STABILITY OF HYDRAULIC STRUCTURES AGAINST LOCAL SCOUR

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ABSTRACT

The problem of local scour created downstream a hydraulic structure founded on a finite pervious layer is studied in this paper. A finite element model was designed to solve the problem. The model was tested and verified. The model was used for computing the effect of the scour pool dimensions on the seepage characteristics. The results were presented in the form of curves. Effects of the different variables on the seepage characteristics are analysed.

NOTATIONS

A the flow domain;
A^e the element domain;
B length of floor;
b distance between upstream Cut-off and the toe of floor;
d_1 depth of upstream cut-off;
d_2 depth of downstream cut-off;
{F} global force vector;
H_1 effective head on the floor;
H_1 upstream water depth;
H_2 downstream water depth;
I number of nodes in element;
$(I_e)_n$ normal exit gradient
k coefficient of permeability;
[K] global stiffness matrix of a domain;
k_{nm}^e element stiffness matrix;
L horizontal length of scouring;
L_1 considered length of the upstream bed;
N_m, N_n linear interpolation functions;
q quantity of seepage per unit width;
q_n normal flux across the boundary of the element;
R residual;
S boundary of the flow domain;
T depth of pervious strata;
V_x, V_y velocity components;
X_a horizontal distance between the apex and the floor end;
ya scouring depth at the apex;
X_n, y_n nodal co-ordinates
ϕ potential head;
$\hat{\phi}$ approximate value of potential;
ϕ_n nodal value of potential;
$\{\phi\}$ global vector of unknown potential;
ψ stream function.

INTRODUCTION

The local scour phenomenon is usually created downstream the floor of the hydraulic structures constructed in alluvial soils. This phenomenon has been investigated by Shukry 1957 [1], (Hartung 1957, Oarstens 1966, Sarma 1971, Farhoudi and Smith 1982, and Uyumaz 1988) [2]. An analysis of the shapes of the scour pool are presented by (Maarten B. de Groot [9] and Uyumaz [2]). From this analysis the shape of the scour pool in the present research was taken as a triangular with downward apex. To make the problem more practical, the floor is provided with two cut-offs, one of them is located at the downstream end of the floor, Figure (1).

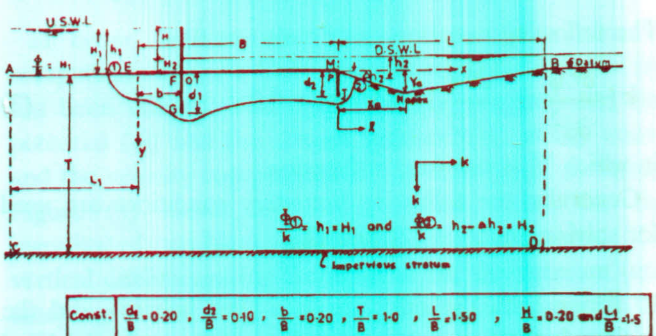

Figure 1.

The effect of the scour pool dimensions on the different seepage characteristics (uplift pressures, exit gradients and quantity of seepage beneath the hydraulic structure) are studied in the present paper. A finite element numerical model has been designed to solve the investigated problem. For the case of seepage beneath a simple flat floor without any vertical projection and without downstream scour pool, the uplift pressures along the floor are calculated twice using the exact mathematical model of Pavlovsky [8], and the designed finite element model. A comparison is made between the obtained results to verify the model.

The results have been plotted in the form of curves. Analysis of these results are presented to illustrate the effect of the scour pool dimensions on the seepage characteristics.

THEORETICAL CONSIDERATIONS

The governing differential equation for steady state flow in an anisotropic and homogeneous soil in the x-y plane is given by Laplace's equation:

$$\frac{\partial}{\partial x}(k_x\frac{\partial \phi}{\partial x}) + \frac{\partial}{\partial y}(k_y\frac{\partial \phi}{\partial y}) = 0.0 \qquad (1)$$

where k_x and k_y are the coefficients of permeability in the x and y directions, respectively; and ϕ is the potential head, in which ϕ = kH, and H is the piezometric head. In the present problem, Figure (1), the flow domain was assumed as isotropic soil ($k_x = k_y = k$), therefore equation (1) will be:

$$k(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}) = \nabla^2 \phi = 0.0 \qquad (2)$$

The following equation can be also written:

$$k(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}) = \nabla^2 \psi = 0.0 \qquad (3)$$

in which Ψ is the stream function.

