SPATIAL LAG EFFECTS IN BED LOAD

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ABSTRACT

Spatial lag effects in bed load of uniform mixture is theoretically and experimentally studied. A semi empirical equation describing the variation of bed load in terms of sediment transport capacity and the probability of erosion is driven. A computation procedure to determine the variation of rate of sediment transport along spatial lag distance is illustrated.

NOTATIONS

A_b area of scour or deposition at a section
C_{SL} spatial lag coefficient (dimensions L^{-1})
C_L unit step length
d grain diameter
d_{50} grain diameter such that 50% of the material by weight is finer than this size
E weight of erosion per unit area
g_s rate of sediment transport
Gs sediment transport capacity
G_* dimensionless rate of sediment transport
i_b fraction of bed area covered by particular grain size
n porosity
P probability of erosion
q probability of deposition
T_* dimensionless bed shear stress
x distance of a point along the spatial lag
L_* dimensionless length of the spatial lag
α_L grain step length constant
τ_b bed shear stress
τ_c critical shear stress
λ constant of Einstein average step length

INTRODUCTION

A dynamic stable channel occurs when the rate of sediment coming to a certain reach is equal to the rate at which the channel can transport the sediment. The capability of the channel to transport sediment is defined as sediment transport capacity. When a constrained sediment boundary conditions are imposed on an alluvial channel, a certain distance is required before the alluvial channel reach an equilibrium state. This phenomenon is viewed as a spatial lag. This phenomenon occurs mainly in the downstream reach of dam spillway in which sediment inflow rates are less than the sediment transport capacity of the flow.

In 1983 Phillips and Sutherland [6] proposed an equation describing the spatial lag as follows

$$(1 - n)\frac{\partial A_b}{\partial t} = C_{SL}(g_s - G_s) \tag{1}$$

where C_{SL} is spatial lag coefficient which is defined as follows

$$C_{SL} = \frac{1}{\alpha_L d_{50}(T_* - T_{*c})} \tag{2}$$

T_* is a dimensionless shear stress which is defined as

$$T_* = \frac{\tau_b}{(\gamma_s - \gamma_w)d_{50}} \tag{3}$$

G_s is the sediment transport capacity which is expressed as

$$G_s = 894(U - U_c)$$
$$U_c = U\frac{\tau_c}{\tau_b} \tag{4}$$

The units in equation (4) is gm, meter and seconds. They determined the step length constant "α_L" from experiments and found that the lower and upper bounds values of "α_L" are equal to 4000 and 9000, respectively.

The big discrepancy in the value of α_L affects the accuracy of the solution of equation (1).

The aim of the present study is to determine the distance in which the alluvial system can compensate the loss of sediment load and to express the variation of bed load along the spatial lag distance.

THEORETICAL ANALYSIS

When clear water flows over a mobile bed and the bed shear stress is greater than the grain critical shear stress, some of the grains will be picked up and transported with the flow. The rate of sediment transport will increase with distance till it reaches its sediment transport capacity. In this condition the weight of picked up grains is equal to the deposited grains causing a stable dynamic bed. It is assumed that the capability of the channel to pick up grains is constant along the channel as long as the flow parameters and the grain size distribution of the bed surface layer do not change. The weight of picked up per grains per unit area having diameter "d" can be determined from the bed load analysis presented by Einstein [2] as follows:

$$E = C\, i_b\, d\, P \sqrt{\frac{g(\gamma_s - \gamma)}{d\gamma_s}} \tag{5}$$

In which "C" is a constant which depends on the grain shape and the specific gravity of sediment and "P" is the probability that the grains having diameter "d" may be picked up. Gessler [3] defined the probability of erosion as the probability that the instantaneous shear stress exceeds the critical shear stress of the grains under consideration. He found that this probability is normally distributed and the standard deviation of the distribution is constant and equal 0.57. He expressed the probability of erosion as follows:

$$P = \frac{1}{\sigma\sqrt{2\pi}} \int_{\infty}^{(\tau_c/\tau - 1)} \exp\frac{-z^2}{2\sigma^2} dz \tag{6}$$

Where τ_c is the critical shear stress of the grain under consideration and can be determined from Shields [8] diagram.

