SUBCRITICAL FLOW THROUGH OPEN CHANNEL CONTRACTION

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ABSTRACT

The hydraulics of flow through contractions is theoretically and experimentally studied. The loss of energy due to contractions is determined empirically as a function of the contracted width of waterway and the kinetic energy of the approach channel. The influence of contraction angle on the loss of energy is experimentally studied. A method to determine the water surface elevation upstream and inside the contraction has been suggested.

NOTATION

B	original bed width
b	contraction bed width
b_{crit}	contraction width which produce critical flow
b/B	contraction ratio
E_1	specific energy upstream contraction
E_{min}	specific energy for critical flow
Fr	Froude number
g	acceleration due to gravity
h_l	head loss due to contraction
h_L	head loss between section n & 1
Q	rate of flow
q_1	unit discharge upstream contraction = Q/B
q_2	unit discharge in contraction = Q/b
v_n	average velocity upstream contraction
v_2	average velocity in the contracted zone
y_n	normal water depth
y_1	water depth just upstream contraction
y_2	water depth in contraction
y_c	critical depth
Δz	difference in bed level between section n & 1
α	contraction angle

INTRODUCTION

The change in cross sectional dimensions of an open channel occurring in a relatively short distance will induce rapidly varied flow. This situation may occur due to constrictions in open channel such as bridges and regulators, etc., or due to reducing the cross sectional area for irrigation purposes.

The purpose of this research is to study the hydraulics of contraction and to provide a method for computing the backwater depth upstream contraction and to determine the loss of energy due to contraction.

The earliest investigation of flow through contraction in open channel was done by Lane [1]. He developed an empirical equation to determine the discharge in term of water depth through contraction and discharge coefficients. He indicated that there may be some relationship between these coefficients and the ratio of the maximum backwater depth produced by contraction to the normal depth.

In 1955 Kindesvater and Carter [2] carried out an experimental investigation on flow through constriction. They presented an empirical discharge equation using correction terms for various geometric conditions to a standard discharge coefficient. In the same year Tracy and Carter [3] presented a method for computing the nominal backwater due to channel constriction. Their solution was based on empirical discharge coefficients and laboratory investigation of the influence of channel roughness, channel shape, and constriction geometry on discharge through constriction.

In 1962 Biery and Delleur [4] presented the results of model testing of semicircular arch bridge. They related the backwater superelevation in terms of the bridge span, the stream width and the Froude number of the approaching flow. They gave design procedures for indirect discharge measurement, for the determination of backwater superelevation and for the determination of the required water area.

In 1973 Skogerboe, Barrett, Walker and Austin [5] compared bridge backwater curve and discharge relations made by Liu, Bradley and Plate at Colorado state *University. They found that the analytical expressions embodied in the current methods of measuring peak* discharge through, or backwater due to, a bridge constriction may be reduced to the form of a submerged flow equation.

In 1979 Vittal [6] proposed dimensionless empirical discharge depth relationships for exponential, trapezoidal and circular channel transitions. In 1983 Fiuzat and Skogerboe [7] recommended the submerged flow method to calculate the discharge through open channel constriction. They used experimental results to calibrate the submerged flow equation.

In 1983 Vittal and Chiranjeevi [8] examined methods for the design of an open channel transition between a rectangular flume and a trapezoidal channel for subcritical flow. They suggested a design method based on suitable boundary functions describing the geometric shape of the transition (namely bed width, bed elevation and side slope) and Hind's transition head loss equations.

In 1990 Shaltot [9] studied experimentally the effect of the rate of flow through horizontal transition and the ratio of the contracted width to the original width on the water depth. She presented empirical equations to predict the rate of flow through construction in term of Froude number.

