EVALUATION OF LENGTH OF A WIDE CHANNEL SUBJECT TO SEEPAGE AND EVAPORATION LOSSES

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ABSTRACT

This paper addresses the analysis of a wide channel subject to seepage and evaporation losses, the channel is being excavated in a permeable soil underlain by a highly permeable aquifer of low piezometric head. Both the cases of constant and variable depths of the top layer are considered. The authors present equations for estimating channel discharge at a general channel section and others for estimating the total length of channel. Because of the implicit nature of the general equation to used in computing channel length a number of non-dimensional curves are given to provide graphical solutions for the problem. The graphs cover a wide range of variables that may be found in practice. The channel length equations are modified for the sake of calculating the hydraulic conductivity of the top soil. Finally three numerical examples are given to show how to use both the equations and the graphs.

NOTATIONS

D depth of the top layer at a general section;
D_o depth of the top layer at a reference section;
h_o piezometric head of the lower aquifer;
i hydraulic gradient of downward seepage;
K hydraulic conductivity of the top layer;
L total length of channel measured from a reference section;
Q_o channel discharge per unit width at a reference section;
Q_x channel discharge per unit width at a general section;
q_e average rate of evaporation at the water surface;
q_{ex} water evaporated per unit channel width and length x;
q_{sx} seepage loss per unit channel width and length x;
S_o difference in slopes of channel bed and the lower surface of interface;
V_s apparent seepage velocity;
x distance downstream from a reference section to a general section;
y water depth of channel at a general section; and
y_o water depth of channel at a reference section.

INTRODUCTION

The problem of seepage from channels has been investigated by many researchers on different lines of approach [1,2,3,4,5].

Solutions for seepage from triangular and trapezoidal channels into soils of finite and infinite depths were provided by Hammad [6], El-Nimr [7], Sharma and Chawla [8], Hathoot [9] and others. In their analyses, the investigators considered seepage taking place from unit length of channel at a specified channel section. The cases in which all the channel discharge is lost by seepage and evaporation and the corresponding effect of the various parameters on the channel length were studied by Hathoot [10,11,12] who assumed straight line surface profiles.

In this paper the objective is to study the combined effect of both seepage and evaporation on the length of wide channels excavated in a permeable soil underlain by a highly permeable aquifer of low piezometric head, Figure (1).

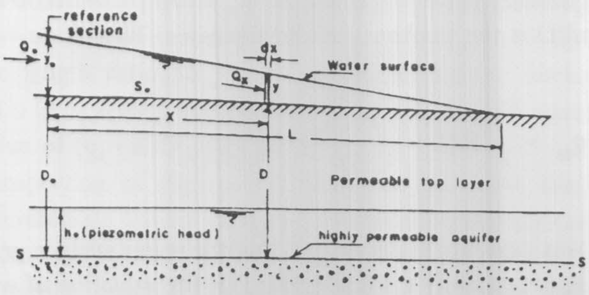

Figure 1. Longitudinal geological section.

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The following assumptions are considered in the paper:

1. The soil is homogeneous and isotropic;
2. Evaporation rate is constant along the water surface;
3. The dynamic effect of surface flow of water is neglected since it should be of minor importance regarding channel length.
4. The water surface profile is assumed a straight line as proved experimentally [13].

SEEPAGE LOSSES

If we consider the longitudinal geological section of Figure (1), the channel discharge per unit width, at a distance x from a reference section, is given by:

$$Q_x = Q_o - q_{sx} \tag{1}$$

in which Q_o is the channel discharge per width at the reference section and q_{sx} is the seepage per unit channel width that takes place between the reference section and the general section. According to Darcy's law the elementary seepage per unit width from a general section may be written as:

$$q_{dsx} = V_s\, dx = Ki\, dx \tag{2}$$

in which V_s is the apparent seepage velocity, K the hydraulic conductivity of the soil and i is the hydraulic gradient. Equation 2 can be put in the following form :

$$dq_s = K\left(1 + \frac{y - h_o}{D}\right)dx \tag{3}$$

in which y is the water depth, h_o is the piezometric head and D is the thickness of the top layer. Therefore:

$$q_{sx} = \int_0^x K\left(1 + \frac{Y - h_o}{D}\right) dx \tag{4}$$

It was experimentally proved that the water surface profile is approximately a straight line [13] the equation of which is given by :

$$y = y_o\,(1 - x/L) \tag{5}$$

in which y_o is the water depth at the reference section and L is the channel length. The value of qx may be obtained for the following cases:

