SKEW RAYS AND PHOTON TURNING POINTS IN GRADED-INDEX FIBERS

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ABSTRACT

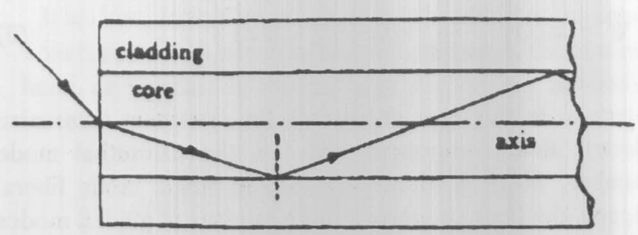
Skew rays transmission in multimode graded index optical fibers is modelled and investigated. The caustic surfaces radii containing these rays are calculated. Dominant linearly polarized modes are studied. The obtained results show that the radii increase with the wavelength of the propagating signal and decrease with the axial refractive index, the fiber core radius and the relative refractive index difference between the core and the cladding.

INTRODUCTION

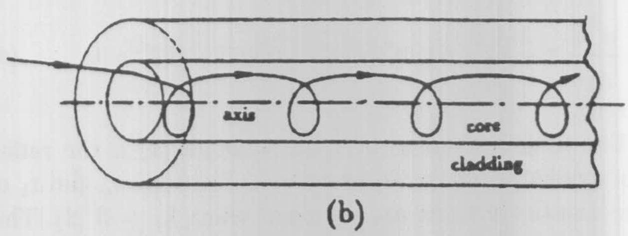
According to the ray theory, there are two types of light propagation within an optical fiber. When the transmitted rays pass through the fiber axis, they are called meriodinal rays, Figure (1-a). However, another category exists when the rays are transmitted without passing through the fiber axis. These rays, which greatly outnumber the meriodinal rays, follow a helical path through the fiber, as illustrated in Figure (1-b), and are called skew rays. When the light input to the fiber is not uniform, skew rays will tend to have a smoothing effect on the distribution of the light as it is transmitted, giving a more uniform output.

Another advantage of the transmission of the skew rays becomes apparent when their acceptance conditions are considered, where the skew rays in a given fiber are accepted at larger axial angles than those of meriodinal rays.

A light ray propagating in a graded index fiber does not necessarily reach every point in the fiber core. The ray is contained within two cylindrical caustic surfaces and for most rays a caustic does not coincide with the core-cladding interface. Hence, the caustics define the classical turning points of the light ray within the graded fiber core. These turning points defined by the two caustics may be designated as occurring at $r = r_o$ and $r = r_1$, i.e. the radii where the wave vector's radial component is zero [1, 2]. The result of the WKB approximation yields an oscillatory field in the region $r_o < r < r_1$ and an evanescent field outside this region [3]. In the region inside the inner caustic defined by $r < r_o$ the field decays towards the fiber axis. The knowledge of the radii r_o and r_1 can be used to get the attenuation of modes in optical fibers and gives information about the most hot parts in the fiber [1]. This is taken into consideration in the fiber design process to avoid harmful effects caused by high temperatures.



(a)



(b)

Figure 1. Light transmission through an optical fiber. a-Meriodinal rays. b-Skew rays.

The core of the fiber under investigation is characterized by a graded refractive index of the form:

$$n(r) = n_1 \{1 - 2\Delta(r/a)^2\}^{1/2} \quad r \leq a \text{ (core)} \qquad (1\text{-a})$$

$$= n_1\{1-2\Delta\}^{1/2} \quad r \geq a \text{ (cladding)}, \tag{1-b}$$

where n_1 is the axial refractive index, Δ is the relative refractive index difference between the core and the cladding and a is the core radius.

MODEL AND ANALYSIS

For a cylindrical waveguide, the wave equation can be written under the form [4]:

$$\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\phi^2} + \{n^2(r)\,k^2 - \beta^2\}\psi = 0 \tag{2}$$

where ψ is the field (E or H), n(r) is the core refractive index, k is the wavevector magnitude in vacuum ($= 2\pi/\lambda$ with λ the wavelength), r and $\phi$are the cylindrical coordinates and β is the propagation constant of the guided mode.