THEORETICAL APPROACH

The depth of flow upstream and through contraction can be theoretically determined by applying the energy equation between section n and 1, and between section 1 and 2 (Figure (1)) as follows:

$$E_n + \Delta z = E_1 + h_L$$

$$y_n + \frac{q_1^2}{2gy_n^2} + \Delta z = y_1 + \frac{q_1^2}{2gy_1^2} + h_L \qquad (1)$$

$$E_1 = E_2 + h_1 + \Delta z_c$$

$$y_1 + \frac{q_1^2}{2gy_1^2} = y_2 + \frac{q_2^2}{2gy_2^2} + h_1 + \Delta z_c \qquad (2)$$

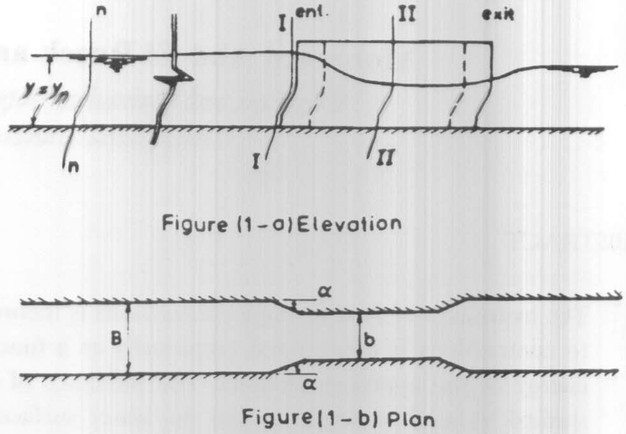
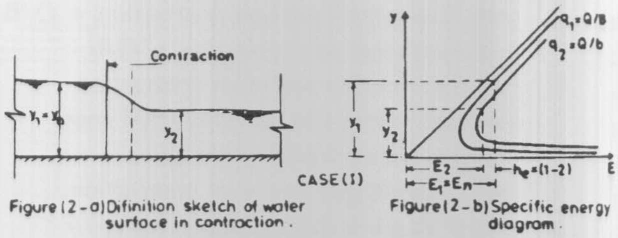


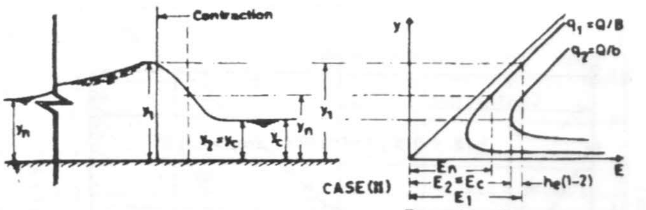
Figure 1. Definition sketch of channel contraction.

Where Δz_o is the difference in bed level between section 1 and section 2 which is too small in comparison with water depth and can be neglected.

Flow through channel contraction behave in two different ways according to the value of the specific energy in the contracted width. The first condition, when the specific energy in the contracted channel is equal to or less than the value of specific energy at the uniform flow ($E_2 \leq E_n$). In this case, the water depth just upstream the contraction will be equal to the normal depth and the water depth in the contraction is greater than the critical depth as shown in Figure (2).



The second condition, the specific energy at the contracted section plus the loss of energy due to contraction is greater than the specific energy in the uniform flow reach upstream the contraction ($E_2 > E_n$). In this case, the specific energy in the contraction is minimum and the water depth equal to the critical depth. The water surface upstream the contraction will rise to maintain enough specific energy to allow the flow to pass through the contraction as shown in Figure (3).



Figure(3-a) Difinition sketch of water surface in contraction.

Figure (3-b) Specific energy diagram.

According to this assumption the energy equation for the second condition may be written as follows:

$$E_1 = E_{min} + h_1 \quad (3)$$

For rectangular section, the minimum specific energy is defined as follows

$$E_{min} = 1.5 \sqrt[3]{\frac{Q^2}{g(b)^2}} \quad (4)$$

Substituting equation (4) into equation (3) it becomes

$$E_1 = 1.5 \sqrt[3]{\frac{Q^2}{gb^2}} + h_1 \quad (5)$$

The maximum contraction width which produces rise of water depth upstream contraction is defined as critical width and can be expressed using equation (5) as follows:

$$b_{crit} = Q \sqrt{\frac{3.375}{g(E_1 - h_1)^3}} \quad (6)$$

Therefore, the value of the loss of energy due to contraction must be known to determine whenever a backwater will be created and the maximum backwater depth. It is assumed that the loss of energy due to contraction is proportion of the kinetic head of the uniform flow and can be expressed as follows:

$$h_l = C \frac{v_n^2}{2g}$$

In which C is a constant and it's value depends on the ratio b/B and the contraction angle "α" i.e. :

$$C = \phi(\frac{b}{B}, \alpha) \quad (8)$$

EXPERMENTAL SETUP

For the purpose of determining the actual loss of energy due to channel contraction a set of experiments was conducted in the hydraulic laboratory, Alexandria University. The testing flume is 50.0 cm wide, 8.50 meters long and 50 cm height. The sides of the channel are made of perspex. Twelve different constriction models were constructed, all of constant length equal 60.0 cm. Three different contraction angles of 90°, 30°, and 11.31° (entrance slope 5:1)were used. For each value of the contraction angle, the width of contraction was changed four times 40 cm, 35 cm, 30 cm, and 25 cm.
In all runs the water surface profile along channel constriction and 1.0 meter upstream contraction was measured. Five different discharges 7.94, 11.449, 14.82, 24.72, and 30.01 liters/sec were used. The flow normal water depth was determined by measuring the average water depth along the channel before installing the constriction in the flume. A total of 60 runs were conducted.

ANALYSIS OF TEST RESULTS:

All experiments showed that the central body of water just upstream the constriction starts to accelerate. As the flow is accelerated between section I and II, the water surface profile dropped rapidly between these two sections to a depth equal to y_2. Further downstream, the water depth started to increase again for a short distance, then started to drop down once again to a depth equal y_2. For constrictions having contraction angle equal 90°, a separation zone was formed in the corners upstream contraction which is not noticed for contraction angle 30° and 11.31°. At the constriction exit, the depth of water starts once again to drop down to form a supercritical flow as shown in Figure (4). A hydraulic jump is created downstream the constriction. The water depth in contraction is equal to the critical depth when the width of contraction is equal or smaller than the critical width which can be determined from equation (6). The backwater depth upstream contraction is affected by the contraction ratio (b/B). The effect of contraction angle on the value of backwater depth is not significant (Figure (7)).

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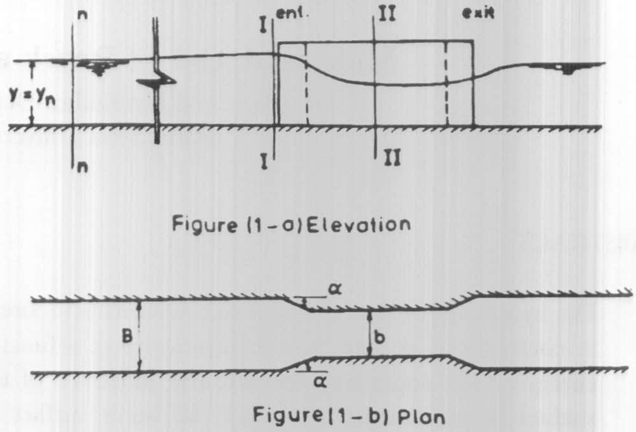

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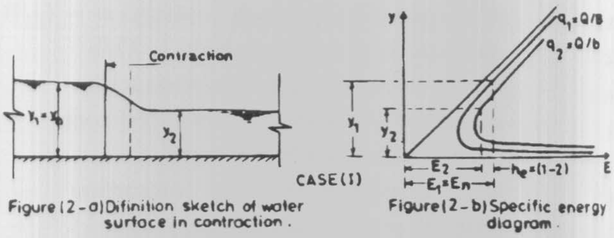

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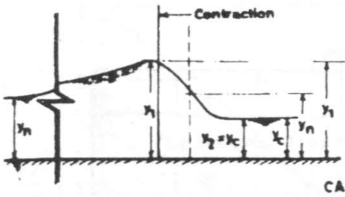

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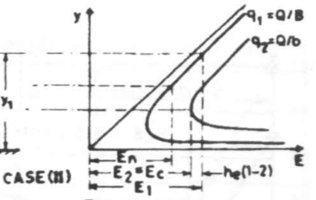

