

# AN INVESTIGATION OF THE PERFORMANCE OF A JOURNAL BEARING WITH A SLIGHTLY - IRREGULAR BORE

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## SUMMARY

A finite journal bearing having a slightly-elliptical bush, and subjected to a set of prescribed loads is numerically analyzed. For each bearing load, the minimum film thickness, the maximum oil pressure and the friction torque are obtained. The effects of the variation of both the ellipticity and the angle between the major axis of the bush and the load line on these operating characteristics are investigated.

## 1. INTRODUCTION

In the journal bearing studied in this work, there is a small deviation in the bearing bore from the regular circular shape.

Goenka and Booker [1] investigated the performance of such bearing, (referred to in their paper as an irregular bearing), for a given cycle with a constant load and sinusoidal angular displacement. They analyzed a bearing having an infinite length and an approximate elliptic shape with one of the axes of the ellipse in the direction of the load. They studied a number of shapes in order to obtain the shape which maximizes the minimum film thickness.

The configuration studied in the present work should not be confused with the two-lobe bearing, which was first investigated by Pinkus [2], and which is frequently referred to as an elliptical bearing also. It is well known that the two-lobe bearing is composed of two circular arcs with two offset centres and two longitudinal grooves. Both bearings, however, have the advantage of being beneficial with regard to oil-film whirl. An elliptic bearing bore would produce more than one positive-pressure zone. Accordingly, this will improve the stability of the rotor-bearing system.

The present analysis deals with a journal bearing having a finite length, a bush with an elliptical bore, and is subjected to a set of prescribed bearing loads. For each load, the maximum oil pressure, the friction torque and the minimum film thickness are computed. The effect of varying the ellipticity of the bearing on these operating characteristics is investigated. To generalize the analysis;

another parameter is also considered, that is, the inclination of the bearing axes on the load line. The effect of the variation of this angle on the foregoing bearing characteristics is also studied. Usually, the minimization of both the friction torque and the maximum oil pressure, and the maximization of the minimum film thickness is a desirable design objective.

## 2. GOVERNING EQUATIONS

The bearing geometry, and the relative positions of the load line, line of centres and the bearing centre line (major axis) are shown in Figure (1). The journal keeps its regular circular shape and the bearing is considered to have an elliptical profile. The major axis is assumed to be inclined to the load with an angle  $\alpha$  (measured in the direction of journal rotation).

The bearing radius  $r_b$  is assumed to be described by the equation

$$r_b = R_{\min} \sin^2 (\theta - \alpha) + R_{\max} \cos^2 (\theta - \alpha) \quad (1)$$

where  $R_{\max}$  and  $R_{\min}$  are the maximum and minimum radii of the bearing respectively.

The film thickness  $h$  is expressed by

$$h = r_b - r_j + e \cos (\theta - \phi) \quad (2)$$

Substituting eqn. (1) yields

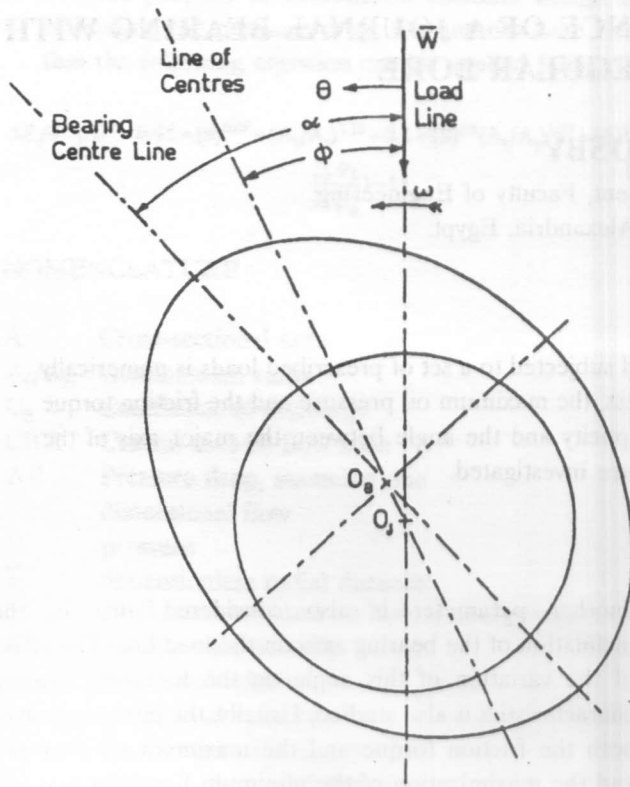


Figure 1. Bearing geometry.

$$h = R_{\min} \sin^2(\theta - \alpha) + R_{\max} \cos^2(\theta - \alpha) - r_j + e \cos(\theta - \phi) \quad (3)$$

Normalizing, the nondimensional film thickness is

$$H = [\sin^2(\theta - \alpha) + \delta \cos^2(\theta - \alpha)] / \psi + 1 - (1/\psi) + \epsilon \cos(\theta - \phi) \quad (4)$$

where

$$H = h/c \quad (5)$$

$$\psi = c/R \quad (6)$$

is the clearance ratio, and

$$\delta = R_{\max} / R_{\min} \quad (7)$$

is the ellipticity.