Solutions of the wave equation are separable under the form [3]:

$$\psi = E(r)\left\{\frac{\cos l\phi}{\sin l\phi}\exp(wt - \beta z)\right\}, \tag{3}$$

where ψ in this case represents the dominant transverse electric field component and l is the azimuthal mode number, which is identically zero in single mode fibers. Hence, the fiber supports a finite number of guided modes of the form of equation (3).

Introducing the solution, given by equation (3) into equation (2) results in a differential equation of the form:

$$\frac{d^2E}{dr^2} + \frac{1}{r}\frac{dE}{dr} + \{(n^2(r)k^2 - \beta^2) - \frac{l^2}{r^2}\}E = 0. \tag{4}$$

The term multiplied by E in equation (4) is the radial component k_r of the wavevector k. The radii r_o and r_1 of the caustic surfaces are obtained when $k_r = 0$ [1]. This gives:

$$n^2(r)\,k^2 - \beta^2 = \frac{l^2}{r^2} \tag{5}$$

Figure (2) represents the graphical solution of equation (5) and clearly illustrates the radii r_o and r_1. Since the value of l is identically zero for the single mode fiber, no solution exists for r_o and r_1 and therefore, the rays in a single mode fiber are only of the meriodinal type.

The propagation constant β for a mode m in a graded index fiber is given by [5]:

$$\beta^2 = n_1^2 k^2 \{1 - 2\Delta(m'/M')^{1/2}\}, \tag{6}$$

where n_1 is the axial refractive index and M′ is the total number of modes given by:

$$M' = \tfrac{1}{2} a^2 n_1^2 k^2 \Delta. \tag{7}$$

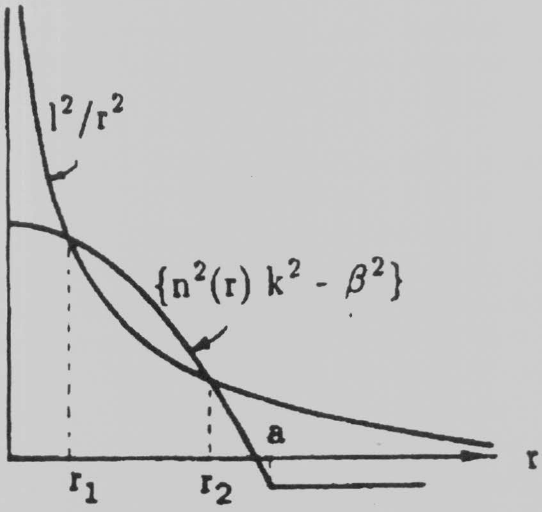


Figure 2. Graphical representation of the functions $\{n^2(r)k^2-\beta^2\}$ and l^2/r^2 giving the definition of the turning points r_o and r_1.

In term of the radial mode number m and the azimuthal mode number l, the mode number m is expressed as [3]:

$$m' = (2m+l+1)^2. \tag{8}$$

RESULTS AND DISCUSSION

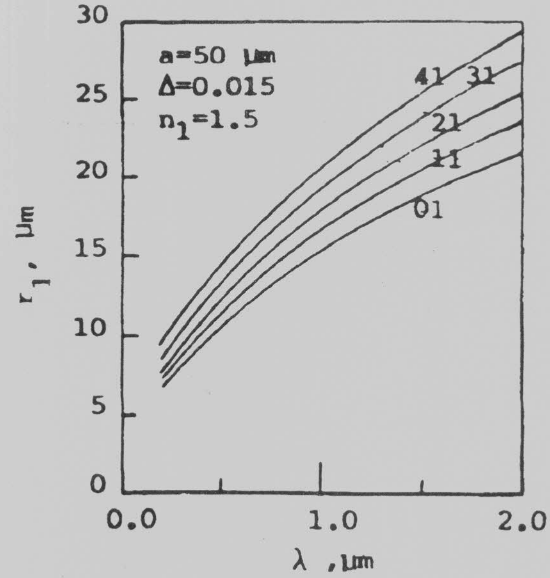
The use of equation (1) and equations (6-8) in equation (5) yields the values of the radii of the cylindrical caustic surfaces, r_o and r_1, for different modes in the graded index fibers. The values of the wavelength λ, the core radius a and the relative index difference Δ are chosen to give a value greater than 3.402 for the normalized frequency V to achieve the multimode condition, where V is given by [3]:

$$V = \frac{2\pi}{\lambda}\, a\, n_1\, (2\Delta)^{1/2}. \tag{9}$$

For a cylindrical waveguide, which is bounded in two dimensions, the two integers l and m are necessary to specify the mode. So, we refer to TE_{lm} and TM_{lm} modes. These modes correspond to meriodinal rays travelling within the fiber [3]. However, the modes having nonzero components of the electric and magnetic fields along the propagation direction are resulting from the skew rays, and are designated HE_{lm} and EH_{lm} depending upon whether the components of H or E make larger contribution to the transverse field. As the relative refractive index difference Δ is very small, both HE and EH mode pairs occur having almost identical propagation constants and are said to be degenerate. The superposition of these degenerating modes characterized by a common propagation constant correspond to a particular linearly polarized (LP_{lm}) mode. Hence, the linear combination of the degenerate modes produces a useful simplification in the analysis.

In the present work, the different dominant modes: LP_{01}, LP_{11}, LP_{21}, LP_{31} and LP_{41} are investigated and the corresponding values of r_o and r_1 are displayed in Figure (3), where it is noted that, for the same fiber, both r_o and r_1 increase with the wavelength λ.

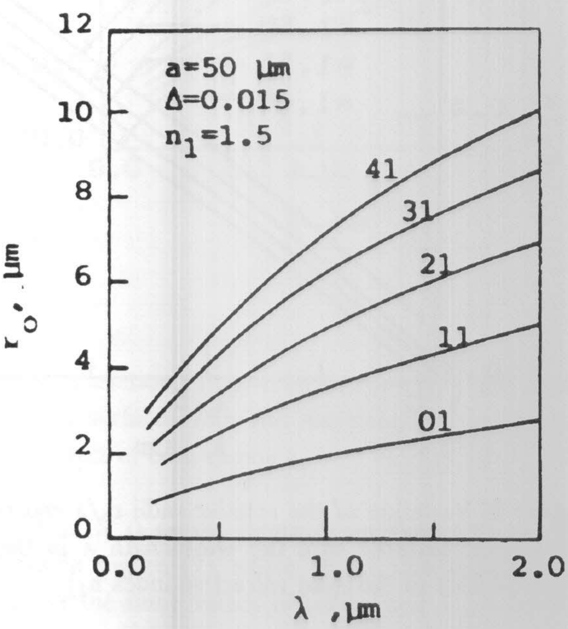
The effect of the radial and azimuthal mode numbers, l and m, is studied through the relative values of the caustic radii as shown in Figure (4).



(b)

Figure 3. Variation of the caustic surfaces radii r_o and r_1 with the wavelength λ for different LP modes.

It is clear, from Figure (4), that as the azimuthal number 1 increases, both the relative radii increase. On the other hand, an increase in the radial mode number m yields an increase in r_1/a and a decrease in r_o/a.



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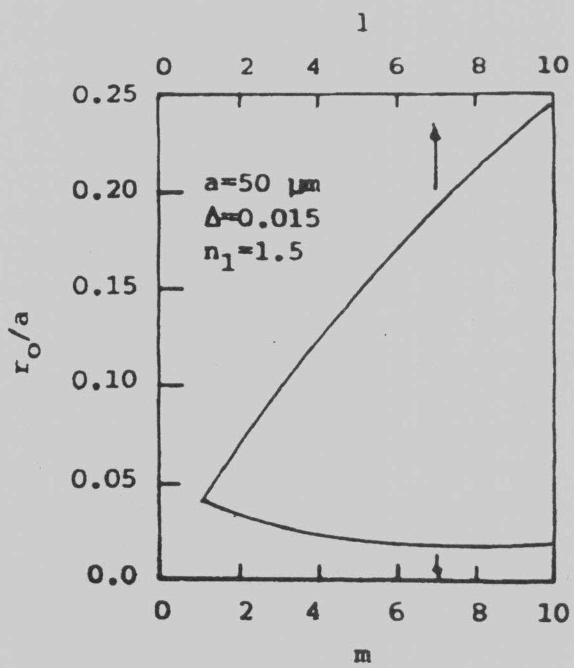