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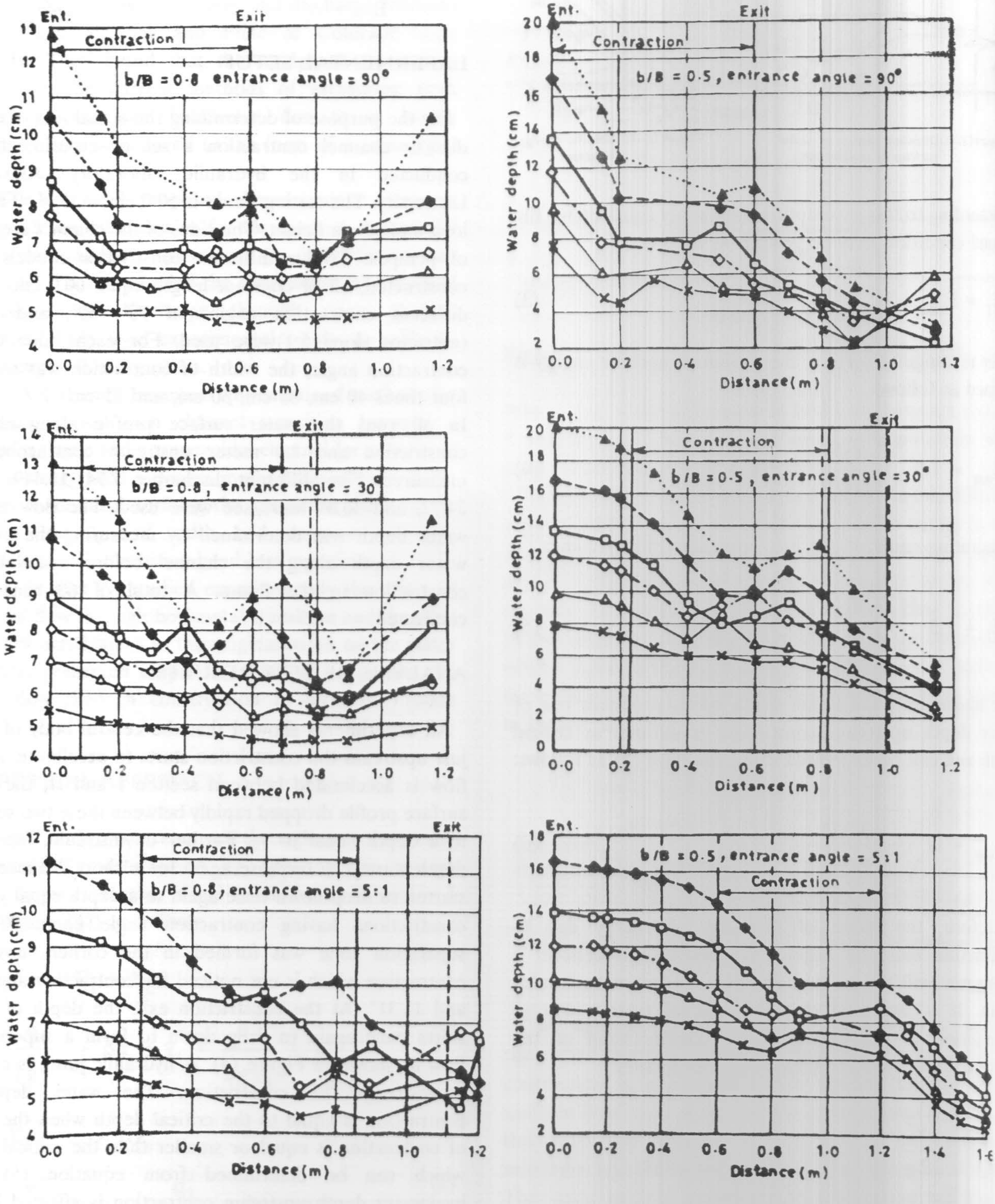

Figure 4. Water surface profile along channel contraction.

