PERFORMANCE PREDICTION OF HEAT EXCHANGERS
Part I: Performance of In-Series Connected Heat Exchangers with Cocurrent and Counter-Current Flow Regimes

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ABSTRACT

Most of heat exchangers of the same or different construction features can be coupled one after another through connecting pipes in order to establish a unit of heat exchangers so that it conforms to specific performance and operational characteristics. This arrangement is practically used when two fluids of relatively low flow rates need to exchange heat over a wide temperature range. The main objective of this study is to deduce general relationships for predicting the overall effectiveness of any number of in-series connected heat exchangers having similar or different structural constructions in both cases of cocurrent and counter-current flow regimes. Furthermore, to study the effect of the different influencing parameters such as the number of together connected heat exchangers, the individual effectiveness of each apparatus and the heat capacity rate ratio as well as the number of transfer units on the performance of such equipments. It is found that the overall effectiveness increases as both the number of connected heat exchangers (n) and the individual effectiveness of each exchanger (ε) increase and the heat capacity rate ratio (C) decreases. It becomes independent of n in case of cocurrent flow regime for $\varepsilon = 0.5$ and with equal capacity rates ($C = 1$). The results show also that the rate of increase of the overall effectiveness with the number of heat exchangers is higher in case of counterflow rather than in case of parallel flow. Furthermore, the effectiveness ratio ($\varepsilon_o/\varepsilon$) increases with n and decreases with NTU at steeply rates for low values of NTU. This analysis might be useful for the proper choice of the number of heat exchangers to be connected together and the choice of the best operation conditions for optimal achieving of a required thermal process.

INTRODUCTION

A heat exchanger is a device which provides for transfer of internal thermal energy between two or more fluids at different temperatures. Heat transfer between the fluids take place through a separating wall. Since the fluids are separated by a heat transfer surface, they do not mix. Common examples of such heat exchangers are the shell and tube exchangers, condensers, evaporators and air preheaters.

Heat exchangers are used in the process, power, automotive, air conditioning, refrigeration, cryogenics, heat recovery, alternate fuels, and manufacturing industries, as well as key components of many products available in the marketplace. These heat exchangers may be classified according to the transfer processes, degree of surface compactness, construction features, flow arrangements, number of fluids, and fluid phase changes or process function.

A heat exchanger of any structural construction is considered as compact if it employs a compact surface on either one or more sides of a two-fluid or a multifluid heat exchanger. The convective heat transfer coefficient for gaseous fluids is generally one or two orders of magnitude lower than water, oil and other liquids. Therefore, a compact heat exchanger on the gas side requires a significantly greater amount of surface area for a specified heat transfer rate than for a liquid as a working fluid. The increase in surface area is achieved by either employing surfaces that have high heat transfer surface area density such as extended surface exchangers (plate-fin and tube-fin types [1-3]) or connecting a number of common heat exchangers in-series such as shell and tube heat exchangers, double pipe heat exchangers, spiral tube heat exchangers, gasketed plate heat exchangers, spiral plate heat exchangers, lamella plate heat exchangers...etc.

In the different application fields, there are heat exchanger units consisting of a number of in-series connected heat exchangers of similar or different types installed for carrying out a certain industrial process. The overall effectiveness of the whole equipment is depending on the individual effectiveness of each heat exchanger and the type of flow arrangement that it is either cocurrent or counter-current flow regime.

The plate heat exchangers are very flexible in the way in which channels may be arranged [4-8]. Among the common flow arrangements, all of the channels may be in-series; this arrangement is used when two fluids of relatively low flow rates need to exchange heat over a wide temperature range. Also, in case of plate heat exchanger units of multipass, each side is subdivided into a series of groups and each group is consequently subdivided into a series of passes consisting of a number of parallel channels [9,10]. Thereby, for predicting the total effectiveness, the unit is considered as a cascade of individual heat exchangers each has a single pass and the problem may be treated as a case of in-series connected heat exchangers.

The study presented here is an analysis of the effect of various influencing parameters on the overall effectiveness of in-series connected heat exchangers having similar or different constructions. The aim of this work is also to deduce general mathematical expressions for predicting the overall effectiveness of the heat exchanger units considered in case of cocurrent and counter- current flow regimes.

ANALYSIS OF PERFORMANCE

I) *Cocurrent Flow Regime*

I.1) *Two Heat Exchangers Connected in-Series*

Figure (1) shows a unit of heat exchangers operated under parallel flow condition and consisting of two heat exchangers, of different effectivenesses, connected in-series.

For steady-state conditions and constant specific heat values, i.e constant C and NTU, the overall effectiveness of the unit considered may be calculated as follows:

$$\varepsilon_1 = \frac{(T_{\iota,out})_1 - T_{\iota,in}}{T_{h,in} - T_{\iota,in}} \tag{1}$$

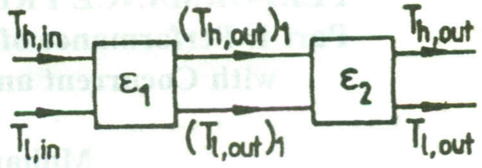

Figure 1. Two connected heat exchangers (cocurrent).

$$C = \frac{(\dot{M}c_p)_\iota}{(\dot{M}c_p)_h} = \frac{T_{h,in} - (T_{h,out})_1}{(T_{\iota,out})_1 - T_{\iota,in}} \tag{2}$$

$$\varepsilon_2 = \frac{T_{\iota,out} - (T_{\iota,out})_1}{(T_{h,out})_1 - (T_{\iota,out})_1} . \tag{3}$$

The overall effectiveness of the unit is defined as

$$\varepsilon_{o,2} = \frac{T_{l,out} - T_{\iota,in}}{T_{h,in} - T_{\iota,in}} \tag{4}$$

The substitution of eqns.(1-3) into eqn.(4) yields

$$\varepsilon_{o,2} = \varepsilon_1 + \varepsilon_2 - (1+C).\varepsilon_1 \varepsilon_2 . \tag{5}$$

I.2) *Three Heat Exchangers Connected in-Series*

The overall effectiveness of the unit can be obtained, as seen from Figure (2), by considering the first two heat exchangers and determining their overall effectiveness using eqn.(5), then combining this unit with the third heat exchanger using also eqn.(5). This gives