The clearance  $c$  used in the normalized expressions is "arbitrarily" defined as

$$c = R_{\min} = r_j \quad (8)$$

Eqn. (4) could be written in the condensed form

$$H = 1 + G \cos^2(\theta - \alpha) + \epsilon \cos(\theta - \phi) \quad (9)$$

where

$$G = (\delta - 1) / \psi \quad (10)$$

is the "noncircularity coefficient".

The lubricating film is assumed to be governed by Reynolds' equation for an incompressible Newtonian isoviscous fluid, which could be written in the normalized form

$$\partial[H^3(\partial P/\partial \theta)]/\partial \theta + \beta^2 \partial[H^3(\partial P/\partial Z)]/\partial Z = 6 dH/d\theta \quad (11)$$

where

$$P = p \psi^2 / \mu \omega \quad (12)$$

$$\beta = 1/d \quad (13)$$

$$Z = 2z/1 \quad (14)$$

This particular bearing configuration does not have a longitudinal oil-admission groove, so that if Swift-Stieber boundary conditions [3] are considered to prevail; it will be assumed that the pressure gradient vanishes at both the leading and trailing boundaries of the load-carrying zone, i.e.

$$P(\theta, Z) = \partial P(\theta, Z) / \partial \theta = 0 \quad (15)$$

at  $\theta = \theta_1, \theta_2$

where  $\theta_1, \theta_2$  are the angular locations of the leading and trailing edges respectively. It should be observed that two positive-pressure load-carrying zones may exist.

Boundary conditions (15) are applicable for a bearing end-lubricated through a circumferential feeding prove. The contribution of the feeding pressure is ignored, so that at both ends of the bush the pressure is assumed to be ambient, i.e.

$$P(\theta, \pm 1) = 0 \quad (16)$$

### 3. SOLUTION PROCEDURE

For a specified set of loads; three operating characteristics are to be computed: the maximum pressure

$P_m$ , the friction torque  $T$ , and the minimum film thickness  $H_{min}$ . The values of the characteristics are to be determined over a range of the noncircularity coefficient  $G$  and the inclination angle  $\alpha$ . The numerical solution is carried on according to the following scheme:

- (i) For prescribed values of  $\alpha$ ,  $G$ ,  $\epsilon$ , and for an assumed value of the attitude angle  $\phi$ ; the pressure distribution is computed. A finite-difference Gauss-Seidel technique with overrelaxation [4] is used to solve eqn. (11). To account for Swift-Stieber boundary conditions, all negative pressures are set equal to zero after each iteration. Thus the conditions that should be satisfied during the numerical computations are

$$P(\theta, Z) \geq 0$$

$$P(\theta, Z) = P(2\pi + \theta, Z)$$

Convergence is attained when

$$\sum |P_n - P_{n-1}| / \sum |P_n| < \epsilon_p$$

$\epsilon_p$  is taken as 0.0001

- (ii) The dimensionless load components  $W_n$  normal to the load line and  $W_p$  parallel to it, are calculated from

$$W_n = \int_{-1}^{+1} \int_0^{2\pi} P \sin\theta \, d\theta dZ \quad (17)$$

$$W_p = \int_{-1}^{+1} \int_0^{2\pi} P \cos\theta \, d\theta dZ$$

If

$$|W_n / W_p| > \epsilon_w$$

where  $\epsilon_w$  is taken as 0.001, then another value of  $\phi$  is assumed until convergence is achieved.

- (iii) Steps (i) and (ii) are repeated to calculate  $W$  for different combinations of  $\alpha$ ,  $G$  and  $\epsilon$ . An interpolation process is performed to get the values of these parameters for the prescribed set of bearing loads.
- (iv) Again, for each of the values of  $\alpha$ ,  $G$  and  $\epsilon$  corresponding to the set of loads - which were calculated in step (iii) - the pressure distribution and

the attitude angle are computed, from which  $P_m$ ,  $T$  and  $H_{min}$  are determined.

