A COMPACT KINEMATIC SYNTHESIS PROCEDURE FOR SERIAL ROBOT'S MANIPULATOR ARMS

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ABSTRACT

A compact simplified kinematic synthesis procedure is presented to carry out the inverse analysis of velocity and acceleration of a general serial robot's manipulator arms. The method uses tensor notation and some important mathematical reductions and manipulations to reveal simple recurrence formulas which are useful to solve the inverse problem. The procedure saves the computing time to a great extent in comparison with the other available techniques such as the inverse Jacobian method. In this analysis the inverse kinematic quantities expressing the joint motors velocities and accelerations are related to the absolute velocities and accelerations of the end-effector using a set of linear algebraic equations. Therefore, the solution of this set of linear algebraic equations leads to a direct and unique solution for any non-redundant robot's manipulators. For the purpose of illustration, three numerical example problems describing a three and two six-degrees of freedom serial robot's manipulator arms are considered.

1-INTRODUCTION

One of the basic problems of point to point motion control of N degrees of freedom manipulator is to estimate the generalized input motion (velocities and accelerations of the manipulator joints). These generalized input motion would direct the robot manipulator to move its end-effector from a given initial location to a final desired one. There are several methods, widely but not universally accepted for solving such problems which have been entitled as "Inverse Kinematic motion or Kinematic Synthesis of manipulators". The inverse velocity and acceleration of six degrees of freedom manipulators have been studied by several researchers, some of these are found in [1-9]. The iterative direct method for solving such problem has been presented in [1]. A group of equations have been arranged in matrix form using the screw coordinate transformations in [2], these equations provide straight forward means to analyze the inverse velocity problem of five and six degrees of serial manipulator arms. Recently, the problem of spatial mechanism synthesis which performs a desired velocity has been implemented and solved in [3] using matrix notations. Also, the inverse velocity and acceleration solutions of serial robot's manipulator arms have been carried out using a canonical form of the Jacobian matrix in [4] and the method of quadratic optimization technique in [5]. Computer-Aided-Synthesis (CAS) design procedure has been presented in [10] and may be applied to design such robot's manipulator arms if the structural errors and precision points are considered. All these previous researches which have been discussed treat the synthesis problem in mechanisms and robot's manipulator arms. The published literatures in the field of kinematics of robot manipulators are often dealing with six-by-six Jacobian matrix [J] and its inverse. Therefore, these methods are probably tedious, time consuming and including some computational errors. Since, the major concern in the previous work is the complexity of the inverse Jacobian matrix, therefore, the purpose of this work is to develop a useful and simple kinematic synthesis procedure to carry out in a direct and unique way the inverse solutions to velocities and accelerations of six degrees of freedom robot's manipulators which coordinates with the required initial and final desired motions of the end-effector. The final location of the end-effector has been determined using the method of consecutive analysis of train components technique which is given in [11].

The present work utilizes tensor notation together with some suitable and necessary mathematical manipulations and reductions to reveal a simple set of linear algebraic equations which relate the joint motor velocities and accelerations in terms of velocities and accelerations of the end-effector. The solution of this set of equations are found to be much simpler in comparison with the other available technique such as the Jacobian matrix. Therefore, the presented approach reduces the computational time to a great extent.

2-INVESTIGATED SYSTEM

A general notation and sequence have been assigned to the configuration of the investigated six degrees of freedom serial robot's manipulator arms as shown schematically in Figure (1). The notation and sequence in Figure (1) are selected to result a simple sequential analysis of the inverse kinematic problem. As shown in Figure (1), the joint motor displacements are either revolute or prismatic or any combination of both.