(a) Constant Depth of the Top Layer

In this case the channel bed slope S_o is approximately equal to that of the surface of interface S-S, Figure (1). Combining Eqs. (4) and (5) we get:

$$q_{sx} = \int_0^x K\left[1 + \frac{y_o(1 - x/L) - h_o}{D}\right] dx$$

from which

$$q_{sx} = Kx\left\{1 + \left[y_o - h_o - \frac{y_o x}{2L}\right] /D\right\} \tag{6}$$

(b) Variable Depth of the Top Layer

In this case S_o will be taken as the bed slope of channel if the surface of interface S-S, Figure (1), is horizontal, otherwise it will be considered as the difference between the slopes of the channel bed and S-S. At a general section, Figure (1), the depth of the top layer is given by:

$$D = D_o - S_o\, x \tag{7}$$

in which D_o is the thickness of the top layer at the reference section. To get the seepage per unit channel width that takes place between the reference section and a general section we combine Eqs. (4), (5) and (7) as follows:

$$q_{sx} = \int_0^x K\left[1 + \frac{y_o(1 - x/L) - h_o}{D_o - xS_o}\right] dx \tag{8}$$

Integrating Eq. (8) and simplifying:

$$q_{sx} = Ky_o\left\{\frac{x}{y_o}\left[1 + \frac{y_o}{LS_o}\right] + \frac{1}{S_o}\left[\frac{h_o}{y_o} + \frac{D_o}{LS_o} - 1\right] \ln\left[1 - \frac{xS_o}{D_o}\right]\right\} \tag{9}$$

By means of Eq. (9), the seepage taking place through channel length x measured downstream from a reference section can be evaluated. It is worthy to point out that Eq. (6) cannot be obtained simply by substituting $S_o = 0$ into Eq. (9) but through an indirect mathematical procedure in which the limit $S_o \rightarrow 0$ is considered.

EFFECT OF EVAPORATION

In some cases of flow in wide channels the evaporation taking place from the free surface may play an important role in affecting the channel length. If we consider the average evaporation q_e at the free surface during a certain season, the water evaporated per unit channel width for a length x is simply given by:

$$q_{ex} = q_e . x \tag{10}$$

CHANNEL DISCHARGE AT A GENERAL SECTION AND CHANNEL LENGTH

Let us consider the combined effect of both seepage and evaporation for the two cases previously mentioned.

(a) Constant Depth of the Top Layer

Generally the discharge at a section x downstream from the reference section is given by:

$$Q_x = Q_o - q_{sx} - q_{ex} \tag{11}$$

Substituting for q_{sx} and q_{ex} from Eqs. (6) and (10), respectively we get

$$Q_x = Q_o - Kx\left\{1 + \left[y_o - h_o - \frac{y_o x}{2L}\right]/D\right\} - q_e . x \tag{12}$$

To obtain the total channel length downstream from the reference section substituting $Q_x = 0$ and $x = L$ into Eq. (12) and solving for L as follows:

$$L = \frac{Q_o}{K\,[1 + (y_o/2 - h_o)/D\,] + q_e} \tag{13}$$

(b) Variable Depth of the Top Layer

Substituting for q_{sx} and q_{ex} as given by Eqs. (9) and (10), respectively, into Eq. (11) we get:

$$Q_x = Q_o - Ky_o\left\{\frac{x}{y_o}\,[1 + \frac{y_o}{LS_o}] + \frac{1}{S_o}\,[\frac{h_o}{y_o} + \frac{D_o}{LS_o} - 1]\ln[1 - \frac{xS_o}{D_o}]\right\} - q_e . x \tag{14}$$

Setting $Q_x = 0$ and $x = L$ in Eq. (14) and rearranging:

$$S_o\left[\frac{Q_o - L(q_e + K)}{Ky_o}\right] = 1 + \left[\frac{h_o}{y_o} + \frac{D_o}{LS_o} - 1\right]\ln\left[1 - \frac{LS_o}{D_o}\right] \tag{15}$$