#### 4. RESULTS AND DISCUSSION

Computations have been carried on for a bearing having a length-to-diameter ratio of 0.5. The eccentricity ratio ranges from 0.2 to 0.95. The inclination angle varies over a range of 180 deg. An increment of 5 deg (reduced to 1 deg in certain cases) was taken. The noncircularity coefficient ranges from 0.1 to 3.0 with an increment varying between 0.02 to 0.2 according to the situation. Thus, more than 100000 cases have been solved for the sake of keeping the error as minimum as possible.

In order to visualize the magnitude of the circularity coefficient; assume for instance that  $\psi = 0.001$  (which is a commonly used clearance ratio), then the ratio  $R_{maj}/R_{min}$  would be 1.0001 if  $G = 0.1$ , and it would be 1.003 if  $G = 3.0$ .

The bearing loads specified for this study are  $W = 0.05, 0.1, 0.2$  and  $0.5$ .  $W$  is the normalized load given by

$$W = \bar{W} \psi^2 / \mu \omega r l \quad (18)$$

A set of low-to-moderate bearing loads have been chosen. It is thought that this may be the probable range of loads to which this type of bearings would be subjected, since as the load becomes smaller, a bearing would be more susceptible to oil-film whirl.

From the computed data giving  $W$  as a function of  $\alpha$ ,  $G$  and  $\epsilon$ ; the values of these parameters for the prescribed bearing loads were calculated. The results are graphically presented in Figures (2-a) to (d). For any selected values of  $G$  and  $\alpha$ ; each figure gives the eccentricity ratio  $\epsilon(G, \alpha)$  corresponding to the bearing load. For a certain inclination angle,  $\epsilon$  increases with the increase of  $G$ . For a certain noncircularity coefficient,  $\epsilon$  is minimum in the vicinity of  $\alpha = 120$  deg. Evidently, if the bearing bore is circular, the  $G$ - $\alpha$  curves will be a straight line parallel to the  $\alpha$ -axis.

Figures (3 a-d) show the variation of the maximum oil pressure  $P_m$  with  $\alpha$  and  $\epsilon$  for each of the specified loads. If we start by examining Figure (3-a) - which corresponds to a load  $W = 0.05$ -, it will be observed that for the same inclination angle; increasing  $G$  will lead to an increase in

$P_m$  (since  $\epsilon$  increases with the increase of  $G$ ). For a circular bearing;  $P_m = 0.044$  (corresponding to  $\epsilon = 0.046$ ). For the same eccentricity ratio, minimum values of  $P_m$  lie in the range  $\alpha = 35 - 50$  deg. Note that  $\epsilon$  is a dependent parameter since it is a function of  $G$  and  $\alpha$ . To show the effect of varying the inclination angle for a certain non-circularity coefficient; consider, for example, that  $G = 1$ . The range of  $P_m$  will lie between 0.22 and 0.36 according to the value of  $\alpha$ . If  $G = 2$ , the range of  $P_m$  will lie between 0.19 and 0.36. Thus, a proper choice of  $\alpha$  may reduce  $P_m$  to about half its value.

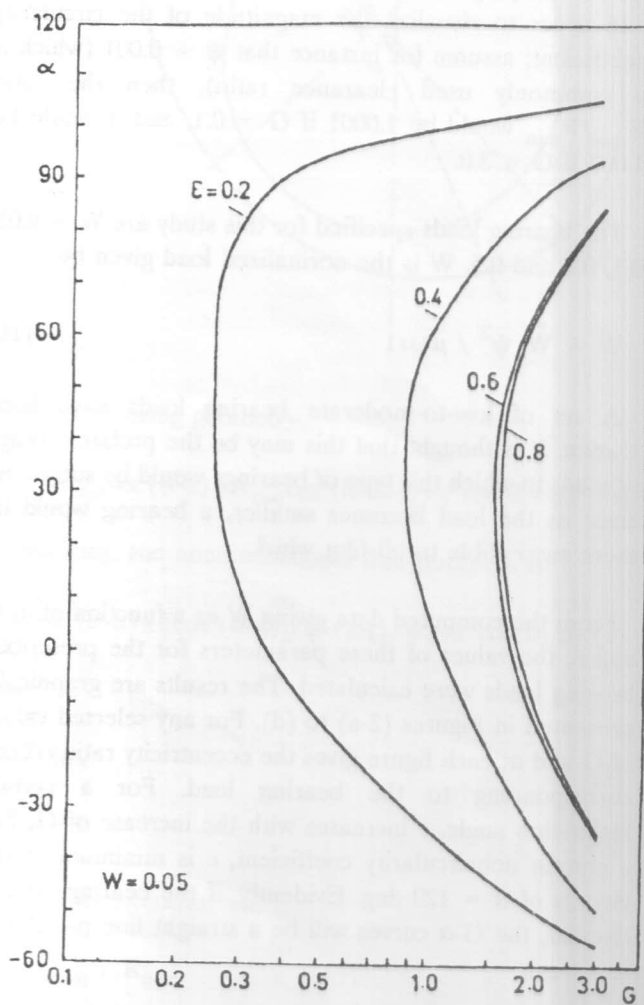


Figure (2-a). Variation of eccentricity ratio with noncircularity coefficient and inclination angle. ( $W = 0.05$ ).