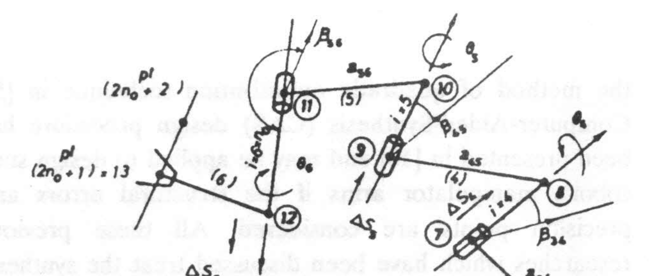

Figure 1. General configuration of six degrees of freedom serial Robot's manipulators arms.

3-KINEMATIC ANALYSIS

The following kinematic analysis is found as a necessary requirement to the proposed synthesis procedure.

3-1-*Position Analysis*

The zero initial location of the manipulator arms is assumed to be known, hence the approach of consecutive analysis of train components technique [11] is used to find out a set of actuators (or joints) displacements which correspond to a desired displaced location of the end-effector in space. Consequently, the displaced location of any auxiliary point j on the arm about an axis i corresponding to rotation θ_i about or translation Δs_i along this axis has been derived in [11]. After performing some mathematical manipulations, this location is defined as;

$$X_{jf}^{k} = y_{ji}^{k} + (X_j^{k} - y_{ji}^{k}) \cos\theta_i - Z_{ji}^{k} \sin\theta_i \quad (1)$$

and

$$X_{jf}^{k} = X_j^{k} + \Delta s_i \gamma_i^{k} \quad (2)$$

for revolute and prismatic joints respectively.

Where,

$$y_{ji}^{k} = X_{2i-1}^{k} + \lambda_{ji} u_i^{k}$$

$$Z^{k}_{ji} = [(X_{2i}^{k2} X_{2i-1}^{k1} - X_{2i}^{k1} X_{2i-1}^{k2}) + (u^{k1}_i X^{k2}_j - u^{k2}_i X^{k1}_j)]/s_i$$

$$K1 = K+1 \quad , \quad K2 = K+2 \quad , K = 1,2,3$$

$$u_i^{k} = (X_{2i}^{k} - X_{2i-1}^{k}), \; s_i = \sqrt{u_i^{m} . u_i^{m}}, \; m = 1,2,3$$

$$\lambda_{ji} = (X_j^{m} - X_{2i-1}^{m}) \, u_i^{m}/s_i^{2} \quad (3)$$

$$\gamma_i^{k} = u_i^{k}/s_i$$

- k Permutation number (k=1,2 or 3. Figures (1)-(4)).
- θ_i Relative angular rotation of arm axis i (due to revolute joint) about an axis i.
- X_j Location of the starting position of any auxiliary point j, j=(2i+1) to (2N+2).
- N Number of the manipulator axes.
- Δs_i Relative translational displacement of arm axis i (due to prismatic joint) along the axis i.

3-2- *Velocity Analysis*

3-2-1- *Absolute Angular Velocity of Arm*

The instantaneous relative angular velocity of the arm about an axis i with respect to the axis i-1 at the final displaced location is given by;

$$\bar{\omega}_{i,i-1} = \dot{\theta}_i \gamma_i^k \tag{4}$$

Therefore, the vectorial summation of the relative angular velocities leads to the absolute angular velocity of the arm axis $\vec{\Omega}_n$, as,

$$\Omega_n^k = \sum_{i=1}^{n} \dot{\theta}_i \gamma_i^k (1 - \delta_{ip}) \tag{5}$$

Where,

δ_{ip} kronecker delta which is unity if p =i and zero if p ≠i, the subscript ρ denotes the number of the arm axis i corresponding to prismatic joint (translational displacement).

3-2-2-*Linear (Translational) Velocity*

Using the concept of the relative linear velocity, the linear velocity of an auxiliary point j corresponding to j=(2i+1) on the manipulator arm relative to point 1 (= X_{2i-1}^k) on the arm axis i is expressed as;

$$\vec{v}_j = \vec{v}_1 + \vec{\Omega}_i \times \vec{R}_{j1} \tag{6}$$

Where,

$\vec{v}_j$ Absolute linear velocity of the point j (= $\dot{X}_{2i+1}^k$).