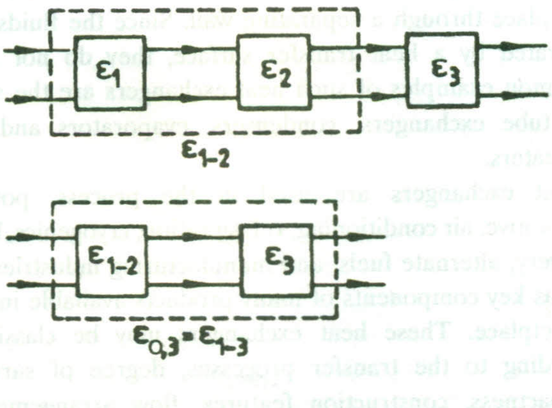

Figure 2. Three heat exchangers connected in-series.

$$\varepsilon_{1-2} = \varepsilon_1 + \varepsilon_2 - (1+C).\varepsilon_1 \varepsilon_2 . \tag{6}$$

$$\varepsilon_{o,3} = \varepsilon_{1-3} = \varepsilon_{1-2} + \varepsilon_3 - (1+C).\varepsilon_{1-2}\varepsilon_3. \qquad (7)$$

Substituting from eqn.(6) into eqn.(7), the following expression results

$$\varepsilon_{o,3} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 - (1+C)(\varepsilon_1\varepsilon_2 + \varepsilon_1\varepsilon_3 + \varepsilon_2\varepsilon_3) + (1+C)^2.\varepsilon_1\varepsilon_2\varepsilon_3 \qquad (8)$$

I.3) *Four Heat Exchangers Connected in-Series*

Figure (3) illustrates the methods of combining the individual heat exchangers together in order to predict the overall effectiveness of the unit. Considering Figure (3-b), one obtains

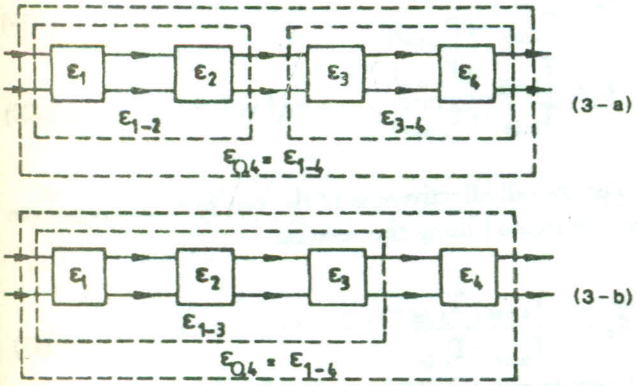

Figure 3. Four heat exchangers connected in-series.

$$\varepsilon_{o,4} = \varepsilon_{1-3} + \varepsilon_4 - (1+C).\varepsilon_{1-3}\varepsilon_4. \qquad (9)$$

The substitution of eqn.(8) into eqn.(9) leads to the following relation:

$$\varepsilon_{o,4} = \varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4-(1+C)(\varepsilon_1\varepsilon_2+\varepsilon_1\varepsilon_3+\varepsilon_1\varepsilon_4 +\varepsilon_2\varepsilon_3+\varepsilon_2\varepsilon_4+\varepsilon_3\varepsilon_4)+(1+C)^2.(\varepsilon_1\varepsilon_2\varepsilon_3+\varepsilon_1\varepsilon_2\varepsilon_4 +\varepsilon_1\varepsilon_3\varepsilon_4+\varepsilon_2\varepsilon_3\varepsilon_4)-(1+C)^3.\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4. \qquad (10)$$

I.4) *Five Heat Exchangers Connected in-Series*

Similarly, as shown in Figure (4), the overall effectiveness of a heat exchanger unit consisting of five in-series connected heat exchangers under cocurrent flow condition may be directly expressed by the following relation:

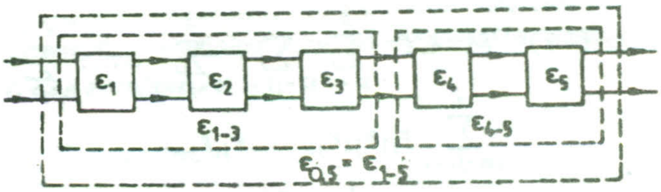

Figure 4. Five heat exchangers connected in-series.

$$\begin{aligned}\varepsilon_{o,5} = {} & \varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4+\varepsilon_5 \\ & -(1+C)(\varepsilon_1\varepsilon_2+\varepsilon_1\varepsilon_3+\varepsilon_1\varepsilon_4+\varepsilon_1\varepsilon_5 \\ & +\varepsilon_2\varepsilon_3+\varepsilon_2\varepsilon_4+\varepsilon_2\varepsilon_5+\varepsilon_3\varepsilon_4+\varepsilon_3\varepsilon_5+\varepsilon_4\varepsilon_5) \\ & +(1+C)^2.(\varepsilon_1\varepsilon_2\varepsilon_3+\varepsilon_1\varepsilon_2\varepsilon_4+\varepsilon_1\varepsilon_2\varepsilon_5+\varepsilon_1\varepsilon_3\varepsilon_4 \\ & +\varepsilon_1\varepsilon_3\varepsilon_5+\varepsilon_1\varepsilon_4\varepsilon_5+\varepsilon_2\varepsilon_3\varepsilon_4+\varepsilon_2\varepsilon_3\varepsilon_5 \\ & +\varepsilon_2\varepsilon_4\varepsilon_5+\varepsilon_3\varepsilon_4\varepsilon_5) \\ & -(1+C)^3.(\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4+\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_5+\varepsilon_1\varepsilon_2\varepsilon_4\varepsilon_5 \\ & +\varepsilon_1\varepsilon_3\varepsilon_4\varepsilon_5+\varepsilon_2\varepsilon_3\varepsilon_4\varepsilon_5)+(1+C)^4.\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4\varepsilon_5.\end{aligned} \qquad (11)$$

I.5) *Overall Effectiveness of n-Different Types Heat Exchangers Connected in-Series Under Cocurrent Flow Regime*