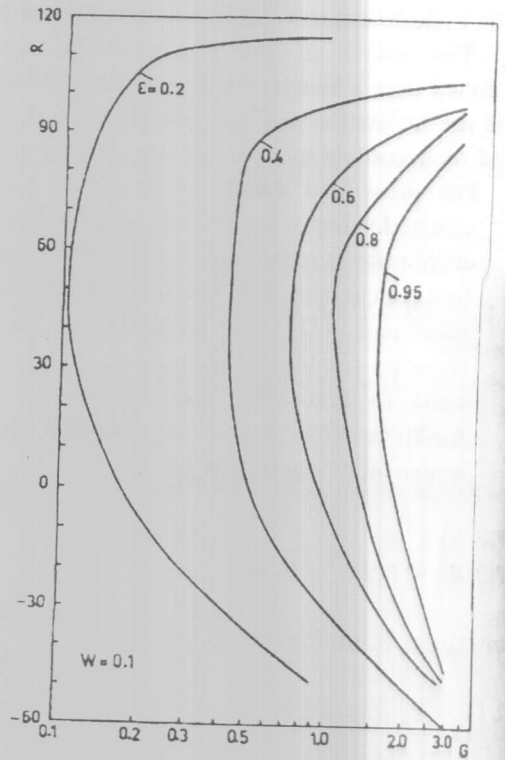


Figure (2-b). Variation of eccentricity ratio with noncircularity coefficient and inclination angle. ( $W = 0.1$ ).

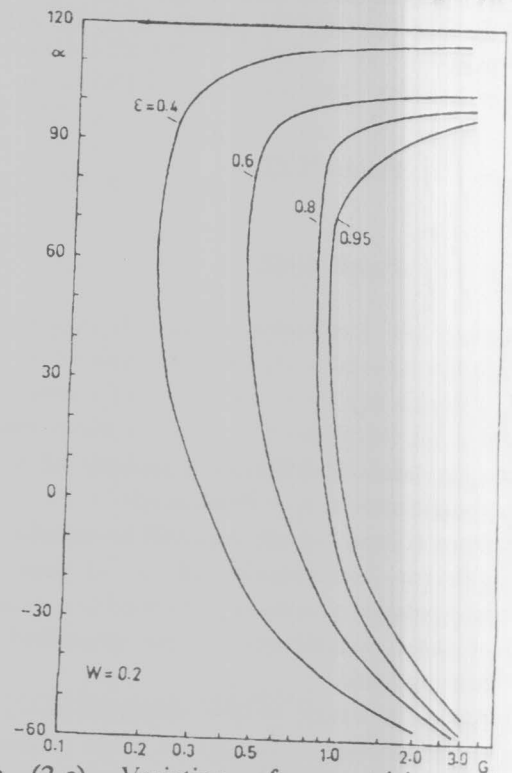


Figure (2-c). Variation of eccentricity ratio with noncircularity coefficient and inclination angle ( $W = 0.2$ ).

