstream filter considering confined seepage beneath a concrete dam with an end sheet pile, Figure (1) For convenience the finite element technique is used in performing the calculations for the above mentioned problem.

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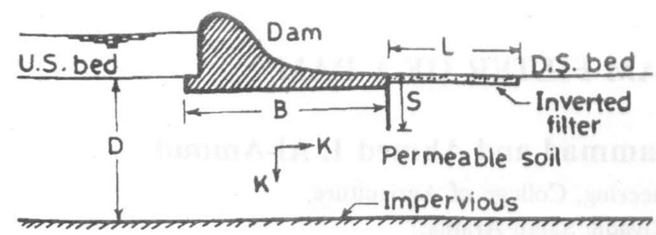

Figure 1. Geometry of the problem.

THEORY

In two dimensional steady flow in homogeneous isotropic soil, the governing differential equation is that of Laplace:

$$k\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) = \nabla^2\phi = 0 \tag{1}$$

in which k is the hydraulic conductivity of soil and ϕ is the velocity potential. In terms of the stream function, ψ, Eq. 1 can be written as:

$$k\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) = \nabla^2\psi = 0 \tag{2}$$

Equation 1 may be solved numerically by applying the finite element method starting from the residual approach [10]:

$$\nabla^2\bar{\phi} = R \tag{3}$$

in which $\bar{\phi}$ is the approximate value of the velocity potential and R is the residual. The value $\bar{\phi}$ can be written as:

$$\bar{\phi} = \sum_{n=1}^{I} N_n \phi_n, \quad (n=1,2,...,I) \tag{4}$$

in which N_n are linear interpolation functions, ϕ_n are the nodal values of ϕ and I is the number of nodes in the element. Substituting the value of $\bar{\phi}$ as given by Eq. 4 into Eq. 3 and considering Galerkin's condition, Eq. 4 becomes:

$$\sum_{n=1}^{I} \int_{A'} \nabla^2 \phi_n \, dA' = 0 \tag{5}$$

in which A' is the flow domain.

Integrating Eq. 5 by parts and considering to Green-Gauss theorem [2,8], Eq. 5 takes the form:

$$\sum_{n=1}^{I} \left\{ \int_{A^e} \left[\frac{\partial N_m}{\partial x}\left(\frac{\partial N_n}{\partial x} + \frac{\partial N_m}{\partial y}\left(\frac{\partial N_n}{\partial y}\right) + \frac{\partial N_n}{\partial x} + \frac{\partial N_n}{\partial y}\right) \right] dxdy \right\} \phi_n - \int_s N_m q_n \, ds = 0, \quad (m=1,2,...I) \tag{6}$$

in which A^e is the element domain, s is the boundary of the flow domain and q_n denotes the normal flux across the boundary of the element. For convenience, Eq. 6 is put in the following form:

$$\sum K^e_{nm} = F^e_n \tag{7}$$

where

$$K^e_{nm} \int_{A^e} \left[\frac{\partial N_m}{\partial x}\left(\frac{\partial N_n}{\partial x} + \frac{\partial N_n}{\partial y}\right) + \frac{\partial N_m}{\partial y}\left(\frac{\partial N_n}{\partial x} + \frac{\partial N_n}{\partial y}\right) \right] dxdy \tag{8}$$

and

$$F^e_n = \int_s N_m q_n \, ds \tag{9}$$

since in our case [K] is symmetric, we have $K^e_{nm} = K^e_{mn}$. Equation 7 is then applied to all the elements of the mesh and finally we get the following set of simultaneous equations:

$$[K]\{\phi\} = \{F\} \tag{10}$$

where [K] is the global stiffness matrix, $\{\phi\}$ is the global vector of unknown potential and {F} is the global nodal force vector. Equation 10 can be used to get unknown ϕ values at different nodes and a similar procedure can be used to get unknown ψ values.