Through comparing the resulting expressions for the overall effectiveness of a heat exchanger unit consisting of two, three, four and five heat exchangers of different constructions arranged in-series with parallel flow condition, which represented by eqns.(5),(8),(10) and (11) respectively and applying the method of mathematical induction, a general formula for the overall effectiveness of any number of connected heat exchangers may be obtained in the following form:

$$\begin{aligned}\varepsilon_{o,n} = {} & \sum_{j1=1}^{n} \varepsilon_{j1} \\ & -(1+C)\sum_{j1=1}^{n-1}\sum_{j2=j1+1}^{n} \varepsilon_{j1}\varepsilon_{j2} \\ & +(1+C)^2\sum_{j1=1}^{n-2}\sum_{j2=j1+1}^{n-1}\sum_{j3=j2+1}^{n} \varepsilon_{j1}\varepsilon_{j2}\varepsilon_{j3} \\ & -(1+C)^3\sum_{j1=1}^{n-3}\sum_{j2=j1+1}^{n-2}\sum_{j3=j2+1}^{n-1}\sum_{j4=j3+1}^{n} \varepsilon_{j1}\varepsilon_{j2}\varepsilon_{j3}\varepsilon_{j4} \\ & + . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ & - . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .\end{aligned}$$

$$+(-1)^{n-2}(1+C)^{n-2}\sum_{j1=1}^{2}\ \sum_{j2=j1+1}^{3}\ \sum_{j3=j2+1}^{4}\cdots\cdots\sum_{j\iota=jk+1}^{n-1}\ \sum_{jm=j\iota+1}^{n}\varepsilon_{j1}\varepsilon_{j2}\varepsilon_{j3}\cdots\cdots\varepsilon_{j\iota}\varepsilon_{jm}$$

$$+(-1)^{n-1}(1+C)^{n-1}\sum_{j1=1}^{1}\ \sum_{j2=j1+1}^{2}\ \sum_{j3=j2+1}^{3}\cdots\cdots\sum_{jm=j\iota+1}^{n-1}\ \sum_{jn=jm+1}^{n}\varepsilon_{j1}\varepsilon_{j2}\varepsilon_{j3}\cdots\cdots\varepsilon_{jm}\varepsilon_{jn} \qquad (12)$$

where $m = n-1$, $\iota = n-2$, $k = n-3$.

I.6) *Case of Similar Heat Exchangers*

When the connected heat exchangers have the same construction features, heat transfer mechanism, degree of surface compactness, flow arrangement, number of fluids and fluid phase changes or process function, their individual effectivenesses will be equal. This means that

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \ldots = \varepsilon_n = \varepsilon .$$

Therefore, the overall effectiveness of the above mentioned two, three, four and five in-series connected heat exchangers respectively may be reduced to

$$\varepsilon_{o,2} = 2\varepsilon-(1+C).\varepsilon^2 \qquad (13)$$

$$\varepsilon_{o,3} = 3\varepsilon -3(1+C).\varepsilon^2 + (1+C)^2\,\varepsilon^3 \qquad (14)$$

$$\varepsilon_{o,4} = 4\varepsilon -6(1+C).\varepsilon^2 + 4(1+C)^2\,\varepsilon^3 - (1+C)^3\varepsilon^4 \qquad (15)$$

$$\varepsilon_{o,5}=5\varepsilon-10(1+C).\varepsilon^2+10(1+C)^2\varepsilon^3-5(1+C)^3\varepsilon^4+(1+C)^4\varepsilon^5 \qquad (16)$$

The foregoing relations can be put in the following general form which holds for any number of in-series connected heat exchangers of equal effectivenesses and operated under cocurrent flow condition.

$$\varepsilon_{o,n} = \frac{1}{1+C}\left\{1-[1-(1+C).\varepsilon]^n\right\} . \qquad (17)$$

II) *Counter-Current Flow Regime*

II.1) *Two Heat Exchangers Connected in-Series*

Referring to Figure (5), the overall effectiveness of a heat exchanger unit consisting of two different type heat exchangers arranged in-series with counter-current flow regime can be predicted in the following manner:

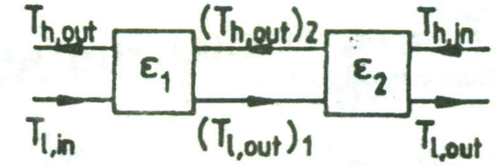

Figure 5. Two connected heat exchangers (counter-current).

$$\varepsilon_1 = \frac{(T_{\iota,out})_1 - T_{\iota,in}}{(T_{h,out})_2 - T_{\iota,in}} \qquad (18)$$

$$C = \frac{T_{h,in} - (T_{h,out})_2}{T_{\iota,out} - (T_{\iota,out})_1} \qquad (19)$$

$$\varepsilon_2 = \frac{T_{\iota,out} - (T_{\iota,out})_1}{T_{h,in} - (T_{\iota,out})_1} \qquad (20)$$

The overall effectiveness of the two heat exchangers can be determined using the relation

$$\varepsilon_{o,2} = \frac{T_{\iota,out} - T_{\iota,in}}{T_{h,in} - T_{\iota,in}} . \qquad (21)$$

Upon eliminating $(T_{\iota,out})_1$ and $(T_{h,out})_2$ from eqns. (18-20) and substituting into eqn.(21), the following expression results

$$\varepsilon_{o,2} = \frac{\varepsilon_1 + \varepsilon_2 - (1+C).\varepsilon_1\varepsilon_2}{1 - C.\varepsilon_1\varepsilon_2} . \qquad (22)$$

II.2) *Three Heat Exchangers Connected in-Series*

In case of a heat exchanger unit consisting of three heat exchangers arranged in-series and operating with counterflow, the overall effectiveness of the unit can be predicted, as in case of parallel flow, by considering two heat exchangers and calculating their effectiveness using eqn.(22), then combining them with the third heat exchanger. In this way, we obtain

$$\varepsilon_{o,3} = \frac{\varepsilon_1+\varepsilon_2+\varepsilon_3-(1+C)(\varepsilon_1\varepsilon_2+\varepsilon_1\varepsilon_3+\varepsilon_2+\varepsilon_2\varepsilon_3)+(1+C+C^2)\varepsilon_1\varepsilon_2\varepsilon_3}{1-C(\varepsilon_1\varepsilon_2+\varepsilon_1\varepsilon_3+\varepsilon_2\varepsilon_3)+C(1+C)\varepsilon_1\varepsilon_2\varepsilon_3} \qquad (23)$$