$\vec{v}_1$ Absolute linear velocity of the point 1 (= $\dot{X}_{2i-1}^k$).

$\vec{\Omega}_i$ Absolute angular velocity of the rigid body i within the manipulator (Eq. 5).

$\vec{R}_{ji}$ Relative position between the two points j and 1(= X_{2i+1}^k-X_{2i-1}^k).

If there is a sliding velocity component due to the presence of a prismatic joint on the axis i, eq. 6 in general form is modified to;

$$\dot{X}_{2i+1}^k = \dot{X}_{2i-1}^k + \Omega_i^{k1} R_{j1}^{k2} - \Omega_i^{k2} R_{j1}^{k1} + (\dot{s}\gamma_i^k)\sigma_{ip} \tag{7}$$

Where,

$R_{j1}^k = (X_{2i+1}^k - X_{2i-1}^k)$, k1 = k+1, k2 = k+2, k=1, 2,3

The absolute linear velocity of the end-effector (gripper) of N degrees of freedom serial robot manipulator arms is deduced by substituting i=1,2,3,.., and N into Eq. 7 as;

$$\begin{aligned}\dot{X}_{2N+1}^k = \dot{X}_1^k &+ (\Omega_2^{k2} - \Omega_1^{k2})X_3^{k1} - (\Omega_2^{k1} - \Omega_1^{k1})X_3^{k2} \\ &(\Omega_3^{k2} - \Omega_2^{k2})X_5^{k1} - (\Omega_3^{k1} - \Omega_2^{k1})X_5^{k2} \\ &\cdots\cdots \\ &+ (\Omega_N^{K2} - \Omega_{N-1}^{k2})X_{2N-1}^{k1} - (\Omega_N^{K1} - \Omega_{N-1}^{k1})X_{2N-1}^{K2} \\ &+ (\Omega_1^{k2} X_1^{k1} - \Omega_1^{k1} X_1^{k2}) + (\Omega_N^{k1} X_{2N+1}^{k2} - \Omega_N^{k2} X_{2N+1}^{k1}) \\ &+ \sum_{i=1}^{N} \dot{s}_i \gamma_i^k \sigma_{ip}\end{aligned} \tag{8}$$

The absolute angular velocity Ω_i^k relative to the absolute angular velocity Ω_{i-1}^k is formulated as;

$$(\Omega_i^k - \Omega_{i-1}^k) = \dot{\theta}_i \gamma_i^k (1 - \sigma_{ip}) \tag{9}$$

Substituting Eq. 9 (for i=1 to N)and Eq. 5 when (n=N) into Eq. 8 and rearranging reveals;

$$\dot{X}_{2N+1}^k = \dot{X}_1^k + \sum_{i=1}^{N} \{\dot{\theta}_i [\gamma_i^{k2}(X_{2i-1}^{k1} - X_{2N+1}^{k1}) - \gamma_i^{k1}(X_{2i-1}^{k2} - X_{2N+1}^{k2})](1 - \delta_{i-p}) + \dot{s}_i \gamma_i^k \sigma_{ip}\} \tag{10}$$

3-3-*Acceleration Analysis*

3-3-1-*Absolute Angular Accelerations*

The differentiation of Eq. 5 with respect to time leads the absolute angular acceleration α_n^k as;

$$\alpha_n^k = \sum_{i=1}^{n} (\ddot{\theta}_i \gamma_i^k + \dot{\theta}_i \dot{\gamma}_i^k)(1 - \sigma_{ip}) \tag{11}$$

Where,

$$\dot{\gamma}_i^k = (\Omega_i^{k1} \gamma_i^{k2} - \Omega_i^{k2} \gamma_i^{k1})$$

$\dot{\theta}_i \dot{\gamma}_i^k$: The Gyroscopic acceleration components.