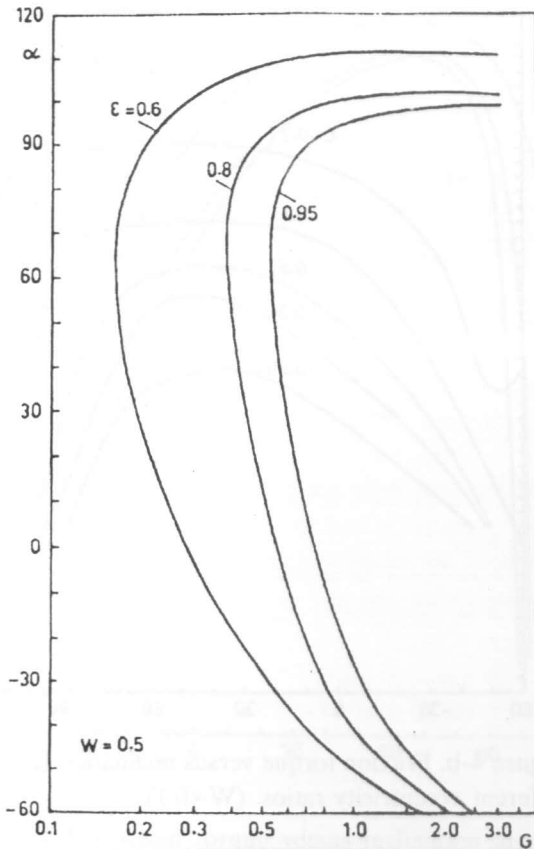


Figure (2-d). Variation of eccentricity ratio with noncircularity coefficient and inclination angle ( $W = 0.5$ ).

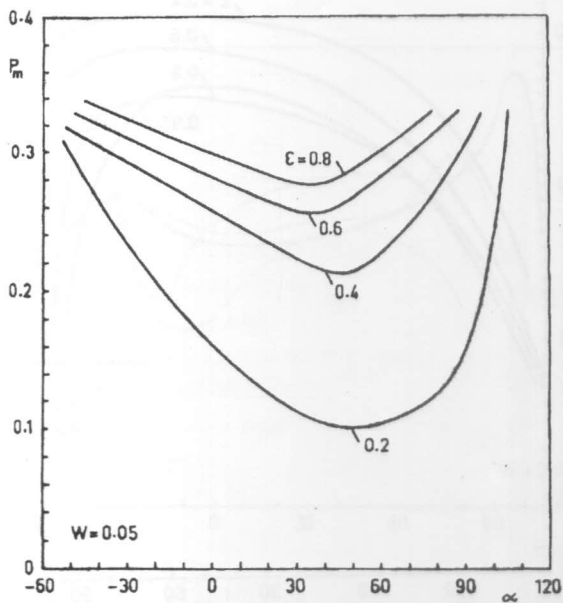


Figure (3-a). Maximum pressure versus inclination angle for different eccentricity ratios. ( $W = 0.05$ ).

Figures (3-b), (3-c) and (3-d) indicate similar trends. In Figure. (3-d), for instance, - where  $W$  is 0.5 - the minimum values of  $P_m$  lie in the range  $\alpha = 35 - 45$  deg. The optimum inclination of the major axis in order to minimize  $P_m$  is approximately 30 - 65 deg for all loads.

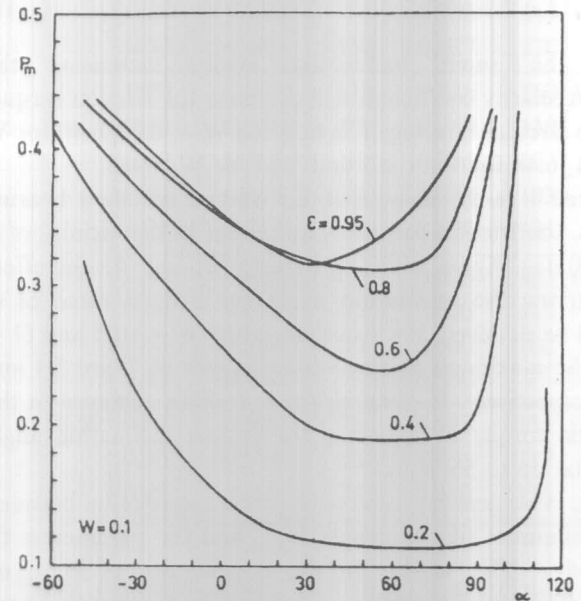


Figure (3-b). Maximum pressure versus inclination angle for different eccentricity ratios ( $W = 0.1$ ).

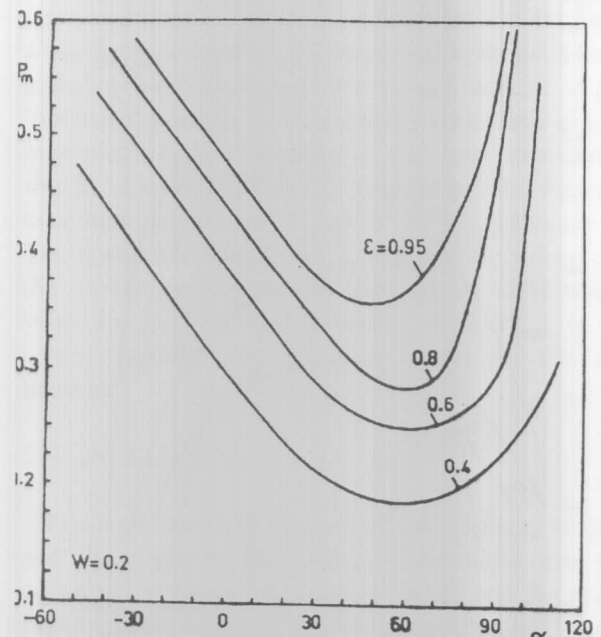


Figure (3-c). Maximum pressure versus inclination angle for different eccentricity ratios. ( $W = 0.2$ )

As the load increases, varying  $\alpha$  becomes more effective. For the same  $G$ , when  $W$  is 0.5, the ratio between the lowest and highest values of  $P_m$  is less than 0.35.