II.3) *Four Heat Exchangers Connected in-Series*

Similarly, as given above, the overall effectiveness in this case may be obtained through combining each two heat exchangers together using eqn.(22) or combining three heat exchangers using eqn.(23) and then adding the last one to them using eqn.(22), this gives

$$\varepsilon_{o,4}=\frac{E_1-(1+C).E_2+(1+C+C^2).E_3-(1+C+C^2+C^3).E_4}{1-C.E_2+C(1+C).E_3-C(1+C+C^2).E_4} \qquad (24)$$

where,

$$E_1=\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4 \qquad (24\text{-a})$$

$$E_2=\varepsilon_1\varepsilon_2+\varepsilon_1\varepsilon_3+\varepsilon_1\varepsilon_4+\varepsilon_2\varepsilon_3+\varepsilon_2\varepsilon_4+\varepsilon_3\varepsilon_4 \qquad (24\text{-b})$$

$$E_3=\varepsilon_1\varepsilon_2\varepsilon_3+\varepsilon_1\varepsilon_2\varepsilon_4+\varepsilon_1\varepsilon_3\varepsilon_4+\varepsilon_2\varepsilon_3\varepsilon_4 \qquad (24\text{-c})$$

$$E_4=\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4. \qquad (24\text{-d})$$

II.4) *Five Heat Exchangers Connected in-Series*

In a similar way, the overall effectiveness for the unit considered can be obtained and expressed directly by the following relation:

$$\varepsilon_{0,5} = [F_1-(1+C).F_2+(1+C+C^2).F_3-(1+C+C^2+C^3).$$
$$F_4 + (1+C+C^2+C^3+C^4).F_5]/[1-C.F_2+C(1+C).F_3$$
$$- C(1+C+C^2).F_4+C(1+C+C^2+C^3).F_5] \qquad (25)$$

where,

$$F_1=\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4+\varepsilon_5 \qquad (25\text{-a})$$

$$F_2=\varepsilon_1\varepsilon_2+\varepsilon_1\varepsilon_3+\varepsilon_1\varepsilon_4+\varepsilon_1\varepsilon_5+\varepsilon_2\varepsilon_3+\varepsilon_2\varepsilon_4 +\varepsilon_2\varepsilon_5+\varepsilon_3\varepsilon_4+\varepsilon_3\varepsilon_5+\varepsilon_4\varepsilon_5 \qquad (25\text{-b})$$

$$F_3=\varepsilon_1\varepsilon_2\varepsilon_3+\varepsilon_1\varepsilon_2\varepsilon_4+\varepsilon_1\varepsilon_2\varepsilon_5+\varepsilon_1\varepsilon_3\varepsilon_4+\varepsilon_1\varepsilon_3\varepsilon_5 +\varepsilon_1\varepsilon_4\varepsilon_5+\varepsilon_2\varepsilon_3\varepsilon_4+\varepsilon_2\varepsilon_3\varepsilon_5+\varepsilon_2\varepsilon_4\varepsilon_5+\varepsilon_3\varepsilon_4\varepsilon_5 \qquad (25\text{-c})$$

$$F_4=\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4+\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_5+\varepsilon_1\varepsilon_2\varepsilon_4\varepsilon_5+\varepsilon_1\varepsilon_3\varepsilon_4\varepsilon_5+\varepsilon_2\varepsilon_3\varepsilon_4\varepsilon_5 \qquad (25\text{-d})$$

$$F_5=\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4\varepsilon_5. \qquad (25\text{-e})$$

II.5) *Overall Effectiveness of n-Different Type Heat Exchangers Connected in-Series Under Counter-Current Flow Regime*

It is clear from eqns.(22-25) that a general formula for the overall effectiveness of any number of different heat exchangers connected in-series and operated under counterflow condition may be obtained in summation forms as follows:

$$\varepsilon_{o,n}=\frac{\phi(C,\varepsilon_j)}{\theta(C,\varepsilon_j)} \qquad (26)$$

where,

$$\phi(C,\varepsilon_j)=\sum_{j1=1}^{n}\varepsilon_{j1}-(1+C)\sum_{j1=1}^{n-1}\sum_{j2=j1+1}^{n}\varepsilon_{j1}\varepsilon_{j2}+(1+C+C^2)$$
$$\sum_{j1=1}^{n-2}\sum_{j2=j1+1}^{n-1}\sum_{j3=j2+1}^{n}\varepsilon_{j1}\varepsilon_{j2}\varepsilon_{j3}-(1+C+C^2+C^3)$$
$$\sum_{j1=1}^{n-3}\sum_{j2=j1+1}^{n-2}\sum_{j3=j2+1}^{n-1}\sum_{j4=j3+1}^{n}\varepsilon_{j1}\varepsilon_{j2}\varepsilon_{j3}\varepsilon_{j4}+$$
$$\cdots\cdots\cdots\cdots\cdots+$$
$$(-1)^{n-1}(1+C+C^2+C^3+....+C^{n-1})\sum_{j1=1}^{1}\sum_{j2=j1+1}^{2}$$
$$\sum_{j3=j2+1}^{3}\cdots\cdots\sum_{jm=jl+1}^{n-1}\sum_{jn=jm+1}^{n}\varepsilon_{j1}\varepsilon_{j2}\varepsilon_{j3}\ldots\varepsilon_{jm}\varepsilon_{jn} \qquad (26\text{-a})$$

and

$$\theta(C,\varepsilon_j) = 1-C.\sum_{j1=1}^{n-1}\sum_{j2=j1+1}^{n}\varepsilon_{j1}\varepsilon_{j2}+C(1+C).$$
$$\sum_{j1=1}^{n-2}\sum_{j2=j1+1}^{n-1}\sum_{j3=j2+1}^{n}\varepsilon_{j1}\varepsilon_{j2}\varepsilon_{j3}-C(1+C+C^2).$$
$$\sum_{j1=1}^{n-3}\sum_{j2=j1+1}^{n-2}\sum_{j3=j2+1}^{n-1}\sum_{j4=j3+1}^{n}\varepsilon_{j1}\varepsilon_{j2}\varepsilon_{j3}\varepsilon_{j4}$$
$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad+$$
$$(-1)^{n-1}C(1+C+C^2+C^3+...+C^{n-2})\sum_{j1=1}^{1}\sum_{j2=j1+1}^{2}$$
$$\sum_{j3=j2+1}^{3}\cdots\cdot\sum_{jm=jl+1}^{n-1}\sum_{jn=jm+1}^{n}\varepsilon_{j1}\varepsilon_{j2}\varepsilon_{j3}\ldots\varepsilon_{jm}\varepsilon_{jn}. \qquad (26\text{-b})$$