NUMERICAL MODEL AND RUNS

The mesh of triangular elements is shown in Figure (2). For convenience, a constant floor length of dam (B = 60.0 m) is considered. The thickness of the permeable soil has the range (D = 10.0 m to D = 60.0 m) and the length of the sheet pile ranges between 0.0 and 4.0 m. The boundary BCD, Figure (2), representing the base of the dam and sheet pile is considered to be the streamline Ψ = 0.0 whereas the boundary GF has ψ = 100.0. The boundaries AB and CE have the property $\partial\psi/\partial n = 0.0$. The program FEM2D [10] is the basis of computation for the cases of a dam without sheet pile, and special subroutines are developed and applied to suit cases in which an end sheet pile of variable length exists. At the beginning a depth D = 10.0 m is considered with sheet pile length S = 0.0, 1.0, ..., 4.0. Then D is increased at intervals of 10.0 m up to D = 60.0 m and for each value of D the above values of S are introduced. Computations are carried out to provide ψ values at the nodal points of elements. At any nodal point on the boundary CE the ψ value indicates the percentage seepage that takes place between point C and the point under consideration. The objective of the following analysis is to find the optimal point, H, on CE that corresponds to a high percentage of seepage and a limited length CH which, in fact, represents the proper length of filter.

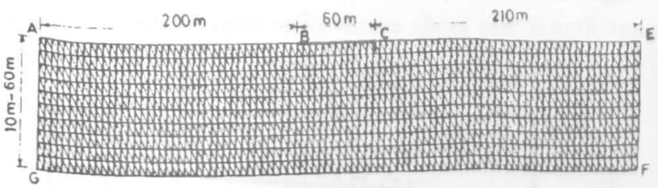

Figure 2. Finite element mesh.

It is worthy to note that the results of the above mentioned runs are checked by considering the boundary HE as impervious [4] and evaluating ϕ values and hence the seepage taking place between C and H, and the comparison indicates satisfactory results. The method followed in checking the results are not used in solving the problem since it is lengthy and cumbersome.

RESULTS AND DISCUSSION

For all runs the ϕ values of the nodes lying on the discharge face CE are examined and, for convenience, points corresponding to ψ = 90%, 92%, 95%, 96%, 97%, 98%, 99%, of the total seepage, q_t, are located by interpolation. In Figures (3) through 8 the filter length ratio L/D is plotted versus the sheet pile length ratio S/D for the seepage percentages listed above. Each of the above figures corresponds to a specific value of B/D, which ranges between 1.0 and 6.0. In all figures the filter length ratio linearly increases on increasing the sheet pile length ratio. Moreover, as B/D decreases, corresponding L/D values decrease, the rate of decrease being lower at smaller B/D values.

Table (1).

increase in seepage	average increase in filter length ratio (L\D)
95% - 96%	8.0%
96% - 97%	9.5%
97% - 98%	12.5%
98% - 99%	19.0%

In Figures (3) through 8 it is evident that as percentage seepage increases, filter length ratio increase, the rate of increase being higher for higher seepage percentages. This may be attributed to the fact that the rate of divergence of streamlines increases at higher seepage percentages. In Table (1) are listed the average percentage increase in filter length ratio corresponding to 1% increase in seepage.

From the values in Table 1 it is clear that the filter length considerably increases between 98 % and 99% with the result that if the 98 % streamline is recommended as a design criterion, though only 2 % of the seepage will not be taken by the filter, a saving of about 19 % in the length of the filter will be achieved. The 98 % seepage curves in Figures (3 through (8)) can be represented by the following linear equation:

$$\left(\frac{L}{D}\right) = A + C\left(\frac{S}{D}\right) \qquad (11)$$

where A and C are constants to be found from Figure (9). Design length of filter may be obtained either from Figures. (3 through (8)) or by applying Eq. 11 which is necessary in cases where B/D values are not available in the above figures. In the following examples is shown the design procedure for evaluating the length of filter.