Part of the picked up particles will be deposited because the drag stress which is responsible for transporting sediment is a function of instantaneous bed local velocity. It is assumed that the probability of deposition of the picked up grains is equal to the probability that drag stress is less than the critical tractive stress of a particle "q" which is expressed as follows

$$q = 1 - P \tag{7}$$

To explain the mechanism of the sediment transport, a unit step distance along the channel is considered to be equal to the average distance travelled by the particles in each jump. The average step distance is expressed by Einstein [1] as follows:

$$C_L = \frac{\lambda d}{1 - p} \tag{8}$$

Einstein assumed the value of λ is constant and equal 100.

If the weight of picked up particles in each unit distance "C_L" is constant and equal "E" then the weight of transported particles in the first unit distance is

$$g_{s_1} = E(1-q)$$

In the second unit distance, the weight of loose particles is "E+E(1-q)" and out of these particles the weight of deposited particles is "q(E+E(1-q))" and the weight of transported sediment is

$$g_{s_2} = [E + E(1 - q)](1 - q)$$

According to the foregoing description of the mechanism of sediment transport, the change in rate of sediment transport can be fairly expressed as follows:

$$dg_s = E(1 - q)^x \tag{9}$$

In which x is the ratio of the distance to the unit step distance.

The rate of sediment transport is

$$g_s = E\int_0^x (1 - q)\,dx$$

$$g_s = E\left[\frac{(1 - q)^x - 1}{\ln(1 - q)}\right] \tag{10}$$

Dividing equation (10) by the weight of erosion per unit distance "E", the dimensionless rate of sediment "$G_* = g_s/E$"

$$G_* = \frac{((1-q)^x - 1)}{\ln(1-q)} \tag{11}$$

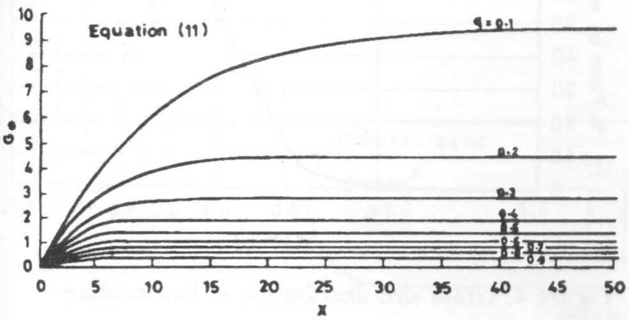

Figure 1. Relationship between dimensionless rate of sediment transport and distance for different probability of deposition.

Equation (11) expresses the variation of the dimensionless rate of sediment transport with distance which increases rapidly with the increase of unit distance till a certain reach where the change in sediment transport can be neglected (Figure (1)). At this reach the channel is considered dynamically stable and the dimensionless rate of sediment transport equal the dimensionless sediment transport capacity. The value of the sediment transport capacity is decreased with the increase of the probability of deposition. The sediment transport capacity can be determined by substituting x in equation (10) by ∞ as follows

$$G_s = \frac{-E}{\ln(1-q)} \tag{12}$$

The minimum length in which an alluvial channel can gain its sediment transport capacity is determined by the condition where the weight of picked up grains is nearly equal to the weight of the deposited grains, in which the change of sediment transport with distance is too small and equal:

$$\frac{dg_s}{E} = 0.0001 \tag{13}$$

Substituting equation (13) into equation (9) it becomes

$$L_* = \frac{\log(0.0001)}{\log(1-q)} \tag{14}$$

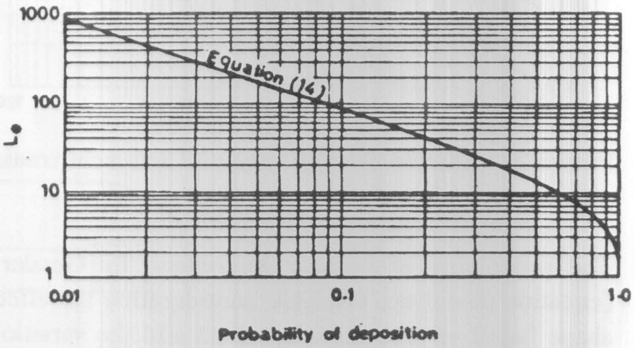

Figure 2. Variation of dimensionless spatial lag length with probability of deposition.