Figures 4 (a,d) give the variation of the friction torque  $T$  with  $\alpha$  and  $\epsilon$ .  $T$  is given by the equation

$$T = \bar{T} c / \mu \omega l r^3 \quad (19)$$

For the same inclination angle, increasing the noncircularity coefficient will decrease the friction torque. For a circular bearing,  $T$  is 6.38 for  $W = 0.05$ , 6.40 for  $W = 0.1$ , 6.48 for  $W = 0.2$  and 7.17 for  $W = 0.5$ .

Figures 4 (a-d) show that for all the specified bearing loads, the friction torque is minimum in the vicinity of  $\alpha = 110$  deg. Again, by using Figures (2) and (4), the effect of varying the inclination angle for a fixed value of  $G$  could be deduced. For example, when  $W = 0.05$  and  $G = 1.0$ , the maximum and minimum values of  $T$  are 5.4 and 2.6 respectively. In general, the reduction achieved in the friction torque by choosing the proper inclination angle may be up to 55 %.

There is no general trend to describe the relation between the minimum film thickness  $H_{min}$  and the parameters  $G$ ,  $\alpha$  and  $\epsilon$ . This is due to the dependence of  $H_{min}$  on several variables changing in different fashions at the same time. An example of the  $H_{min} - \alpha$  relationship for different eccentricity ratios is illustrated in Figure (5) for the case  $W = 0.05$ .

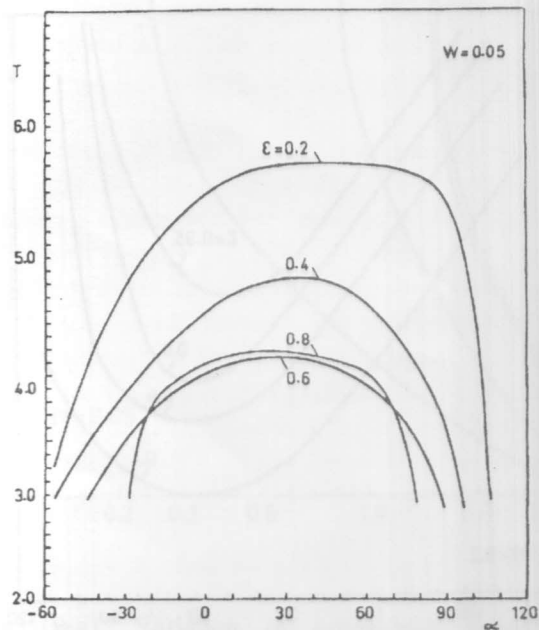


Figure 4-a. Friction torque versus inclination angle for different eccentricity ratios. ( $W=0.05$ ).

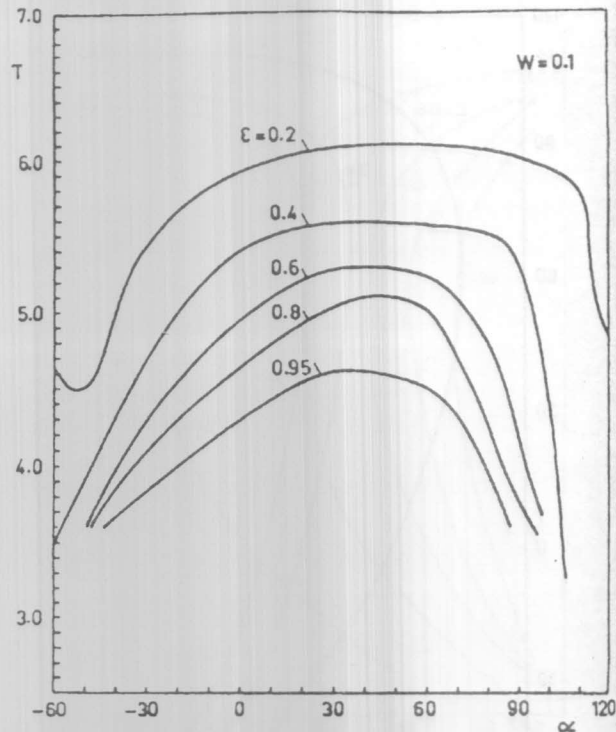


Figure 4-b. Friction torque versus inclination angle for different eccentricity ratios. ( $W=0.1$ ).