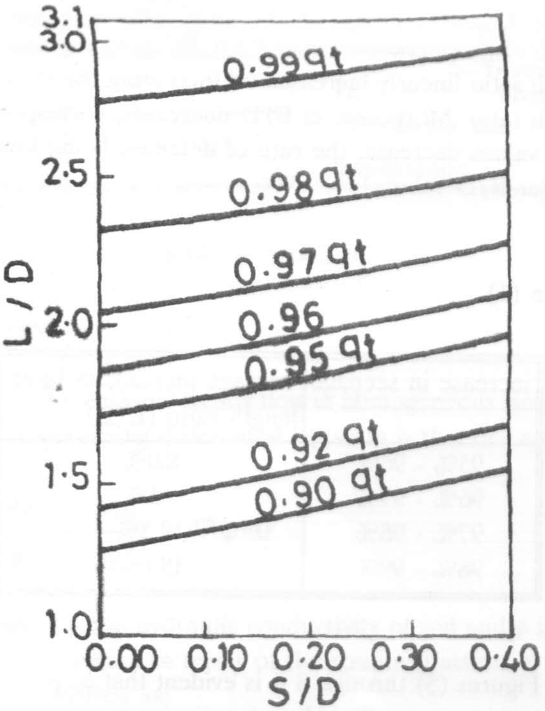

Figure 3. Filter length ration versus sheet pile length ratio for B/D = 6.0.

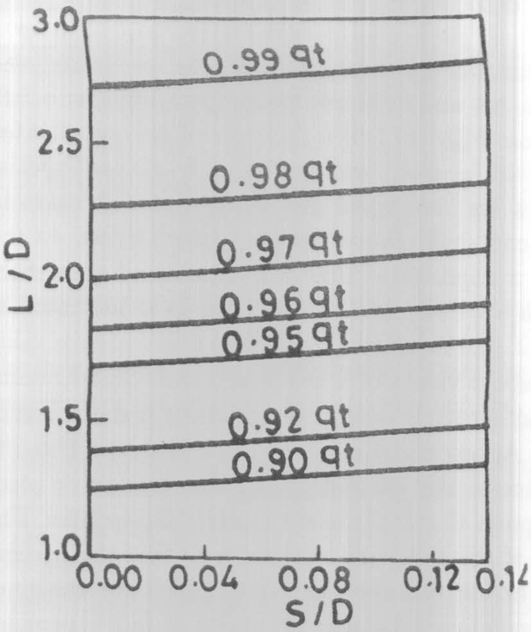

Figure 5.Filter length ration versus sheet pile length ratio for B/D = 2.0.

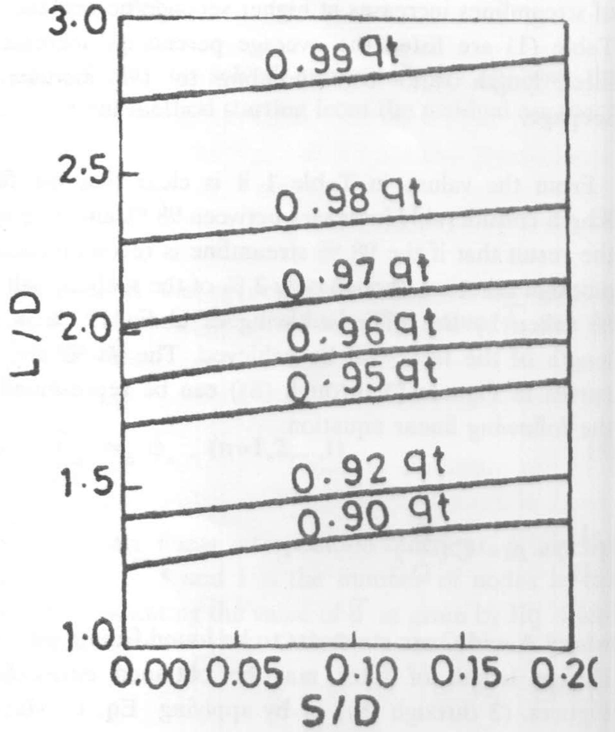

Figure 4. Filter length ration versus sheet pile length ratio for B/D = 6.0.