It is evident that the channel length, L, cannot be estimated directly from Eq. (15) but through a trial and error procedure. For convenience, Eq. (15) is put in the following form:

$$\frac{Q_oS_o}{Ky_o} = 1 + \left[\frac{LS_o}{D_o}\right]\frac{(q_e + K)D_o}{Ky_o} + \left[\frac{h_o}{y_o} + \frac{D_o}{LS_o} - 1\right]\ln\left[1 - \frac{LS_o}{D_o}\right] \tag{16}$$

Equation (16) contains the following non-dimensional parameters: $(q_e + K)D_o/(Ky_o)$, h_o/y_o, the discharge ratio, Q_oS_o (Ky_o) and the length ratio LS_o/D_o. Because of the implicit nature of Eq. (16) it is necessary to establish graphical solution for the channel length. Figures (2) through (12) are plotted on the basis of Eq. (16). The values of the above mentioned non-dimensional parameters considered in the figures are so chosen to cover all the possible practical conditions. Each figure corresponds to an individual value of h_o/y_o. In all figures the length ratio LS_o/S_o is plotted versus the discharge ratio $Q_oS_o/(Ky_o)$ and each curve corresponds to a certain value of $(q_e + K)D_o/(Ky_o)$.

Inspection of Figures (2) through (12) shows that for intermediate values of h_o/y_o, linear interpolation can be successfully used to estimate channel length. The following numerical examples illustrate how to estimate channel length for different cases.

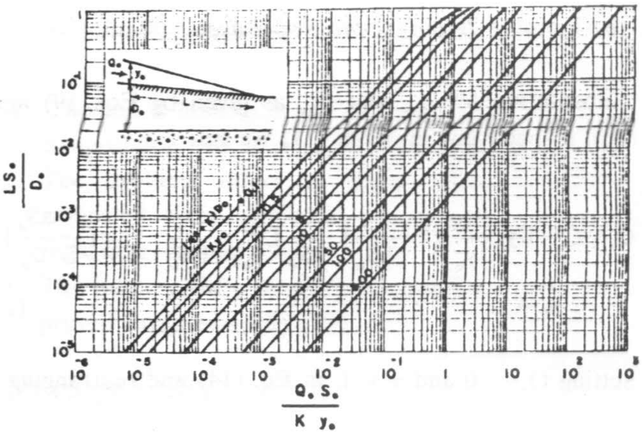

Figure 2. Length ratio (LS_o/D_o) versus discharge ratio (Q_oS_o/Ky_o) for h_o/y_o = 0.

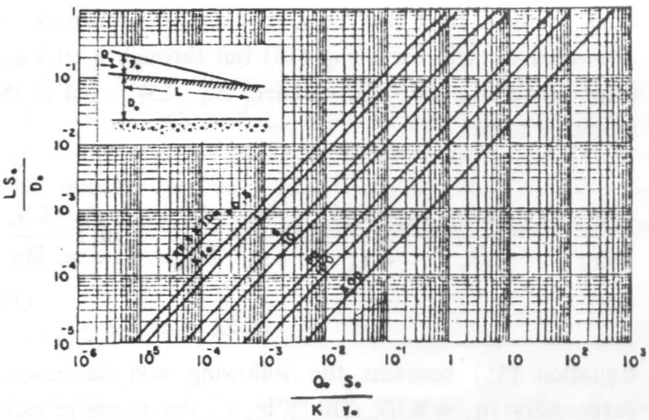

Figure 3. Length ratio (LS_o/D_o) versus discharge ratio (Q_oS_o/Ky_o) for h_o/y_o = 0.2.

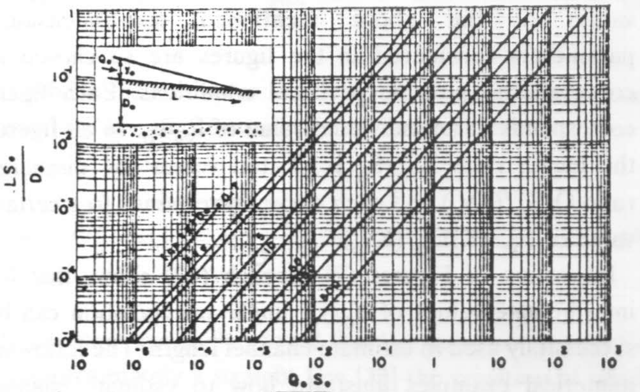

Figure 4. Length ratio (LS_o/D_o) versus discharge ratio (Q_oS_o/Ky_o) for h_o/y_o = 0.5.