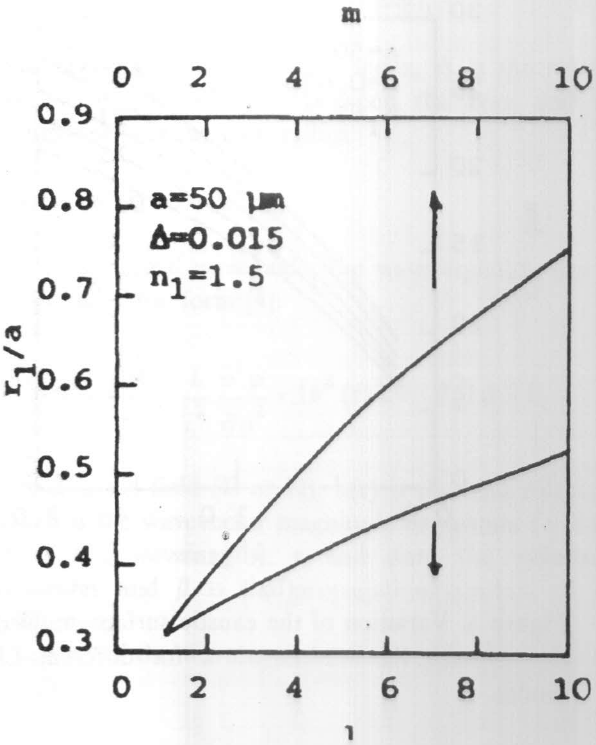

Figure 4. Effect od the azimuthal and radial mode numbers, l and m, on the relative radii, r_o/a and r_1/a, of the caustic surfaces.