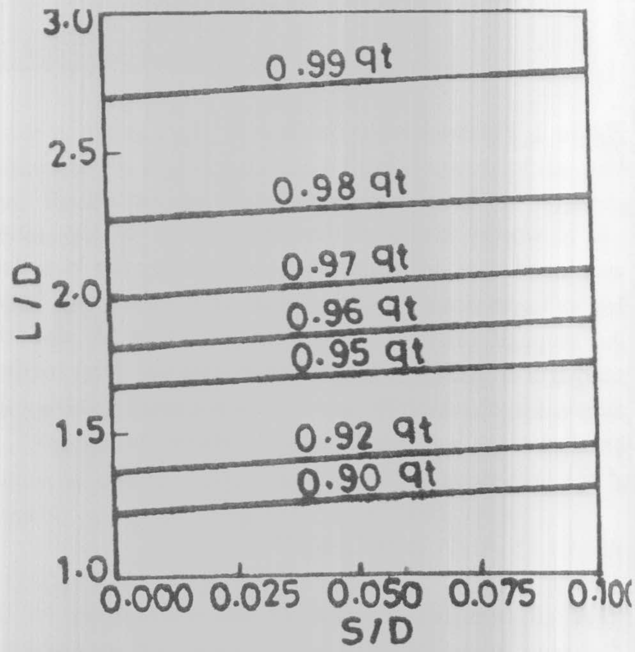

Figure 6. Filter length ratio versus sheet pile length ratio for B /D = 1.5.

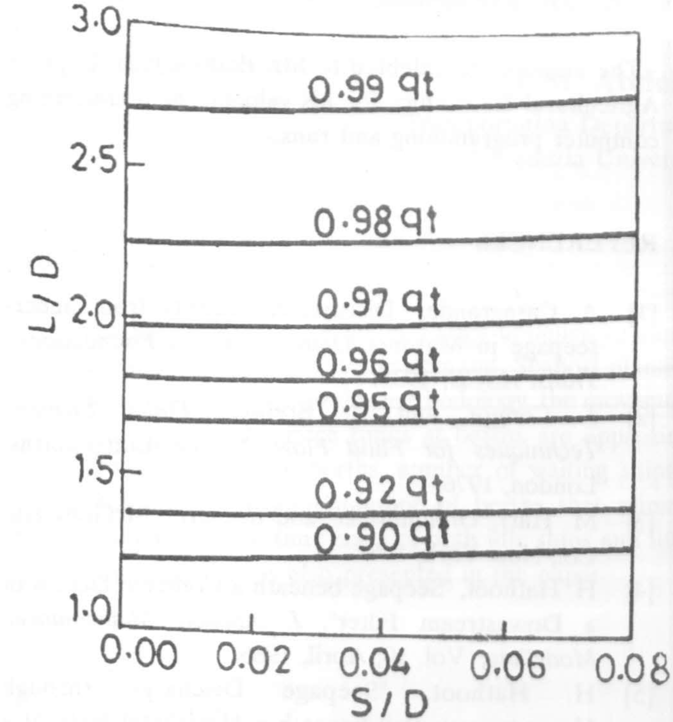

Figure 7. Filter length ratio versus sheet pile length ratio for B/D = 1.2.