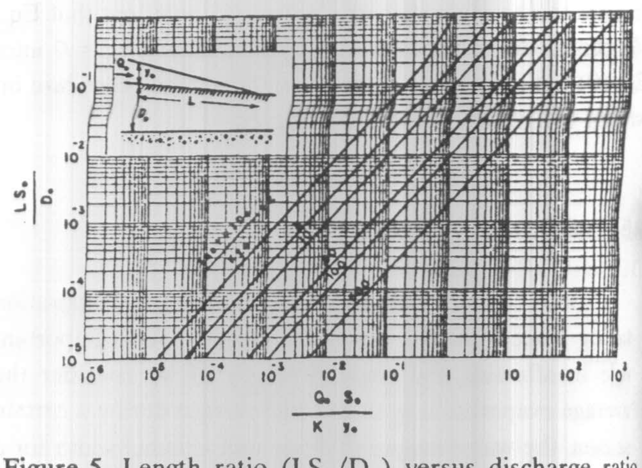

Figure 5. Length ratio (LS_o/D_o) versus discharge ratio (Q_oS_o/Ky_o) for h_o/y_o = 1.0.

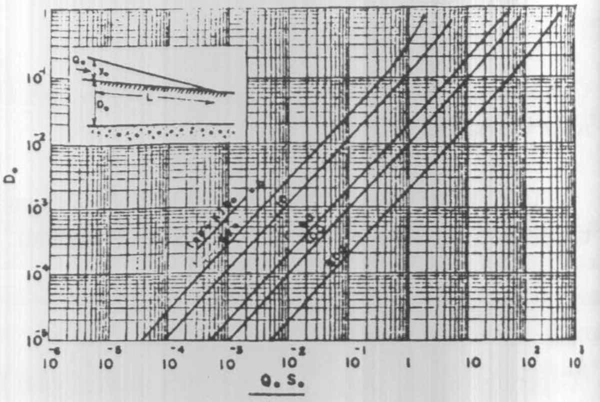

Figure 6. Length ratio (LS_o/D_o) versus discharge ratio (Q_oS_o/Ky_o) for h_o/y_o = 2.0.

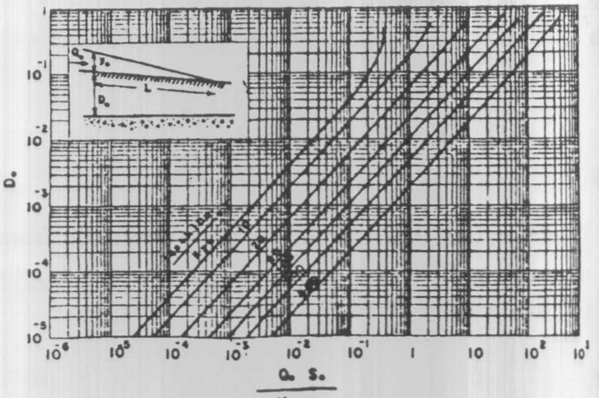

Figure 7. Length ratio (LS_o/D_o) versus discharge ratio (Q_oS_o/Ky_o) for h_o/y_o = 5.0.

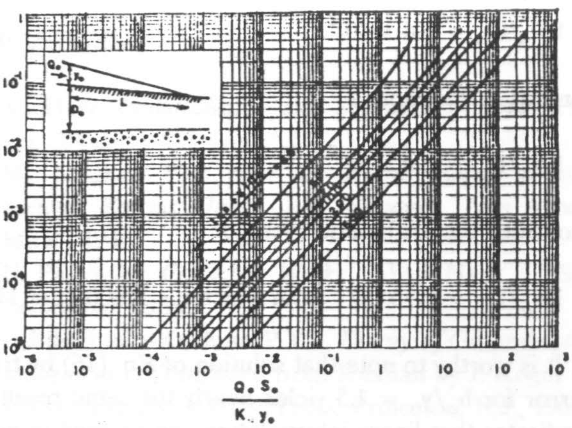

Figure 8. Length ratio (LS_o/D_o) versus discharge ratio (Q_oS_o/Ky_o) for h_o/y_o = 10.0.