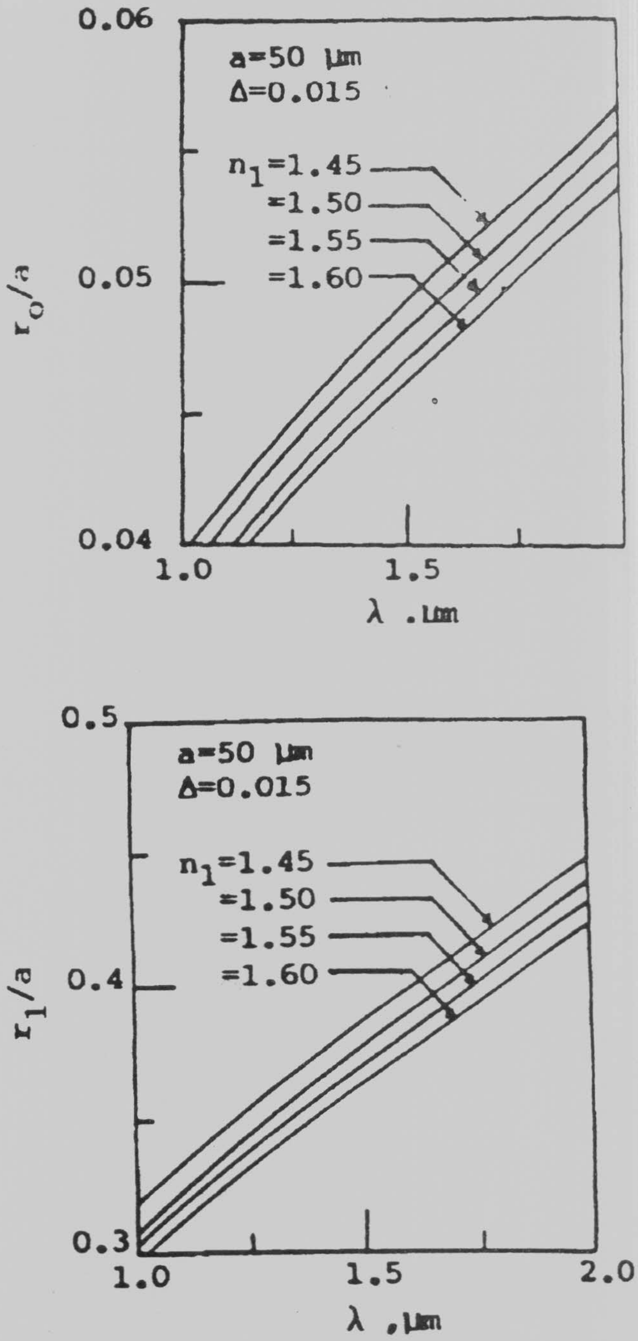

Figure 5. Variation of the relative radii r_o/a and r_1/a of the caustic surfaces with the wavelength λ in the LP_{ol} mode {Effect of the axial refractive index n_1}.

More emphasis is now given to the most dominant mode LP_{01}[6] which corresponds to the traditional mode HE_{11}. Through this mode, the effect of the fiber parameters $\{n_1$, a and $\Delta\}$ on the relative radii of the caustic surfaces is studied.

From the ray theory, it is well known that the increase of the axial refractive index n_1 must be followed by a ray refraction more close to the fiber axis which yields a decrease in the caustic surfaces radii. This gives a good explanation for the results obtained in Figure (5).

The variation of the relative radii of the caustic surfaces, r_o/a and r_1/a, with λ at different values of core radius, is displayed in Figure (6), where it is clearly seen that for the same fiber, i.e. same values of a and Δ, both the relative radii increase with the wavelength λ. From Figure (6) also, it is observed that for the same values of Δ and λ, the relative radii decrease with the core radius a.

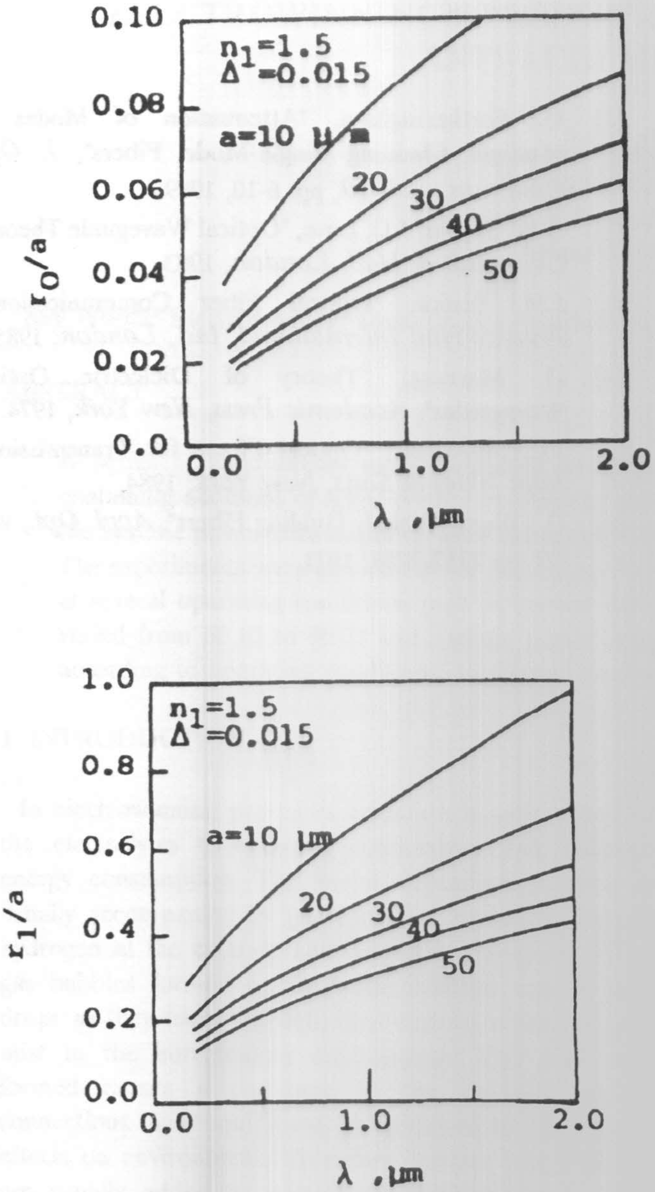

Figure 6. Variation of the relative radii r_a/a and r_1/a of the caustic surfaces with the wavelength λ in the LP_{01} mode { effect of core radius a}.