3-2-2-*Linear (Translational) Acceleration*

The acceleration of any auxiliary point j ($\ddot{X}^{K}_{2i+1}$) is derived by differentiating Eq. 7 with respect to time. Therefore,

$$\ddot{X}^{k}_{2i+1} = \ddot{X}^{k}_{2i-1} + \alpha_i^{k1} R^{k2}_{j1} - \alpha_i^{k2} R^{k1}_{j1} + \Omega_i^{k1} \dot{R}^{k2}_{j1} - \Omega_i^{k2} \dot{R}^{k1}_{j1} + (\ddot{s}_i \gamma_i^{k} + \dot{s}_i \dot{\gamma}_i^{k}) \sigma_{ip} \tag{12}$$

Differentiating Eq. 10 with respect to time reveals;

$$\begin{aligned} \ddot{X}^{k}_{2N+1} = \ddot{X}^{k}_{1} + \sum_{i-1}^{N} \{ & \ddot{\theta}_i [\gamma_i^{k1} (\dot{X}^{k2}_{2N+1} - \dot{X}^{k2}_{2i-1}) - \gamma_i^{k2} (\dot{X}^{k1}_{2N+1} - \dot{X}^{k1}_{2i-1}) \\ & + \dot{\gamma}_i^{k1} (X^{k2}_{2N+1} - X^{k2}_{2i-1}) - \dot{\gamma}_i^{k2} (X^{k1}_{2N+1} - X^{k1}_{2i-1})](1 - \sigma_{ip}) \\ & + \ddot{\theta}_i [\gamma_i^{k1} (X^{k2}_{2N+1} - X^{k2}_{2i-1}) - \gamma_i^{k2} (X^{k1}_{2N+1} - X^{k1}_{2i-1})](1 - \sigma_{ip}) \\ & + (\ddot{s}_i \gamma_i^{k} + \dot{s}_i \dot{\gamma}_i^{k}) \sigma_{ip} \} \end{aligned} \tag{13}$$

KINEMATIC SYNTHESIS PROCEDURE

This procedure mainly depends on the preceding velocities and accelerations equations and consists of the following steps;

a- The initial location together with the required motion of the investigated system are defined. Hence the successive points 1,2,.... and (2N+2) and the final location of the end-effector are determined using Eqs. 1 and 2.

b- The end-effector velocities Ω^{k}_{n} and $\dot{X}^{k}_{2N+1}$ which are given by Eqs. 5 (when n=N) and 10 are written in matrix form as;

$$\bar{A}\dot{\phi} = f^{-} \tag{14}$$

Where,

$$\bar{A} = \left[\frac{a_{ki}}{a_{qi}} \right] \quad i = 1,2,....N \quad k = 1,2,3, q = k+3$$

$$\dot{\phi}^{-T} = \{\dot{\phi}_1, \dot{\phi}_2, \dot{\phi}_N\}$$

$\dot{\phi}_i = \dot{\theta}_i$ (corresponding to revolute joint)

$\dot{\phi}_i = \dot{s}_i$ (corresponding to revolute joint)

and $F^{-T} = \{ F_1, F_2, ..., F_6\}$ (15)

The elements a_{ki}, a_{qi} and F to F are obtained from Eqs. 5 and 10 as;

$$a_{ki} = \gamma_i^{k} (1-\sigma_{ip}) \quad , F_k = \Omega_N^{k} \tag{16}$$

and

$$a_{qi} = L_i^{k} (1-\sigma_{ip}) + \dot{\gamma}_i^{k} \sigma_{ip} \quad , F_q = (\dot{X}_{2N+1}{}^{k} - \dot{X}_1{}^{k}) \tag{17}$$