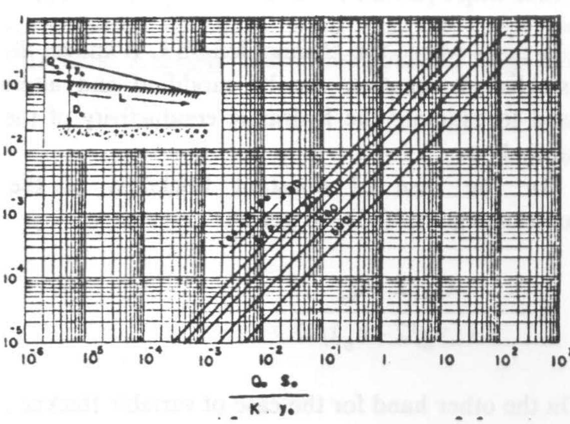

Figure 9. Length ratio (LS_o/D_o) versus discharge ratio (Q_oS_o/Ky_o) for h_o/y_o = 20.0.

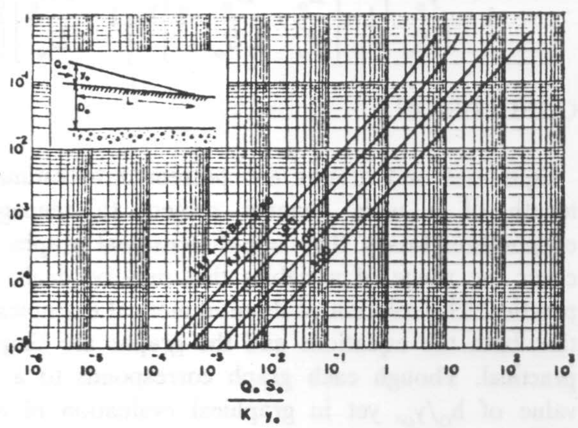

Figure 10. Length ratio (LS_o/D_o) versus discharge ratio (Q_oS_o/Ky_o) for h_o/y_o = 50.0.

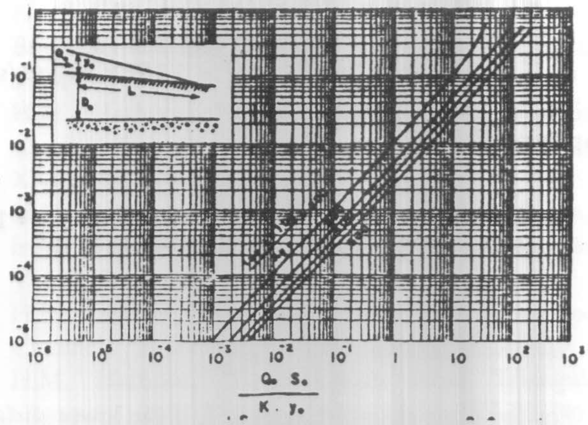

Figure 11. Length ratio (LS_o/D_o) versus discharge ratio (Q_oS_o/Ky_o) for h_o/y_o = 100.0.

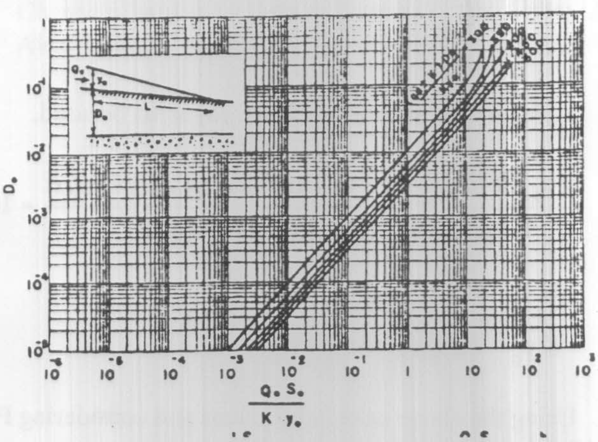

Figure 12. Length ratio (LS_o/D_o) versus discharge ratio (Q_oS_o/Ky_o) for h_o/y_o = 200.0.