II.6) *Case Of Similar Heat Exchangers*

In this case, the above predicted overall effectivenesses for two, three, four and five connected heat exchangers respectively become,

$$\varepsilon_{o,2} = 1 - \frac{(1-\varepsilon)^2}{1 - C\varepsilon^2} \qquad (27)$$

$$\varepsilon_{o,3} = 1 - \frac{(1-\varepsilon)^3}{1 - 3C\varepsilon^2 + C(1+C)\varepsilon^3} \qquad (28)$$

$$\varepsilon_{o,4} = 1 - \frac{(1-\varepsilon)^4}{1 - 6C\varepsilon^2 + 4C(1+C)\varepsilon^3 - C(1+C+C^2)\varepsilon^4} \qquad (29)$$

$$\varepsilon_{o,5} = 1-(1-\varepsilon)^5/[1-10C\varepsilon^2+10C(1+C)\varepsilon^3$$
$$-5C(1+C+C^2)\varepsilon^4+C(1+C+C^2+C^3)\varepsilon^5] \qquad (30)$$

Upon comparing eqns.(27-30) and applying the mathematical induction method, a general expression for the overall effectiveness of n in-series connected heat exchangers having equal individual effectivenesses and operating under counterflow condition may be recasted in the following form:

$$\varepsilon_{o,n} = 1 - \frac{(1-\varepsilon)^n}{\Gamma(C,\varepsilon)} \qquad (31)$$

where,

$$\Gamma(C,\varepsilon) = 1 - \frac{n(n-1)}{2!}.C\varepsilon^2 + \frac{n(n-1)(n-2)}{3!}.C(1+C)\varepsilon^3$$
$$- \frac{n(n-1)(n-2)(n-3)}{4!}.C(1+C+C^2)\varepsilon^4$$
$$+ \,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.$$
$$+ (-1)^{n-1}.C\,(1+C+C^2+C^3+ \,...\, C^{n-2})\varepsilon^n. \qquad (32)$$

RESULTS AND DISCUSSION

The foregoing analysis yields the overall effectiveness of in-series arranged heat exchangers as a function of the number of heat exchangers connected together and the heat capacity rate ratio as well as the individual effectiveness of each heat exchanger.

The variation of the overall effectiveness with both the number of heat exchangers and the heat capacity rate ratio for cocurrent flow regime is presented in Figures (6)-(9) for ε = 0.2, 0.3, 0.4 and 0.5 respectively. It is clear from the figures that the overall effectiveness increases with n, but the rate of increase seems to be smaller as both C and ε increase. It becomes independent of n when $\varepsilon = 0.5$ and with equal capacity rates (C=1).

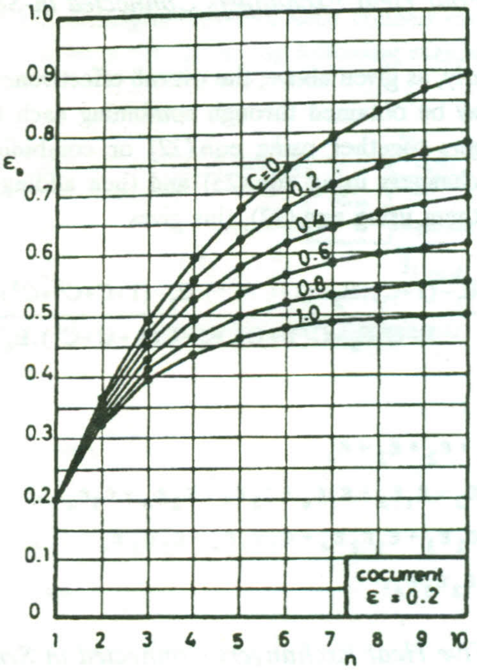

Figure 6. Total effectiveness against the number of connected heat exchangers with different heat capacity rate ratios (cocurrent, $\varepsilon = 0.2$).

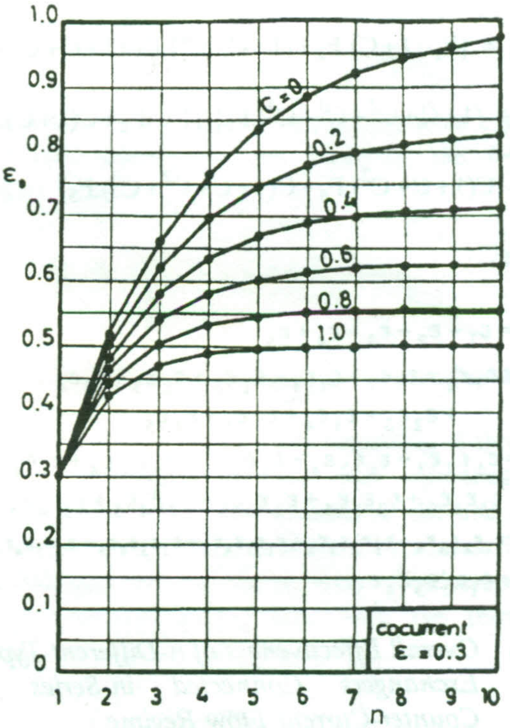

Figure 7. Total effectiveness against the number of connected heat exchangers with different heat capacity rate ratios (cocurrent, $\varepsilon = 0.3$).

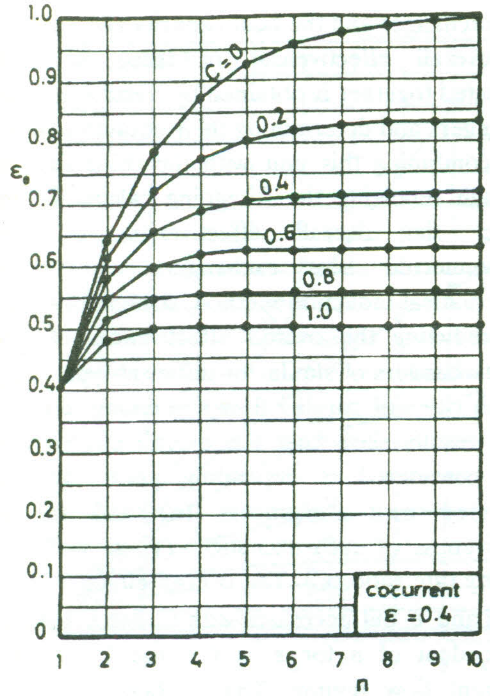

Figure 8. Total effectiveness against the number of connected heat exchangers with different heat capacity rate ratios (cocurrent, $\varepsilon = 0.4$).

