OPTIMIZATION TECHNIQUES IN APPROXIMATE COORDINATE CALCULATION

Sayed El-Naghi
Transportation Engineering Department, Faculty of Engineering,
Alexandria University, Alexandria, Egypt.

ABSTRACT

This paper offers a direct and straightforward solution for computation of the approximate initial evaluation of coordinates of geodetic stations. Such approximation are necessary in some computation techniques such as variation of coordinates and intersection problems. This will lead to minimize the effort and time in their adjustments. Error propagations were evaluated and treated. The use of appropriate initial values enhances the process of their adjustments to be more convenient and easier.

INTRODUCTION

The geodetic task of position computations, as well as computing any associated geometry quantity between may involve working with a single terrain point, to terrain points or dealing with many terrain points that form a network. In addition, the geodetic task of position computation in two dimensions using cartesian coordinate, which represents one of the main objectives in geodesy implies us to use initial or approximate values for these coordinates as in many geodetic computation techniques such as, variation of coordinates and intersection problems [1,2].

Evidently, it will be wise, comfort and practically to perform such initial geodetic computations to be very close to their most probable values.

By starting with such approximation the repeating of adjustment and the iteration will be minimum.

PRINCPLES

Here in, a single method for computing the approximate coordinates (initial coordinates) for a new point is presented, as follows :

Assuming two known points A and B, where their coordinates are (X_A, Y_A) and (X_B, Y_B) respectively, and it is required to determine the initial coordinates for the new point E (X_E, Y_E).

From Figure (1), the distance AB = S is measured or computed from the coordinates of A and B, and distances from A and B to the new point a and b are measured respectively. From the geometry of the figure, the coordinates of E will be as follows :

$$\left.\begin{aligned} X_E &= X_A + S_1 \cos - h \sin \\ Y_E &= Y_A + S_1 \sin + h \cos \end{aligned}\right\} \quad (1)$$

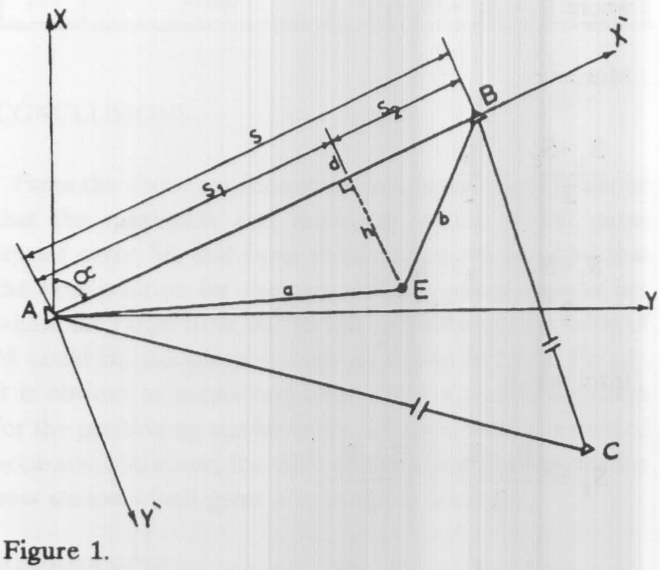

Figure 1.

These equations can be used if the new point is on the right of line AB when going from A to B. If the new point lies on the opposite side, then Equation (1) will deformed to:

$$\left.\begin{aligned} X_E &= X_A + S_1 \cos + h \sin \\ Y_E &= Y_A + S_1 \sin - h \cos \end{aligned}\right\} \quad (2)$$

In Equations (1) and (2), S and h can be computed from the geometry of the figure as follows :

$$\left.\begin{aligned} a^2 &= h^2 + S_1^2 \\ b^2 &= h^2 + S_2^2 \end{aligned}\right\} \quad (3)$$

and for $S = S_1 + S_2$, Equation (3) will be:

$$a^2 = h^2 + (S - S_2)^2$$

$$b^2 = h^2 + (S - S_1)^2$$

from which

$$\left.\begin{aligned} S_1 &= \frac{a^2 - b^2 + S^2}{2S} \\ S_2 &= \frac{b^2 - a^2 + S^2}{2S} \end{aligned}\right\} \quad (4)$$

where $S = S_1 + S_2$.

Moreover,

$$\frac{S_1 + S_2}{2} = \frac{S}{2},$$

$$\frac{S_1 - S_2}{2} = \frac{a^2 - b^2}{2S},$$

from which

$$S_1 = \frac{S_1 + S_2}{2} + \frac{S_1 - S_2}{2},$$

$$S_2 = \frac{S_1 + S_2}{2} - \frac{S_1 - S_2}{2},$$

where $S = S_1 + S_2$

the value of h can be obtained from the formula :

$$h = \sqrt{a^2 - S_1^2} = \sqrt{b^2 - S_2^2} \quad (5)$$

Estimate of Accuracy

To compute the mean square error for the new point E, two new axes X' and Y' are assumed and the coordinates of E with respect to the new axes namely :