Numerical Example 1

A sandy Layer overlies a layer of coarse sand and gravel of piezometric head h_o = 2.0 m. The surfaces bounding the upper sandy layer are parallel with a common slope of 5.0 cm/km and the thickness of the sandy layer is 22.0 m, Water flows in a wide channel excavated in the sandy layer at a depth y_o = 1.0 m at a given section with a corresponding velocity V_o = 0.5 m/s. Find the expected length of channel if the hydraulic conductivity of the sand K = 4.0 m/day and evaporation at the water surface q_e = 10.0 mm/day.

Solution

$$Q_o = V_o y_o = 0.5\ (1.0) = 0.5\ m^3/s/m$$

$K = 4.0/[24 \times 60 \times 60] = 4.63 \times 10^{-5}$ m/s.

$q_e = 10.0/[1000 \times 24 \times 60 \times 6] = 1.157 \times 10^{-7}$ m/s.

applying Eq. (13)

$$L = \frac{0.5}{4.63\times10^{-5}[1+(0.5-2.0)/20.0]+1.157\times10^{7}} = 11643\,m$$

$= 11.643$ km

Numerical Example 2

For the same data of Example 1, if the lower surface of the sand layer is horizontal with an initial layer thickness underneath channel bed, $D_o = 10.0$ m, find the channel length.

Solution

$h_o/y_o = 2.0$, therefore Figure (6) is to be used.

$$\frac{(q_e+K)D_o}{Ky_o} = \frac{(1.157\times10^{-7}+4.63\times10^{-5})\,10.0}{4.63\times10^{-5}(1.0)} = 10.025$$

$$\frac{Q_oS_o}{Ky_o} = \frac{0.5(5.0\times10^{-5})}{4.63\times10^{-5}(1.0)} = 0.54$$

Using the above calculated values and considering Figure (6) we get:

$$\frac{LS_o}{D_o} = 6.4 \times 10^{-2}$$

from which

$L = 12800$ m $= 12.8$ km.

Numerical Example 3:

For the same data of example 2 but for $h_o = 1.5$ m find the channel length.

Solution

In this case $h_o/y_o = 1.5$ m, which is not available in either figures. However let us consider Figures (5) and (6) for which h_o/y_o are 1.0 and 2.0 , respectively. From Figure (5), we get:

$LS_o/D_o = 5.4 \times 10^{-2}$

and as in Example 2

$LS_o/D_o = 6.4 \times 10^{-2}$

consider the arithmetic average:

$LS_o/D_o = 5.9 \times 10^{-2}$ from which $L = 11800$ m $= 11.8$ km.

It is worthy to note that solution of Eq. (16) by trial and error for $h_o/y_o = 1.5$ yields nearly the same result which indicates that linear interpolation can be used to estimate channel lengths for intermediate values of h_o/y_o.

ESTIMATION OF THE HYDRAULICS CONDUCTIVITY

In cases where the channel length L is known the above established equation can be modified and alternatively used to estimate the hydraulic conductivity of the upper sandy layer.

For the case of constant thickness of the layer, rearranging Eq. (13) we get :

$$K = \frac{Q_o/L - q_e}{1+(y_o/2-h_o)/D} \qquad (17)$$

On the other hand for the case of variable thickness of the layer, Eq. (16) is put in the following form :

$$K = \frac{Q_o/L - q_e}{1+\frac{y_o}{LS_o}\left\{1+\left[\frac{h_o}{y_o}+\frac{D_o}{LS_o}-1\right]\ln\left[1-\frac{LS_o}{D_o}\right]\right\}} \qquad (18)$$

CONCLUSION

Equations and graphs are presented to estimate the length of a wide channel subject to seepage and evaporation losses. The non-dimensional graphs mostly cover the range of variables that may be dealt with in practice. The solution of three numerical examples shows that both the equations and the graphs are simple and practical. Though each graph corresponds to a certain value of h_o/y_o, yet in graphical evaluation of channel length for intermediate h_o/y_o values, simple linear interpolation proves adequate and sufficient.

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