In which "L_*" is the minimum dimensionless length of spatial lag effects. According to equation (14), the spatial lag length decreases with the increase of the probability of deposition (as shown in Figure (2)) . If the sediment transport capacity of the channel is known, the variation of rate of sediment transport can easily be determined by dividing equation (10) by the sediment transport capacity of the channel determined from equation (12) and can be expressed as follows:

$$\left(\frac{g_s}{G_s}\right)_{th} = 1 - (1-q)^x \tag{15}$$

Figure (3) shows the relationship between the ratio of the rate of sediment transport to the sediment transport capacity with distances and for different values of probability of deposition.

According to the foregoing analysis, the spatial lag effects in bed load which are defined as the inability of an alluvial channel to immediately overcome constrained boundary conditions, can be characterized by the probability of erosion of bed material, unit step length and sediment transport capacity.

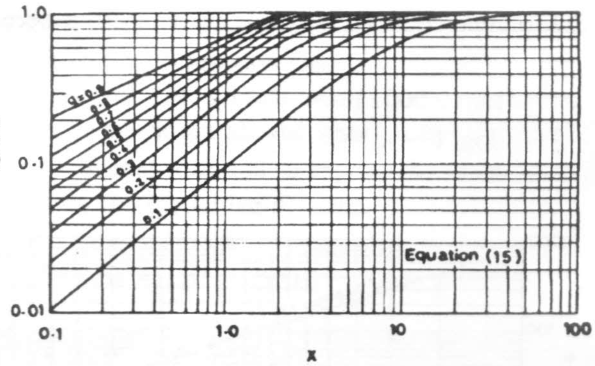

Figure 3. Variation of relative rate of sediment transport with dimensionless distance.

The probability of erosion determined by Gessler [3] (equation 6) did not take into consideration the effect of shape factor of grains, hiding effect and the variation of standard deviation of the velocity fluctuations. Therefore, the theoretical equations describing the variation of rate of sediment transport along the channel must be experimentally verified.

EXPERIMENTAL SET-UP

For the study of the spatial lag effects, a set of experiments were conducted in the hydraulics laboratory, of the Iowa Institute of Hydraulic Research (IIHR), University of Iowa, USA. The experiments were performed in a recirculating tilting flume 27.45 meter (90 ft) long and 91.5 cm (3 ft) wide and 30.5 cm (1 ft) deep. The sediment free water was discharged into a tail box, then pumped back in the inlet box through one foot spiral-weld return pipes. For the recirculation of water, variable speed pumps were used to control the discharge and to avoid any kind of disturbance. The slope of the flume can be adjusted by means of four synchronized motor-driven cam jacks.

A sediment mixture having mean diameter of 4.027 mm and geometric standard deviation 1.549. The thickness of the sediment mixture was 10 cm. Figure (4) shows the grain size distribution of the bed material. Three bed load samplers were put in the testing flume to create spatial lag effects in two reaches as shown in Figure (5). Two of these bed load sampler were put in the upstream and downstream end of the testing flume. For each run, the distance between the third sediment sampler and upstream sediment sampler was changed to 1.15, 2.95 and 7.00 meters. The aim of installing sediment sampler is to trap all transported sediment so that the rate of sediment transport at different distances can be calculated. Table (I) illustrates the test program.

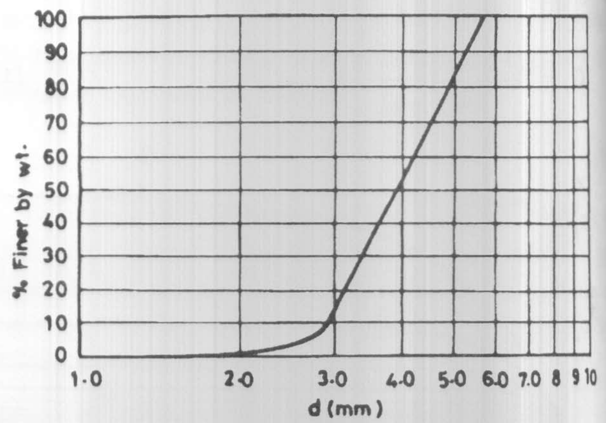

Figure 4. Grain size distribution of bed mixture.