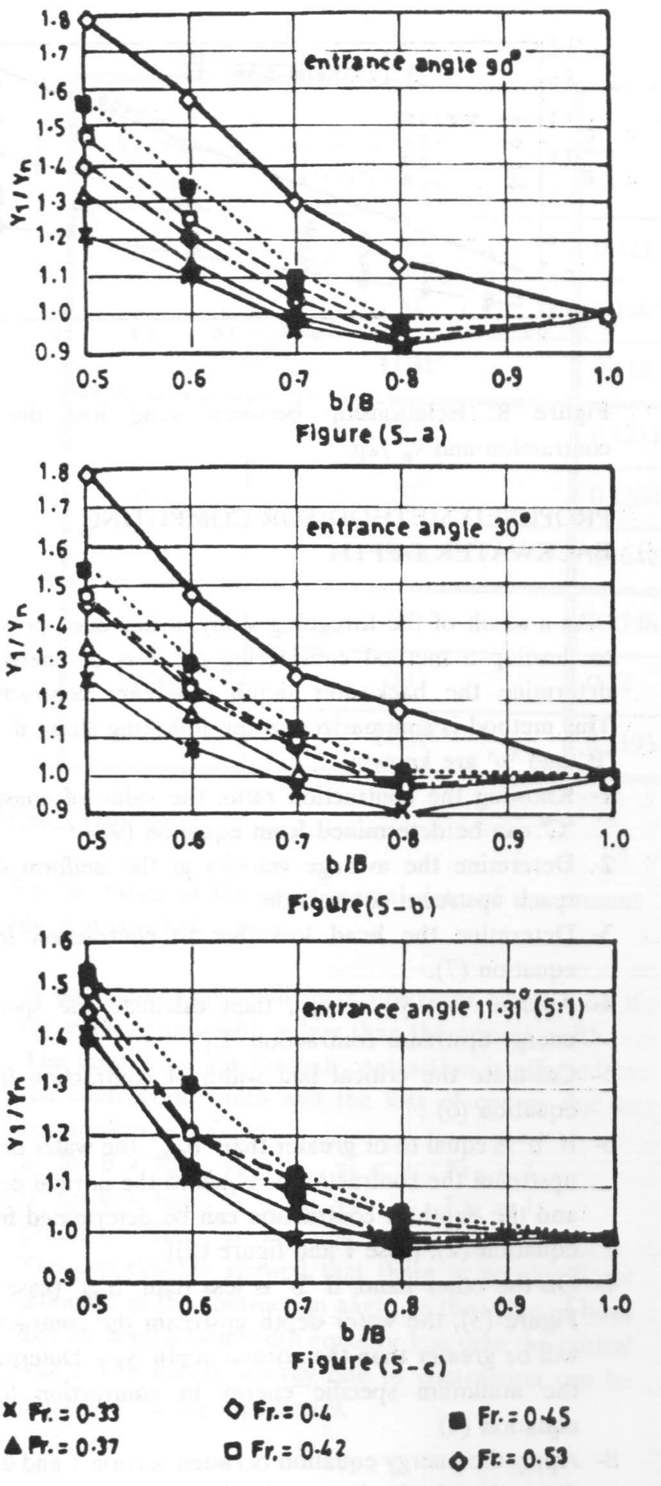

Figure 5. Variation of relative depth upstream contraction Y_1/Y_n with contraction ratio b/B.

In the condition when "b" is less or equal "b_{crit}", the backwater depth upstream contraction increases with the increase of Froude number. The relationship between the backwater depth upstream contraction with Froude number for different contraction ratios is shown in figure (5-a, 5-b and 5-c). Using the measured water depth and knowing the rate of flow, the total energy line along the constriction is determined. Figure (6) shows the variation of the total energy line and water surface profile along one of the tested constrictions.