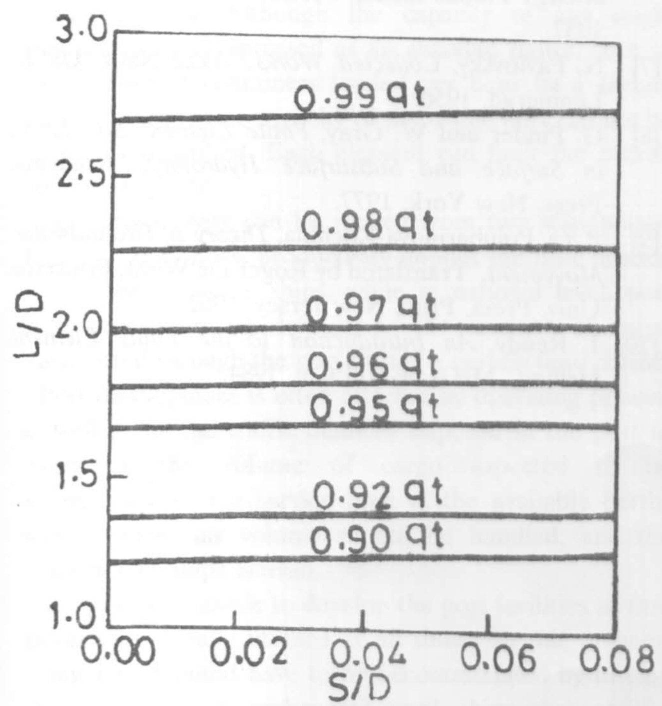

Figure 8. Filter length ration versus sheet pile length ratio for B/D = 1.0.

Numerical Example 1

It is required to design the length of a filter downstream from a dam of floor length 30.0 m if a sheet pile 1.0 m long is provided at the end of the downstream apron. The top soil is underlain by an impervious layer 20.0 m deep from the channel bed.

Solution

$$\frac{B}{D} = \frac{30.0}{20.0} = 1.5,$$

$$\frac{S}{D} = \frac{1.0}{20.0} = 0.05$$

From Figure (6). we have

$$\frac{L}{D} = 2.254, \text{ from which}$$

$$L = 2.254(20.0) = 45.08\text{m}.$$

Numerical Example 2:

It is required to design the length of filter for the following data:

S = 1.5 m, B = 60.0 m and D = 25.0 m.

Solution

$$\frac{B}{D} = \frac{60.0}{25.0} = 2.4,$$

Since B/D does not correspond to any of the Figures (3 through (8)), Eq. 11 is to be applied. From Figure (9) we have:

A = 2.255 and C = 0.826.

Applying Eq. 11:

$$\left(\frac{L}{D}\right) = 2.255 + 0.826(0.025)$$

$$= 2.276, \text{from which}$$

$$L = 2.276(25.0) = 56.9\text{m}$$

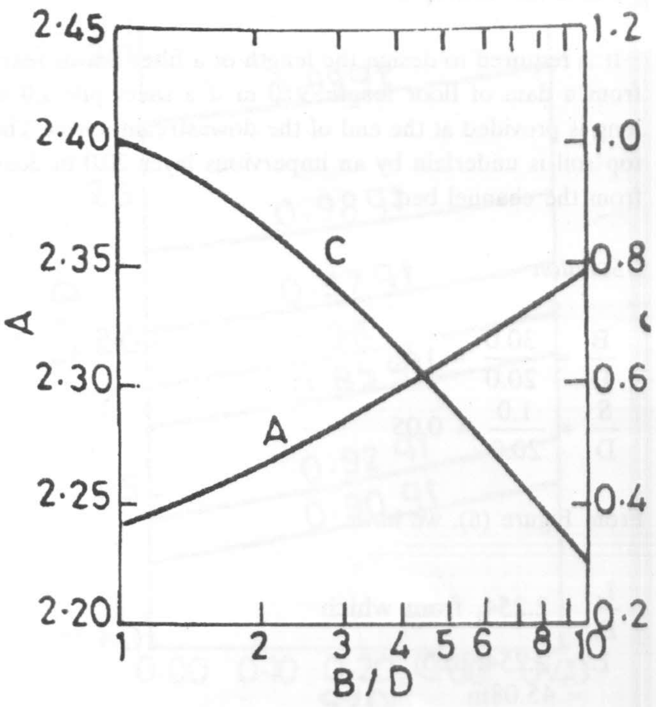

Figure 9. The constants A and C of Eq. (11) versus floor length ratio of dam.

CONCLUSION

For the problem of seepage beneath a dam with an end sheet pile it is found that the length of a downstream filter depends upon the length of floor, the length of sheet pile and the depth of the permeable soil. It is concluded that the point of intersection of the streamline corresponding to 98 % seepage with the downstream bed constitutes the design criterion for the length of filter. In general it is recommended that the length of filter be in the range 2.242 to 2.525 of the depth of the permeable soil underneath the dam. The solution of two numerical examples show that both design charts and equation provided in the paper are simple and practical.

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