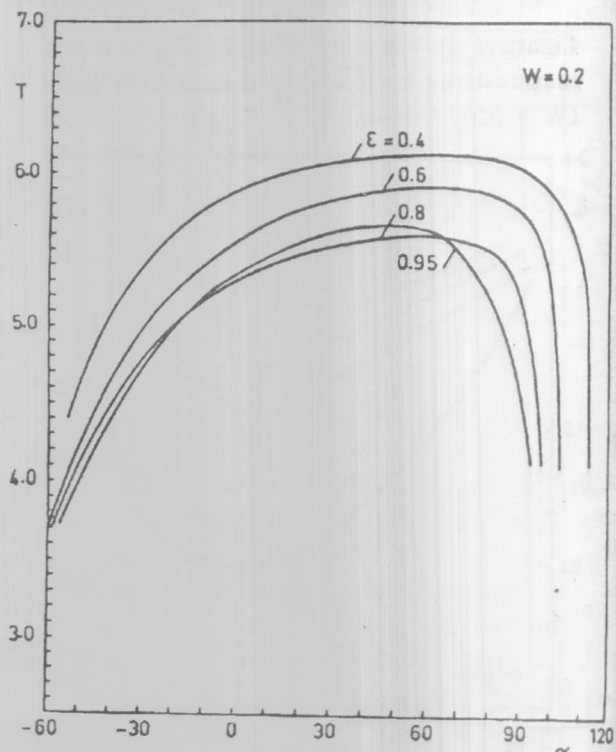


Figure 4-c. Friction torque versus inclination angle for different eccentricity ratios. ( $W=0.2$ ).

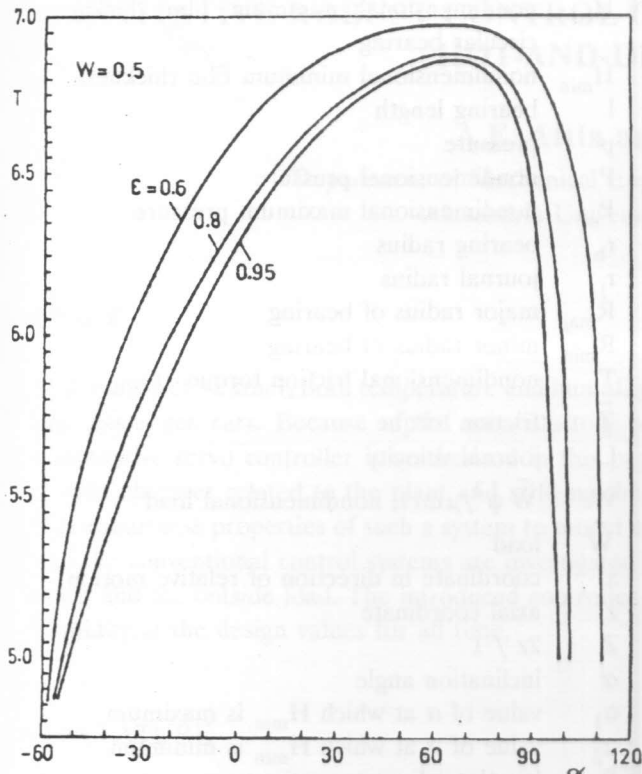


Figure 4-d. Friction torque versus inclination angle for different eccentricity ratios. ( $W=0.5$ ).

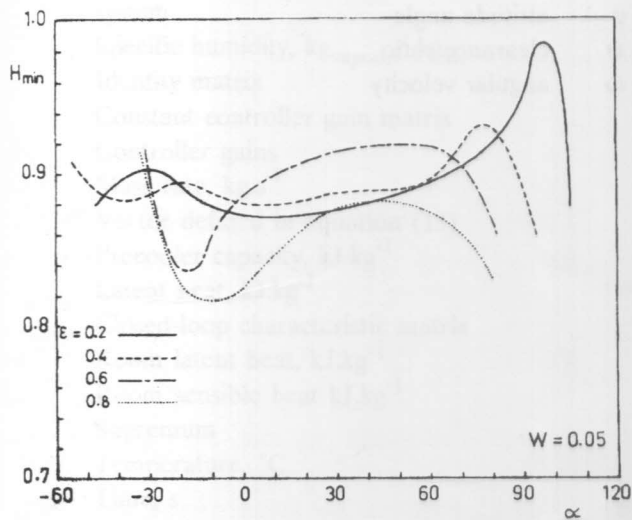


Figure 5. Minimum film thickness versus inclination angle for different eccentricity ratios ( $W=0.05$ ).