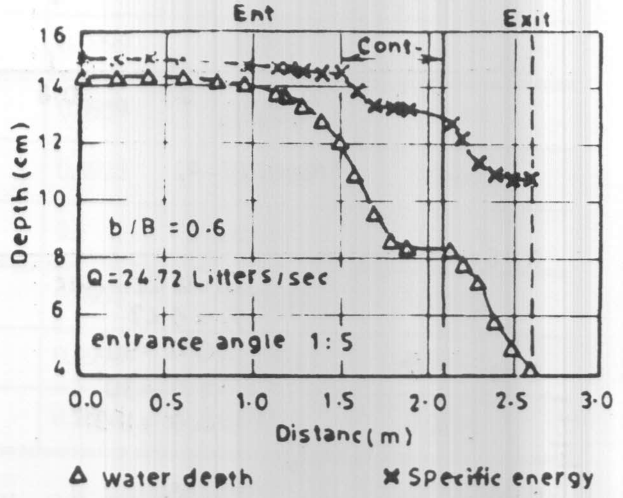

Figure 6. Water surface profile and total energy line.

The difference between the total energy at entrance and the average total energy along the contracted width is defined as the loss of energy due to channel contraction. For the same Froude number, the loss of energy due to contraction increases with the decrease of the contraction ratio (b/B) as shown in Figure (7).

It is noticed that, there is no significant effect of contraction angle on the value of head loss for contraction width less or equal to the critical width as recorded in Table 1 and also shown in Figure (7).

Based on the data obtained from experiments, the value of the constant "C" in equation (7) is determined using the method of least square, which is expressed as follows

$$C = \frac{1}{2.932(b/B) - 0.997} \qquad (9)$$

The correlation coefficient of equation (7) equals 0.878. Figure (8) shows the variation of the observed and the calculated loss of energy due to contraction with the kinetic energy $[v_n^2/2g]$ at uniform flow for different values of contraction ratios "b/B".

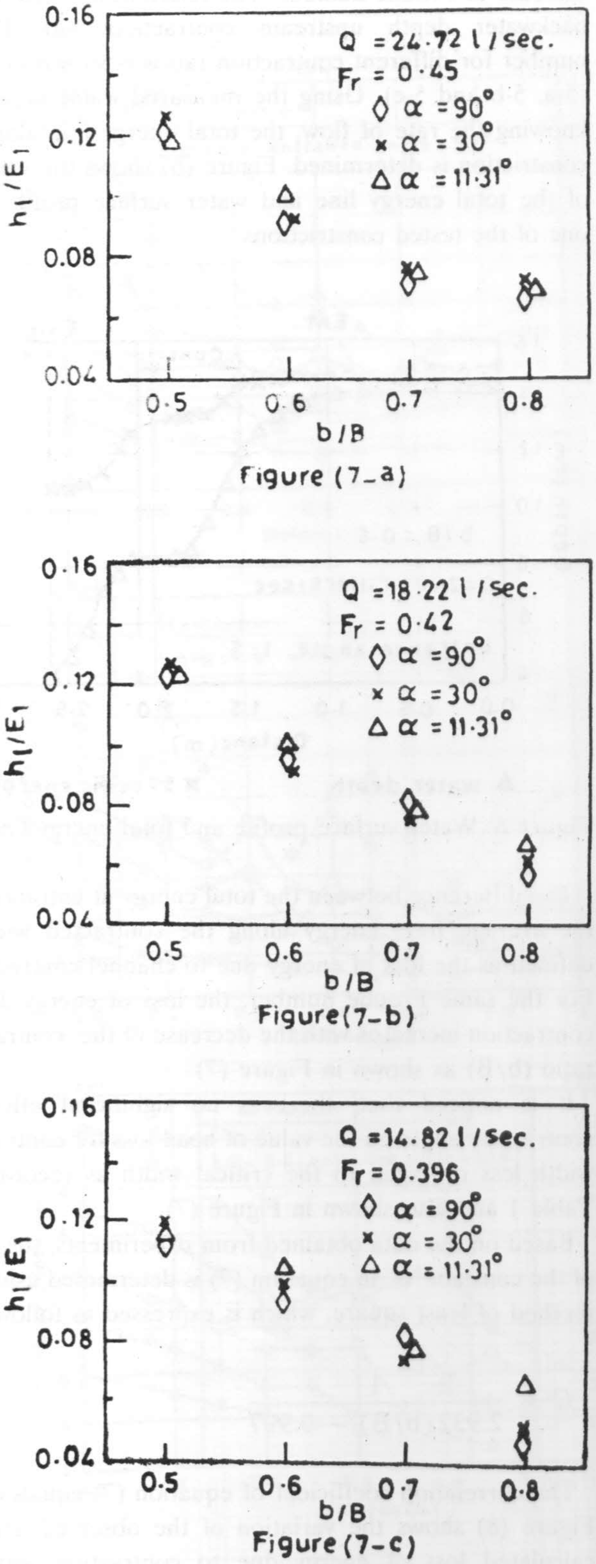

Figure 7. Head loss (h_l/E_1) versus contraction ratio (b/B).