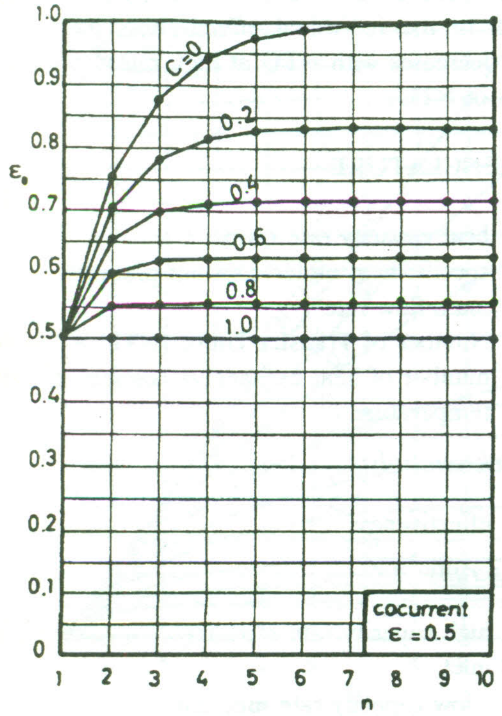

Figure 9. Total effectiveness against the number of connected heat exchangers with different heat capacity rate ratios (cocurrent, $\varepsilon = 0.5$).

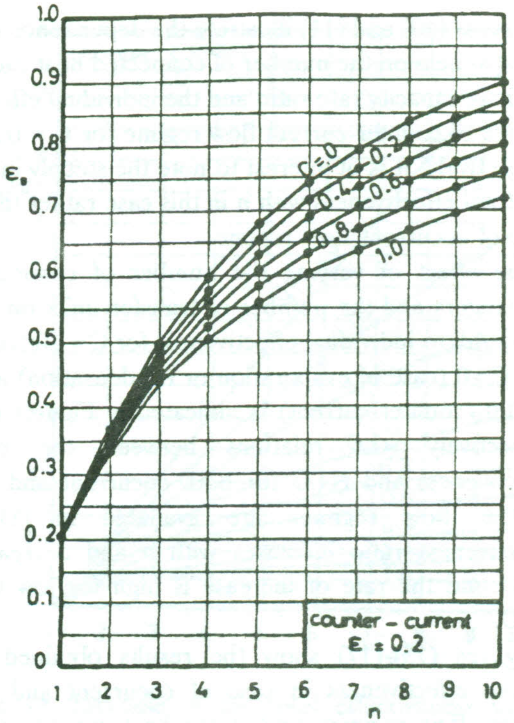

Figure 10. Total effectiveness against the number of connected heat exchangers with different heat capacity rate ratios (counter-current, $\varepsilon = 0.2$).

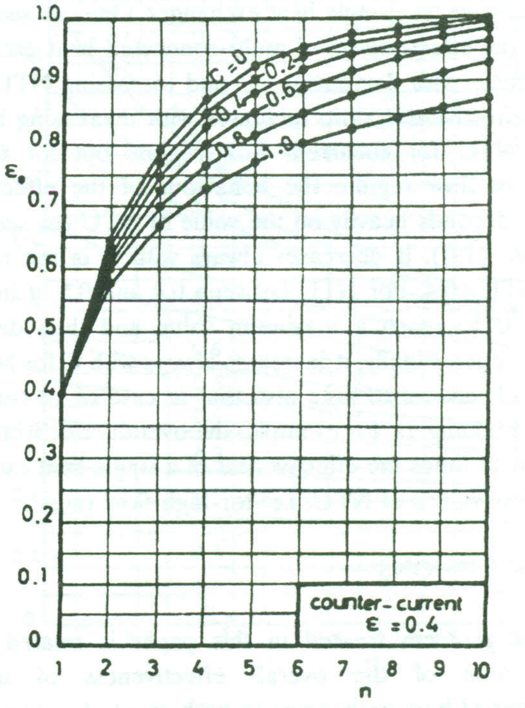

Figure 11. Total effectiveness against the number of connected heat exchangers with different heat capacity rate ratios (counter-current, $\varepsilon = 0.4$).

Figures (10) and (11) illustrate the dependence of overall effectiveness on the number of connected heat exchangers, the heat capacity rate ratio and the individual effectiveness in case of counter-current flow regime for $\varepsilon = 0.2$ and 0.4 respectively. It is of interest to note the steeply increase of the total effectiveness with n in this case rather than in the case of cocurrent flow regime.

The effect of varying the number of connected heat exchangers and the number of transfer units on the ratio of overall to individual effectiveness for $C=1.0$ (cocurrent) and $C=0$ (case of evaporation or condensation) as well as $C=1.0$ (counter-current) is indicated in Figures (12)-(14) respectively (the relations between the individual effectiveness and NTU for both cocurrent and counter-current flow regimes are available in [11]). The effectiveness ratio increases with n and decreases with NTU and the rate of increase is high for low values of NTU.

Figures (15)-(17) show the results obtained for the overall effectiveness in case of cocurrent and counter-current flow regimes against the heat capacity rate ratio with the number of transfer units as parameters, while Figures (18) and (19) represent the corresponding curves for the effectiveness ratio of ten connected heat exchangers to a single heat exchanger. One can see clearly that the effectiveness of multi-connected heat exchangers increases with decreasing C and increasing NTU, while the effectiveness ratio increases with decreasing both C and NTU for cocurrent flow regime but for counter-current flow regime the behaviour of the effectiveness ratio depends heavily on the value of NTU (as seen from Figure (19)). It decreases always with C in the range of $0<NTU<0.4$. For NTU between 0.4 and 0.5, it increases with C to reach a maximum value and then decreases once more. Finally, it increases always with C for $NTU>1$. It is of interest to note also that in case of 10 connected heat exchangers, for example, the overall effectiveness can reach 10 times the effectiveness of a single heat exchanger for low values of NTU, i.e. for high flow rates.