Where

$$L_i^{k} = \gamma_i^{k2} (X_{2i-1}{}^{k1} - X_{2N+1}{}^{k1}) - \gamma_i^{k1} (X_{2i-1}{}^{k2} - X_{2N+1}{}^{k2})$$

and

$$\gamma_i^{k} = u_i^{k} / s_i \tag{18}$$

Therefore, in case of six degrees of freedom Robot's Manipulator (i.e.) N=6, Eqs. 14 and 15 are used directly to find out the joint rates ($\dot{\phi}_i$,i =1,2,..) for specified absolute angular and linear velocities of the end-effector $\Omega_N{}^{k}$ and $\dot{X}_{2N+1}$ respectively. Also, $\dot{X}_1{}^{k}$= 0.0, since the base point (1) is fixed.

c- Next, the absolute angular velocities of connecting links Ω^{k}_{n} (n =1,2 ,N-1) are calculated using Eqs. 5. Also, the absolute velocities of the auxiliary points ($\dot{X}^{k}_{2i=1}$), (i= 1,2 ,...,N-1)are evaluated using Eqs. 7.

d- Similar to step b, the absolute angular and linear end-effector accelerations α^{k}_{n} and $\ddot{X}_{2N+1}{}^{k}$ which are given by Eqs. 11 (when n=N) and 13, respectively, are written in matrix form as;

$$\bar{A}\ddot{\phi}^{-} = f^{-} \tag{19}$$

Where,

A and $\ddot{\phi}^{-}$ are as step b. Therefore,

$$\ddot{\phi}^{-} = \{\ddot{\phi}_1, \ddot{\phi}_2, ... \ddot{\phi}_N\}$$

and

$$f^{-T} = \{ f_1, f_2, ..., f_6\} \tag{20}$$

The elements f_1 to f_6 are obtained from eqs. (11) and (13) as;

$$f_k = \alpha^k{}_N - \sum_{i=1}^{N} \dot{\theta}_i \dot{\gamma}_i^k{},1(1-\sigma_{ip}), \quad k=1,2,3 \tag{21}$$

and

$$f_q = (\ddot{X}^k{}_{2N+1} - X_i^k) - \sum_{i=1}^{N} [\dot{\theta}_i (Q_i^k + w_i^k)(1-\sigma_{ip}) + \dot{S}_i \gamma_i^k \sigma_{ip}] \tag{22}$$

where,

$$Q^k{}_i = \gamma_i{}^{k1}\,(\dot{X}^{k2}_{2N+1} - \dot{X}^{k2}_{2i-1}) - \gamma_i^{k2}(\dot{X}^{k1}_{2N+1} - \dot{X}^{k1}_{2i-1})$$

$$w^k{}_i = \dot{\gamma}^{k1}{}_i\,(X^{k2}_{2N+1} - \dot{X}^{k2}_{2i-1} - \gamma_i^{k2}(X^{k1}_{2N+1} - X^{k1}_{2i-1}) \tag{23}$$

Consequently, for a six degrees of freedom Robot's manipulator arms eqs. 19 and 20 are used directly to find out the joint rates $\ddot{\phi}_i$ (i=1,2,.. and 6) for specified absolute angular and linear end-effector accelerations $\alpha_N{}^k$ and $\ddot{X}_{2N+1}{}^k$ respectively. Where $\ddot{X}_{2N+1}{}^k = 0.0$.

5-IMPLEMENTATION

The previous procedure is applied to solve the inverse problem of three types of serial robot's manipulator arms. The first is the Puma 560 and the others are six degrees of freedom robot's manipulators are shown in Figures (2),(3) and (4) respectively. The base fixed point 1 has coordinates $X_1{}^k$,(0,0,0) and $\dot{X}_1{}^k = \ddot{X}_1{}^k = 0$

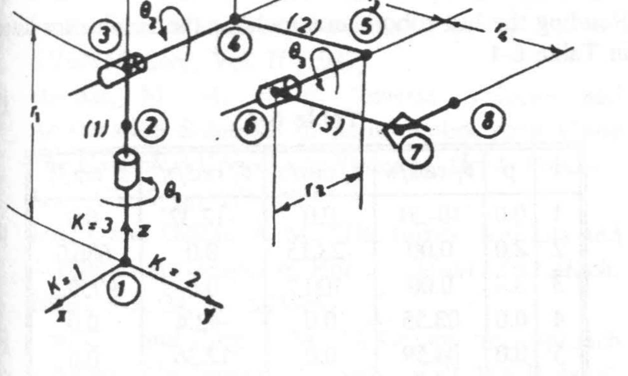

Figure 2. Puma 650 robot's manipulator.