The effect of Δ on the values of the radii r_1/a and r_o/a is shown in Figure (7), which shows that they decrease with Δ for the same values of both a and λ.

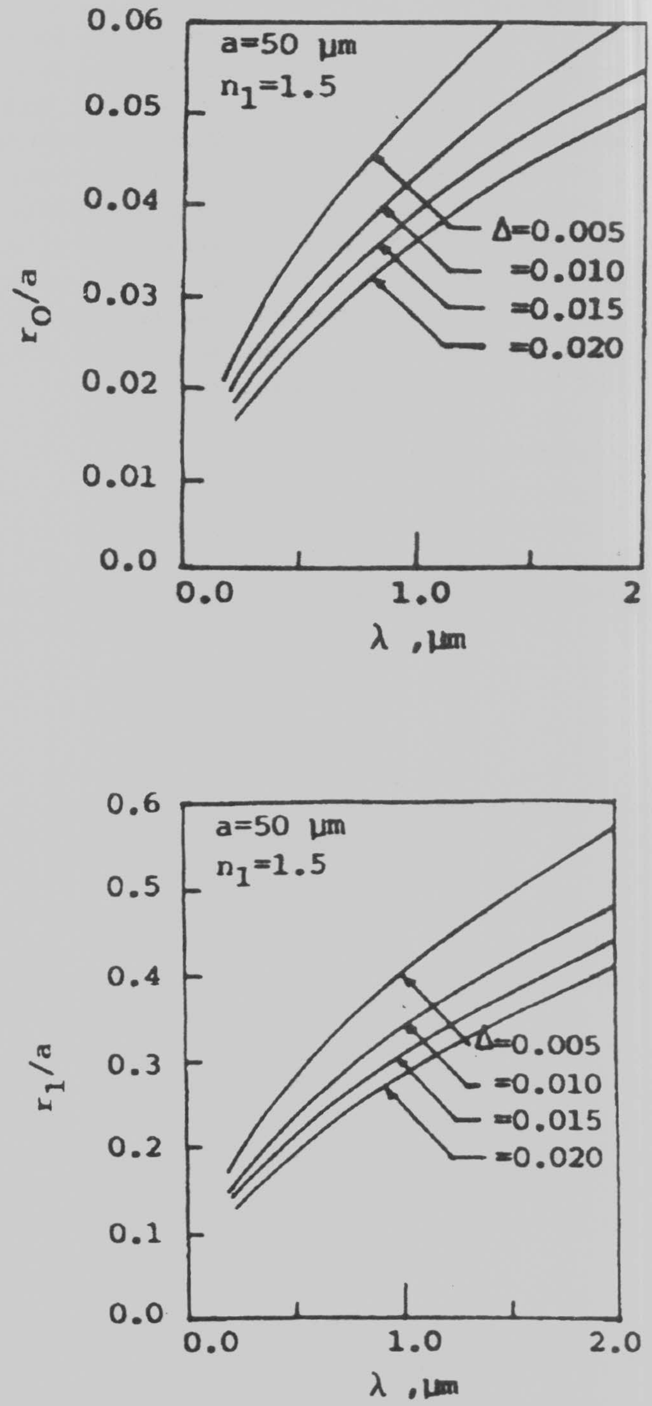

Figure 7. Variation of the relative radii r_o/a and r_1/a of caustic surfaces with the wavelength λ in the LP_{01} mode {effect of the relative refractive index difference Δ}.

CONCLUSION

Ray transmission in graded-index optical fibers is modelled and investigated. It was shown that, skew ray transmission is possible only in multimode fibers, while in single mode fibers only meriodinal rays can exist. The caustic surfaces radii containing the skew rays are calculated for the dominant LP modes. The effect of the wavelength $\lambda$and the fiber parameters (axial refractive index n_1, core radius a, and relative refractive index difference Δ between core and cladding) was studied, where it was found that the caustic surfaces radii increase with λ and decrease with n_1, a, and Δ .

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