CONCLUSIONS

The problem treated in this paper is related to the prediction of the overall effectiveness of in-series connected heat exchangers in both cases of cocurrent and counter-current flow regimes. Thereby, the equivalent effectiveness of two connected heat exchangers is calculated as a function of the individual effectiveness of each exchanger and the heat capacity rate ratio. After this, the overall effectiveness of three heat exchangers connected together is obtained by considering the first two exchangers and determining their equivalent effectiveness, then combining this unit with the third heat exchanger. Through repeating the foregoing calculation method to predict the overall effectiveness of four, five, six,...connected heat exchangers and applying the mathematical induction method, straightforward formulae for predicting the overall effectiveness of n-connected heat exchangers of similar or different types are prevailed in both cases of parallel flow and counterflow.

The results show that the overall effectiveness of the units considered is depending upon the number of connected heat exchangers (n) and the individual effectiveness of each exchanger (ε) as well as the heat capacity rate ratio (C). The overall effectiveness increases with n and ε but decreases with C, and it becomes totally independent of n for $\varepsilon = 0.5$ and $C = 1$ in case of cocurrent flow regime. That is because the individual effectiveness reaches thereby the maximum value. Moreover, steeply increase of the overall effectiveness with n can be remarked in case of counterflow rather than in case of parallel flow. Also, the effectiveness ratio of the overall to the individual effectiveness increases with n while decreases with NTU at high rates specially for low values of NTU.

NOMENCLATURE

C	heat capacity rate ratio
c_p	specific heat under constant pressure
$\dot{M}$	mass flow rate
NTU	Number of Transfer Units, ($NTU = UA/\dot{M}c_p$)
n	number of heat exchangers connected together
T	temperature

Greek symbols:

ε	effectiveness

Subscripts:

h	high capacity rate medium
in	inlet
ι	low capacity rate medium
n	number of heat exchangers connected together
o	overall
out	outlet
t	total

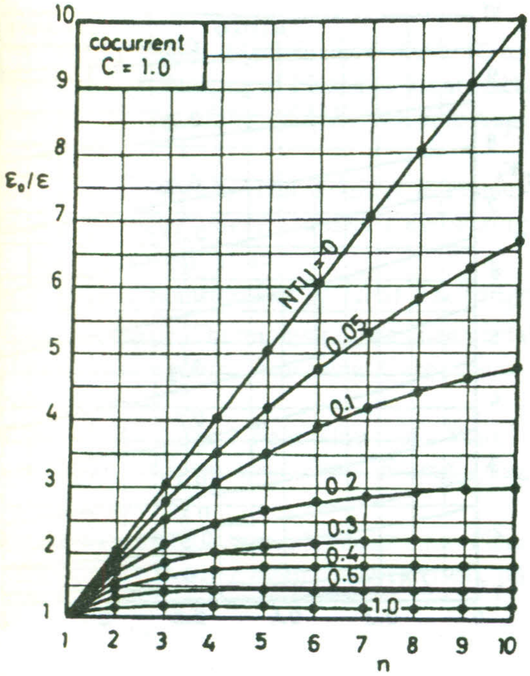

Figure 12. Increase of the total effectiveness with the number of connected heat exchangers for different NTU (cocurrent, C=1.0).

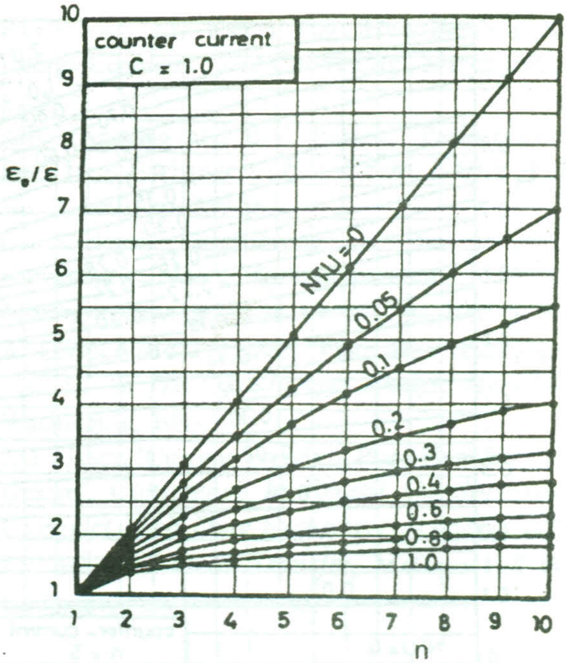

Figure 14. Increase of the total effectiveness with the number of connected heat exchangers for different NTU (counter-current, C=1.0).

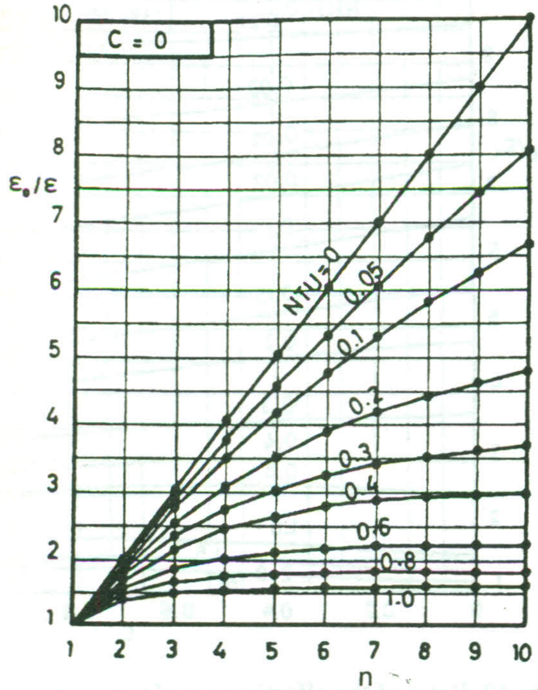

Figure 13. Increase of the total effectiveness with the number of connected heat exchangers for different NTU (evaporation or condensation, C=0).

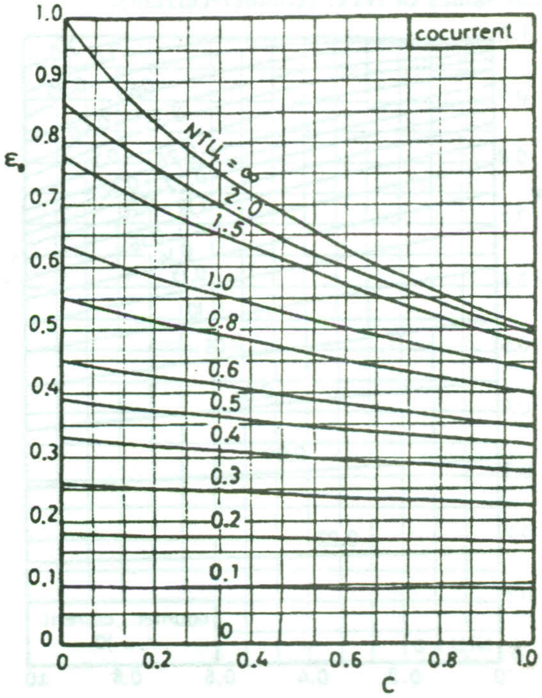

Figure 15. Variation of the total effectiveness with the heat capacity rate ratio for different values of NTU_t (cocurrent flow regime).