5-1 *The Puma 560*

Figure (2) shows the Puma 560 of three degrees of freedom with the following data;

i	p	θ	Δs_i	remarks
1	0	30	0	$r_1=2, r_2=1$
2	0	45	0	$r_3=2\ ^2m$
3	0	60	0	

End-effect.\k	1	2	3
$\dot{X}_7{}^k$(m/s)	-3.0	0.5	2.0
$\ddot{X}_7{}^k$ (m/s_2)	6.0	-1.0	4.0

i.e $\sigma_{ip} = 0$ (since all joints are revolutes)

5-2- *First six degrees of freedom serial robot's manipulator*

Figure (3) shows the first six degrees of freedom serial robot's manipulator arms with revolute joints and has the following data;

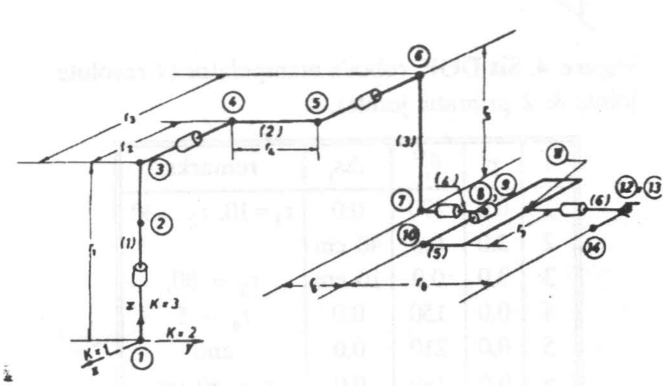

Figure 3. Six DOF. robot's manipulator (revolute joints).

i	p	θ_i^o	Δs_i	remarks
1	0	330	0.0	$r_1=20$, $r_2=7$
2	0	270	0.0	$r_3=14$, $r_4=40$
3	0	150	0.0	$r_5=3$, r46 =42
4	0	2400	0.0	r_7 = 5 and
5	0	90	0.0	r_8 = 40 cm
6	0	300	0.0	

End-effect\k	1	2	3
Ω_6^k	8.0	-10.0	12.0
α^k_6(rad/s^2)	45.0	12.0	-35.0
$\dot{X}^k_{13}$(cm/s)	-400.0	-100.0	-350.0
$\ddot{X}^k_{13}$(cm/s^2)	-1500.	-8000	-4000

i.e. $\sigma_{ip}=0$ (all joints are revolutes)

5-3- *Second six degrees of freedom robot manipulator*

Figure (4) shows the third solved example problem which presents six degrees of freedom serial robot's manipulator arms with four revolute joints and two prismatic ones. The robot has the following data;

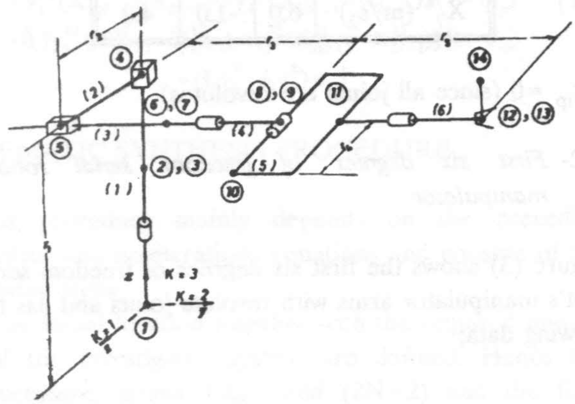

Figure 4. Six DOF. robot's manipulator (4 revolute joints & 2 primatic joints).