Table I. Test Program

	Run 1	Run 2	Run 3
Discharge (m^3/sec)	$9.53x10^{-2}$	$8.8x10^{-2}$	0.1023
Slope	$2.98x10^{-3}$	$2.01x10^{-3}$	$2.49x10^{-3}$
Depth (m)	0.128	0.1366	0.1475

EXPERIMENTAL PROCEDURE

Sediment mixture was properly placed in the flume and all sediment traps were filled with sand. The flume was filled with 10 cm depth of water. The channel slope was adjusted to the designed slope. The discharge was increased gradually to the design discharge to avoid bed disturbance.

The water surface and the bed elevation were measured along the testing flume. The flow was kept running till the flow becomes uniform and the channel reach its dynamic stable condition. The bed slope of the channel was then measured. After reaching the uniform flow condition, the flow of the channel was stopped and the sediment traps were uncovered. A clear water was running once again under the same discharge and the same slope to study the spatial lag in two reaches. The length of the first reach was 1.15 m and the second reach was 18.8 m. After fifteen minutes, the flow was stopped and the collected sediments in the downstream end of each reach were dried and weighed. The rate of sediment transport at these two distances was determined. This procedure was repeated for the same discharge and bed slope but changing the position of the intermediate sediment trap to determine the variation of rate of sediment transport along the channel. To obtain the effect of the probability of erosion on the rate of sediment transport, the same experiment procedure was repeated for the other two discharges stated in Table (I).

ANALYSIS OF RESULTS

Experiments shows that the rate of sediment transport increases with distance and the value of the rate of sediment transport increases with the increase of the probability of erosion of bed particles. The variation of the observed rate of sediment transport with distance is shown in Figure (6).

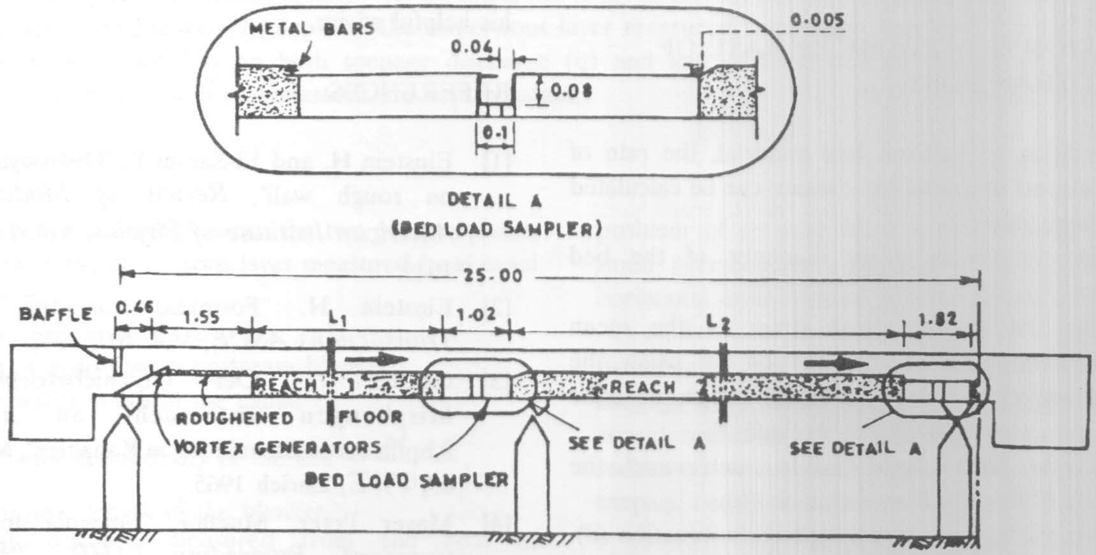

Figure 5. Schematic diagram of the experimental set-up.

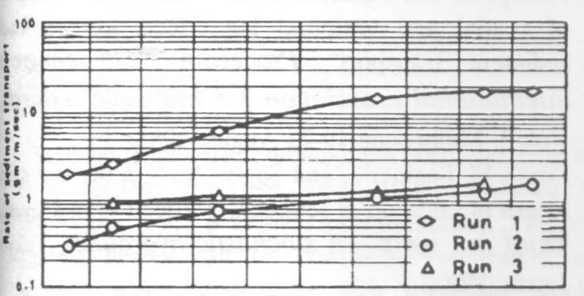

Figure 6. Variation of rate of sediment transport with distance.