Table (1). Minimum and maximum values of the minimum film thickness and corresponding inclination angles.

W	$\epsilon$	max. $H_{min}$	$\alpha_1$	min. $H_{min}$	$\alpha_2$	$H_{cir}$
0.05	0.20	0.986	103	0.872	8	0.954
	0.40	0.932	83	0.879	7	
	0.60	0.920	52	0.836	163	
	0.80	0.889	150	0.817	170	
0.10	0.20	0.961	113	0.816	160	0.860
	0.40	0.926	93	0.740	110	
	0.60	0.905	83	0.756	107	
	0.80	0.854	71	0.678	103	
	0.95	0.842	50	0.698	135	
0.20	0.40	0.838	104	0.678	133	0.745
	0.60	0.822	93	0.600	110	
	0.80	0.787	93	0.560	100	
	0.95	0.752	73	0.550	97	
0.50	0.60	0.588	75	0.425	111	0.555
	0.80	0.581	63	0.341	112	
	0.95	0.569	74	0.340	116	

Table (1) summarizes the  $H_{min} - \alpha$  relationship for different bearing loads. It indicates the maximum and minimum values of  $H_{min}$ , and the inclination angles at which these maxima and minima occur. These angles are denoted in the table by  $\alpha_1$  and  $\alpha_2$  respectively. Corresponding values of the minimum film thickness for a circular bearing  $H_{min}$  (a unique value for each load) are included for comparison. We cannot indicate a general "optimum" position or zone for the major axis in order to maximize  $H_{min}$ . However, as the load increases, care should be taken in choosing  $\alpha$  since the ratio between the maximum and minimum values of  $H_{min}$  becomes larger. The maximum value of  $H_{min}$  is moderately decreased by the increase of  $\epsilon$  (i.e. with the increase of  $G$  while  $\alpha$  is being fixed). For smaller values of  $G$ ,  $H_{min}$  is slightly larger than the corresponding values for the circular bearings.

### 5. CONCLUSIONS

This study shows the effects of the ellipticity of the bush and the angle of inclination of its major axis on the maximum pressure, the friction torque and the minimum film thickness for a journal bearing with a slightly-elliptical bush. In summary, the following remarks are pointed out:

- (i) For the same  $\alpha$ ,  $P_m$  increases with increase of  $G$ .

The optimum value of  $\alpha$  in order to minimize  $P_m$  is

approximately 30 - 65 deg. A proper choice of  $\alpha$  may lead to a reduction in  $P_m$  of more than 50 %.

- (ii) For the same  $\alpha$ , T decreases with increase of G. T is minimum in the vicinity of  $\alpha = 110$  deg. The reduction attained in T by the proper choice of  $\alpha$  is up to 55 %.
- (iii) As the load increases,  $H_{min}$  becomes more influenced by  $\alpha$ . The maximum  $H_{min}$  is moderately decreased with the increase of G.

## REFERENCES

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## NOMENCLATURE

c	clearance
d	journal diameter
e	eccentricity
G	noncircularity coefficient
h	film thickness
H	nondimensional film thickness

$H_{cir}$	nondimensional minimum film thickness for circular bearing
$H_{min}$	nondimensional minimum film thickness
l	bearing length
p	pressure
P	nondimensional pressure
$P_m$	nondimensional maximum pressure
$r_b$	bearing radius
$r_j$	journal radius
$R_{maj}$	major radius of bearing
$R_{min}$	minor radius of bearing
T	nondimensional friction torque
$\bar{T}$	friction torque
U	journal velocity
$\bar{W}$	$\bar{W} \psi^2 / \mu \omega r_1$ , nondimensional load
$\bar{W}$	load
x	coordinate in direction of relative motion
z	axial coordinate
Z	$2z / l$
$\alpha$	inclination angle
$\alpha_1$	value of $\alpha$ at which $H_{min}$ is maximum
$\alpha_2$	value of $\alpha$ at which $H_{min}$ is minimum
$\beta$	length-to-diameter ratio
$\delta$	ellipticity
$\epsilon$	eccentricity ratio
$\theta$	angular coordinate
$\theta_1$	angular location of the leading edge
$\theta_2$	angular location of the trailing edge
$\mu$	viscosity
$\phi$	attitude angle
$\psi$	clearance ratio
$\omega$	angular velocity