i	p	θ_i^o	Δs_i	remarks
1	0.0	270	0.0	$r_1=10$, $r_2=30$
2	2.0	0.0	40 cm	
3	3.0	0.0	10 cm	$r_3 = 80$,
4	0.0	150	0.0	$r_4 = 5$
5	0.0	210	0.0	and
6	0.0	180	0.0	$r_5= 40$ cm

End-effect\k	1	2	3
Ω_6^k(rad/s)	0.0	5.0	10.0
α_6^k (rad/s^2)	-50.0	10.0	-5.0
$\dot{X}_{13}^k$ (cm/s)	200.0	600.0	150.0
$\ddot{X}_{13}$(cm/s^2)	4000	1000	600.0

$\sigma_{ip} = 0$ for i = 1,4,5 and 6
$\sigma_{ip} = 1$ for i = 2 and 3

6-RESULTS AND DISCUSSION

The solutions of Eqs. 14,15,19 and 20 have been carried out. The results of the first example (Puma 560 manipulator) are given in Tables 6-1 and 6-2

Table 6-1.

i	$\dot\theta_i$ (rad/s)	$\ddot\theta_i$(rad/s^2)
1	0.986	-3.60
2	-1.40	12.62
3	-1.23	-60.71

Table 6-2.

End-effect \k	1	2	3
Ω_3^k(rad/s)	0.0600	0.030	0.987
α_3^k(rad/s^2)	-63.52	-36.6	-3.6000

The results of table 6-1 have been carried out using the parts of Eqs. 14 and 19 corresponding to Eqs. 10 and 13. Then results of table 6-2 are computed using Eqs. 5 and 11, respectively, for (n=N=3). Also, the results of the second and third examples are obtained. Table 6-3 indicates the results of the serial robot's manipulator arms in the second example.

Table 6-3.

i	$\dot\theta_i$(rad/s)	$\ddot\theta_i$(rad/s^2)
1	30.01	-634.22
2	09.80	-305.55
3	-10.86	219.80
4	13.30	-646.51
5	-16.70	137.15
6	-02.91	-055.80

$\dot s_i = \ddot s_i = 0.0$.

Reading the last robot's manipulator the results are listed in Table 6-4.

Table 6-4.

i	p	$\dot\theta_i$ rad/s	$\dot s_i$ cm/s	$\ddot\theta_i$ rad/s^2	$\ddot s_i$ cm/s^2
1	0.0	10-.51	0.0	-12.32	0.0
2	2.0	0.00	-23.15	0.0	686.0
3	3.0	0.00	-100.9	0.0	9759.3
4	0.0	03.55	0.0	-43.4	0.0
5	0.0	04.59	0.0	12.36	0.0
6	0.0	04.10	0.0	-42.31	0.0

All of these results corresponding to the joint motor velocities and accelerations in tables 6-1 ,2,3 and 6-4 have been used to compute the end-effector velocity and acceleration quantities using the method of direct kinematic analysis of consecutive train components. The computed quantities are found to be very close to the specified values of the input data to the inverse kinematic problem. This implies that the presented procedure is simple and very useful for the inverse solution of velocities and accelerations problems of any serial robot's manipulator arms.

7-CONCLUSIONS

The selected general configuration which has been used in this work results a simple sequential analysis of the inverse kinematic problem. Tensor notation together with some important mathematical reductions and manipulations reveal simple recurrence formulas which are found very useful to solve the inverse problem with minimal time requirements in comparison with the other available techniques. One of the main advantages of the presented procedure is to eliminate the usage of Jacobian matrix approach which is widely used to solve such inverse problem which consumes a considerable computing time to evaluate the Jacobian matrix elements.

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