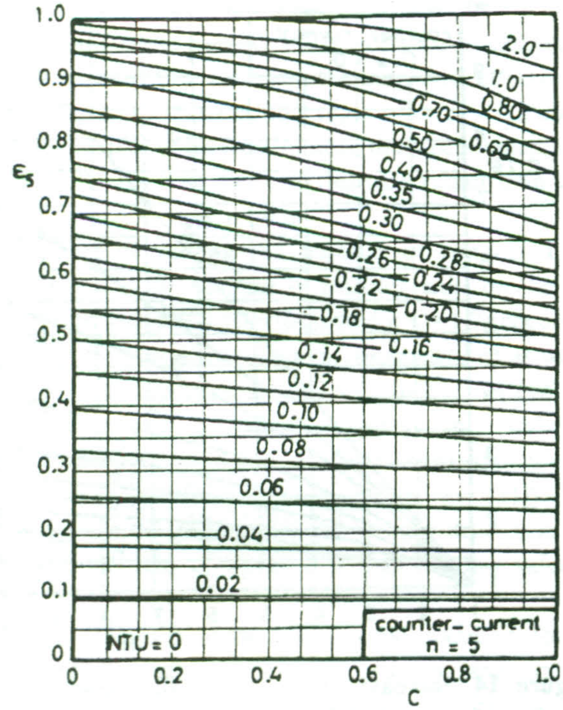

Figure 16. Variation of the total effectiveness of five connected heat exchangers with the heat capacity ratio for different values of NTU (counter-current).

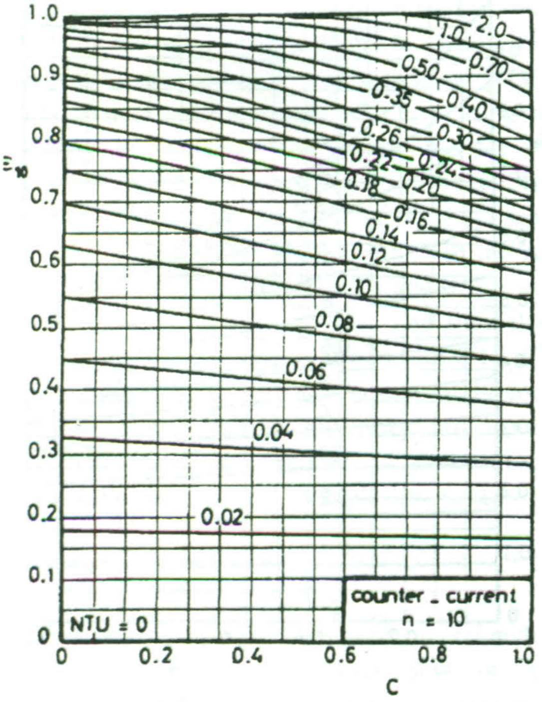

Figure 17. Variation of the total effectiveness of ten connected heat exchangers with the heat capacity ratio for different values of NTU (counter-current).

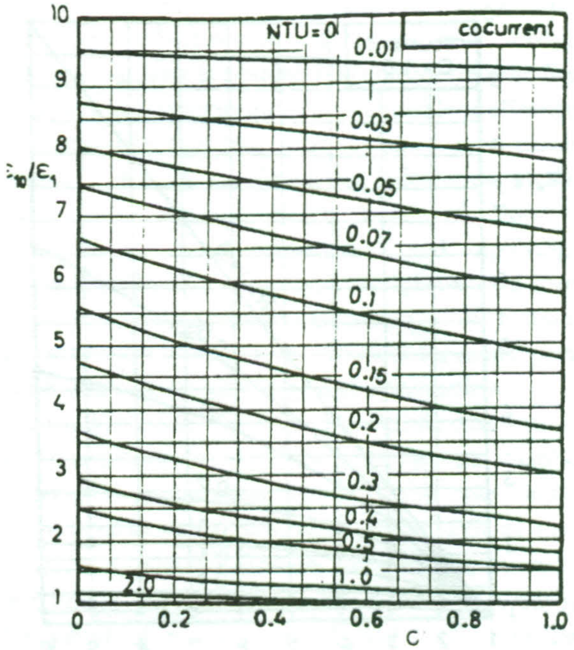

Figure 18. Ratio of the effectiveness of ten connected heat exchangers to a single heat exchanger against the heat capacity rate ratio and NTU as a parameter (cocurrent).

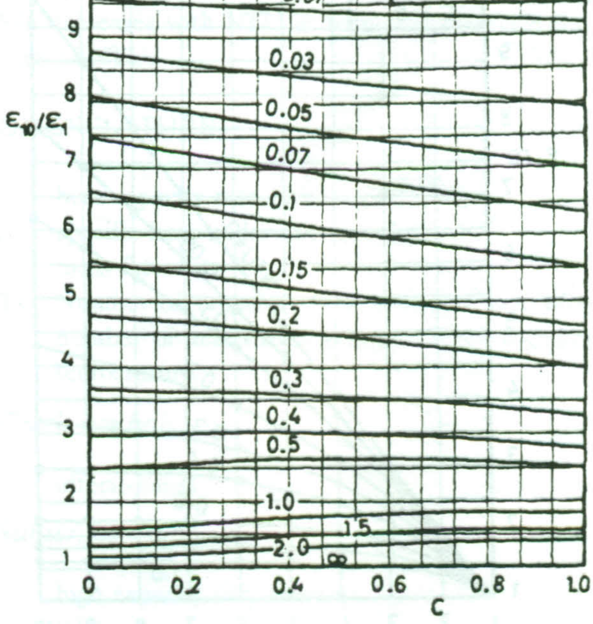

Figure 19. Ratio of the effectiveness of ten connected heat exchangers to a single heat exchanger against the heat capacity rate ratio and NTU as a parameter (counter-current).

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