$$\left.\begin{aligned} X' &= S_1 = \frac{a^2 - b^2 + S^2}{2S} \\ Y' &= h = \sqrt{a^2 - S_1^2} = \sqrt{a^2 - \left(\frac{a^2 - b^2 + S^2}{2S}\right)^2} \end{aligned}\right\} \quad (6)$$

and the mean square errors σ_x' and σ_x' in x'-direction and y'-direction respectively are as follows:

$$\left.\begin{aligned} \sigma_{x'}^2 &= \left(\frac{\delta S_1}{\delta a}\right)^2 \sigma_a^2 + \left(\frac{\delta S_1}{\delta b}\right)^2 \sigma_b^2 \\ \sigma_{y'}^2 &= \left(\frac{\delta h}{\delta a}\right)^2 \sigma_a^2 + \left(\frac{\delta h}{\delta b}\right)^2 \sigma_b^2 \end{aligned}\right\} \quad (7)$$

where

$$\left.\begin{aligned} &\frac{\delta S_1}{\delta a} = \frac{a}{S}, \quad \frac{\delta S_1}{\delta b} = -\frac{b}{S} \\ &\frac{\delta h}{\delta a} = \frac{a(S - S_1)}{hS} \quad \text{and} \quad \frac{\delta h}{\delta b} = -\frac{bS_1}{hS} \end{aligned}\right\} \quad (8)$$

Substituting from (8) to (7), we obtain :

$$\left.\begin{aligned} \sigma_{x'}^2 &= \frac{a^2}{S^2} \sigma_a^2 + \frac{b^2}{S^2} \sigma_b^2, \\ \sigma_{y'}^2 &= \frac{a^2 (S - S_1)^2}{h^2 S^2} \sigma_a^2 + \frac{b^2 S_1^2}{h^2 S^2} \sigma_b^2 \end{aligned}\right\} \quad (9)$$

The mean square error M for the position of E will be given as :

$$M = \sqrt{\sigma_x^2 + \sigma_y^2} = \sqrt{\sigma_{x'}^2 + \sigma_{y'}^2} \quad (10)$$

substituting values of $\sigma_{x'}$ and $\sigma_{y'}$ we get :

$$M^2 = \sigma_{x'}^2 + \sigma_{y'}^2 = \frac{a^2 b^2}{h^2 S^2} (\sigma_a^2 + \sigma_b^2) \quad (11)$$

$$M = \frac{ab}{hS}\sqrt{\sigma_a^2+\sigma_b^2}$$

or finally

$$M = \frac{\sqrt{\sigma_a^2+\sigma_b^2}}{\sin\alpha} \qquad (12)$$

this is because:

$$S_1 = \frac{ab\sin\alpha}{2} = \frac{hS}{2}$$

which means :

$$\frac{ab}{hS} = \frac{1}{\sin\alpha}$$

assuming that σ_x and σ_y are equal in value, then:

$$\sigma_a = \sigma_b = \sigma$$

Equation (12) can be rewritten as:

$$M = \frac{\sigma}{\sin\alpha}\sqrt{2} \qquad (13)$$

It is clear from Equation (13) that the mean square error M for the position of the new point E will be minimum if $\alpha = 90^\circ$. Therefore; $M_{min} = \sigma\sqrt{2}$ (when $\alpha = 90^\circ$).

To clarify the value M with respect to the value of angle α, different values of σ will be used (e.g., σ = 1 mm, 5 mm and 10 mm) and using Equation (13) value M can be obtained from the following Table:

Table 1: The values of M for different values of angle α

α^o	$M = (\sigma / \sin\alpha)\sqrt{2}$		
	σ = 1 mm	σ = 5 mm	σ = 10 mm
3	27.02	135.09	270.18
5	16.22	81.12	162.24
10	8.14	40.72	81.43
20	4.13	20.67	22.00
40	2.20	11.00	26.33
60	1.63	8.16	15.05
70	1.50	7.52	14.36
80	1.44	7.18	14.14
90	1.41	7.07	14.36
100	1.44	7.18	15.05
110	1.50	7.52	16.33
120	1.63	8.16	22.00
140	2.20	11.00	41.34
160	4.31	20.67	81.43
170	8.14	40.72	162.24
175	16.22	81.12	270.18
177	27.02	135.09	

CONCLUSIONS

From the above-mentioned table it is very clear to notice that the maximum and minimum values of the mean square error M, and from these values we can find that the best position for the new point is when angle α lies within the range from 40° to 140°. Otherwise, the value of M could be considerably high as shown in table. Finally, it is obvious as mentioned before that a closest approach for the positioning can be easily achieved with its expected accuracy. Moreover, the table offers a best position for the new station which gives a reasonable accuracy.

REFERENCES

[1] A.L. Allan, J.R. Hollwey, J.H.B. Maynes, *Practical Field Surveying and Computations*, London, 1975.

[2] Hazay, D.Sc. (Techn.)- Corresponding Member of the Hungarian Academy of Sciences, Adjusting Calculation in Surveying, Budapest, 1970.