It was found that the distance after which the rate of sediment transport becomes constant increases with the increase of the shear stress. The values of the calculated ratio of rate of sediment transport to the sediment transport capacity determined by equation (15) show discrepancy with the data obtained from experiments. Using a least square method, a semi empirical equation is developed to fit the theoretical values with the corresponding actual data obtained from experiments which is expressed as follows:

$$\left(\frac{g_s}{Gs}\right)_{actual} = 0.92\left(\frac{g_s}{Gs}\right)_{th}^{0.777} \qquad (16)$$

The correlation coefficient of equation (16) is 0.87. Figure (7) shows the actual values of the relative rate of sediment transport and the values calculated from equation (16).

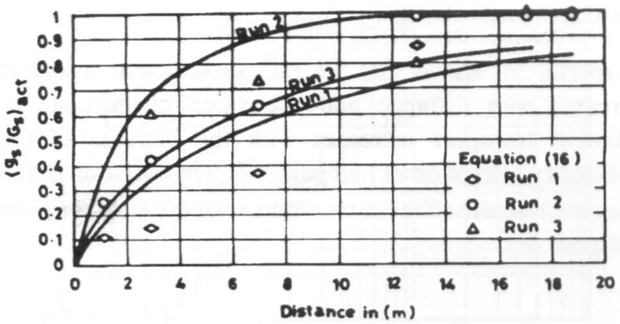

Figure 7. Relationship between the observed and calculated values of relative rate of sediment transport.

CALCULATION PROCEDURE OF RATE OF SEDIMENT TRANSPORT

In the condition of uniform bed material, the rate of sediment transport in spatial lag distance can be calculated by the following steps:

1- Determine the mean grain diameter of the bed sediments.
2- Determine the critical shear stress of the mean diameter from Shield's diagram or by using the equations developed by Yassin [9] which describe Shield's criterion for incipient motion.
3- Knowing the bed shear stress, determine the probability of erosion from equation (6).
4- Determine the average step length from equation (8).
5- Using equation (14) to calculate the spatial lag distance.
6- Compute the sediment transport capacity using Einstein method or Mayer Peter and Mueller [4] formula.
7- Determine the theoretical relative rate of sediment transport from equation (15).
8- Calculate the rate of sediment transport from equation (16).

CONCLUSION

The spatial lag effects which is defined as the inability of an alluvial channel to immediately overcome constrained boundary conditions, can be characterized by the probability of erosion of bed material, unit step length and sediment transport capacity. It is found that the spatial lag distance increases with the increase of the probability of erosion and can be estimated from equation (14). The variation of rate of sediment transport depends mainly on the unit step length and the probability of erosion. More experiments are needed to determine accurately the unit step length and the probability of erosion.

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REFERENCES:

[1] Einstein H. and El-Samni E.,"Hydrodynamic Forces on rough wall", *Review of Modern Physics, American Institute of Physics*, Vol. 21, No. 3, July 1949.

[2] Einstein H. "Formulas for the bed load", *Transactions ASCE*, Vol. 107, 1942.

[3] Gessler J., "Der Geschiebetriebbeginn bei Mischungen untersucht an natuerlichen Abpflasterunsersheinung in Kanaelen", Mitteilung No 69, *VWE*, Zurich 1965.

[4] Mayer Peter, Mueller, "Formula for bed load transport", *Proceeding IAHR, 2nd Congress*, Stockholm, 1948.

[5] Phillips B., *Ph.D. Thesis*, University of Canterbury, Christchurch, New Zealand, July 1984.

[6] Phillips B. and Sutherland A., "Numerical modelling of spatial and temporal lag effects in bed load sediment transport", *Proceeding 21st congress international association for hydraulic research*, Vol. 3, Melbroune 19-23 August 1985.

[7] Poreh M, Seginer J. and Sagiv A., "On the unit step length of saltation of sediment particles in bed layer", *Third int. symp. on stochastic hydraulics*, Tokyo Japan, 1980.

[8] Shields, "Anvendung der Aehnlichkeitsmechanik und der Turbulenzforschung auf die Geschiebebewegung", *Mitteilung der Preussischen Versuchsanstalt fuer Wasserbau und Schiffbau*, Heft 26 Berlin 1936.

[9] Yassin A., "Incipient motion of uniform bed material", *The Bulletin of the Faculty of Engineering*, Alexandria University, 1982.