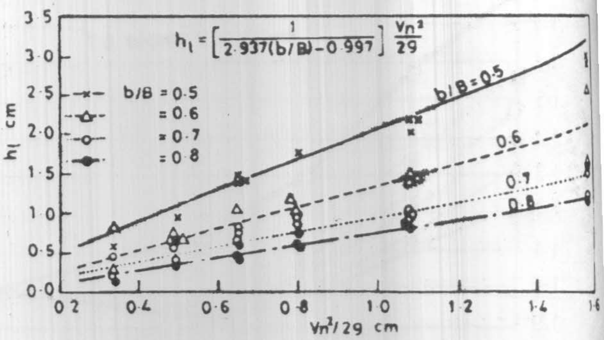

Figure 8. Relationship between head loss due to contraction and $v_n^2/2g$.

PROPOSED METHOD FOR COMPUTING BACKWATER DEPTH

As a result of the foregoing study, it has been possible to develop a method considering the loss of energy to determine the backwater depth upstream contraction. This method is summarized in the following steps, if "Q", "B" and "b" are known:

1- Knowing the contraction ratio, the value of constant "C" can be determined from equation (9).
2- Determine the average velocity at the uniform flow reach upstream contraction.
3- Determine the head loss due to contraction from equation (7).
4- Assume that $y_1 = y_n$, then calculate the specific energy upstream contraction "E_1".
5- Calculate the critical bed width of contraction from equation (6) .
6- If "b" is equal to or greater than "b_{crit}", the water depth upstream the contraction is equal to the normal depth and the depth in contraction can be determined from equation (2), [case I and figure (2)].
7- On the other hand, if "b" is less than "b_{crit}" (case. II, Figure (3), the water depth upstream the contraction will be greater than the normal depth "y_n". Determine the minimum specific energy in contraction form equation (4)
8- Apply the energy equation between section 1 and 2, to determine the backwater depth upstream contraction as follows:

$$y_1 + \frac{q_1^2}{2gy_1^2} = E_{min} + h_l \tag{10}$$

Table 1. Values of ratio of loss of energy due to contraction to specific energy upstream contraction

Froude No	b/B	0.5	0.6	0.7	0.8
	Entrance angle α	h_l/E_1			
0.45	90°	0.123	0.093	0.073	0.0679
	30°	0.1265	0.0928	0.07746	0.07411
	11.31°	0.118	0.0999	0.0755	0.07
0.42	90°	0.1251	0.0959	0.08215	0.05898
	30°	0.1266	0.0925	0.07683	0.06091
	11.31°	0.1242	0.1	0.07687	0.06796
0.396	90°	0.1148	0.0957	0.08168	0.04988
	30°	0.12	0.0996	0.07491	0.05329
	11.31°	0.1193	0.1040	0.07558	0.0674

CONCLUSION

For the range of the experimental data of the present study, it is concluded that:

1- A backwater curve will form upstream contraction if the contraction width is less than the critical width.

2- The flow condition through contraction is affected by the contraction width and the loss of energy due to contraction as follows:

a- for $b > b_{crit}$, $y_1 = y_n$, $y_2 < y_1$, $Fr_2 < 1$,

b- for $b = b_{crit}$, $y_1 = y_n$, $y_2 = y_c$, $Fr_2 = 1$,

c- for $b < b_{crit}$, $y_1 > y_n$, $y_2 = y_c$, $Fr_2 = 1$

3- The experiments showed that there is no significant influence of the contraction angle on the value of head loss for contraction width equal or less than the critical width. The loss of energy due to contraction can be determined using equation 9.

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