Generally, the following boundary conditions are used for solving equation (2) and (3):

1. Along both the entrance face (line source), and the exit face, (line sink}, the values of the potential head are $\phi_1 = kH_1$, $\phi_2 = kH_2$, respectively. Or

$$\frac{\phi_1}{k} = H_1, \ ..., \ \frac{\phi_2}{k} = H_2 \qquad (4)$$

2. Along the subsurface contour of the floor, the value of the stream function, Ψ = 0.0.

3. Along the top line for the impervious layer, the normal velocity v_n equals zero $\partial\phi/\partial n = 0.0$ and the value of stream function Ψ = constant = q, in which q is the quantity of seepage per unit width.

FINITE ELEMENT MODELING

In this paper, the finite element technique is used for solving the governing equation (1). The values of velocity potential (ϕ) and stream function (Ψ) can be obtained at any point through the flow domain by solving the partial differential equations (2) and (3), respectively. These equations can be solved approximately by the finite element method as follows:

Applying the residual approach [6] to equation (2) gives:

$$\nabla^2 \bar{\phi} = R \qquad (5)$$

in which R is the residual and ϕ is the approximate value of the potential;

$$\bar{\phi} = \sum_{n=1}^{I} N_n \phi_n, \ (n = 1, 2, ..., I) \qquad (6)$$

in which N_n are linear interpolation functions, ϕ_n is the nodal values of ϕ at x_n and y_n and I is the number of nodes in the element, and x_n, y_n are the nodal co-ordinates.

Substituting the value of $\bar{\phi}$ from equation (6) into equation (5) and according to the Galerkin's condition [6] equation (5) will be:

$$\sum_{n=1}^{I} \int_A \nabla^2 \phi_n \, dA = 0.0 \qquad (7)$$

in which A is the flow domain.

Integrating equation (7) by parts and according to Green-Gauss theorem [3,5,6] equation (7) becomes:

$$\sum_{n=1}^{I} \left\{ \int_{A^e} \left[\frac{\partial N_m}{\partial x}(\frac{\partial N_n}{\partial x} + \frac{\partial N_n}{\partial y}) + \frac{\partial N_m}{\partial y}(\frac{\partial N_n}{\partial x} + \frac{\partial N_n}{\partial y}) \right] dx\,dy \right\} \phi_n - \int_s N_m q_n \, ds = 0.0, \quad (m = 1, 2, ..., I) \qquad (8)$$

in which A^e is the element domain, S is the boundary for the flow domain and q_n denotes the normal flux across the boundary of the element. Equation (8) can be also written as follows:

$$\sum K^e_{nm} \phi^e_n = F^e_n \qquad (9)$$

where

$$k^e_{nm} = \int_{A^e} \left[\frac{\partial N_m}{\partial x}\left(\frac{\partial N_n}{\partial x} + \frac{\partial N_n}{\partial y}\right) + \frac{\partial N_m}{\partial y}\left(\frac{\partial N_n}{\partial x} + \frac{\partial N_n}{\partial y}\right) \right] dx\,dy \qquad (10)$$

and

$$F^e_n = \int_s N_m \, q_n \, ds \qquad (11)$$

For the present case [K] is symmetric ($k^e_{nm} = k^e_{mn}$). Equation (9) represents the finite element model of equation (2). The procedure to obtain equation (9) is repeated for all elements in the domain. Then, all equations are combined into a set of simultaneous equations:

$$[K]\,\{\phi\} = \{F\} \qquad (12)$$

where [K] is the global stiffness matrix, $\{\phi\}$ is the global vector of unknown potential head to be determine and {F} is the global nodal force vector. The final solution can be obtained from equation (12) after applying the boundary conditions. The values of ψ can also be obtained by the same procedure for equation (3).

BOUNDARY CONDITIONS

Referring to the geometry of the model in Figure (1), the associated boundary conditions for the problem are:

1. Along the entrance face AE; $\phi/k = H_1$ and $V_x = 0.0$ ($\partial\phi/\partial x = 0.0$).
2. Along the exit face MNB; $\phi_2/K = H_2$; and $(\partial\phi/\partial t) = (\partial\psi/\partial n) = 0.0$ where t and n stand for tangential and normal directions of the boundary.
3. Along the base of the floor EFGOPJM, $\psi = 0.0$.
4. Along the impervious boundary CD, $V_y = 0.0$ ($\partial\phi/\partial y = 0.0$)

MODEL TESTING AND VERIFICATION

The finite element model was arranged for the case of simple floor founded on a finite depth of permeable strata. The uplift pressures along the floor were calculated and compared with the results of the exact solution which was done by pavlovsky [8]. Figure (2) shows a comparison between the finite element method and exact method for the case of simple floor. The figure shows a good agreement between the two methods.

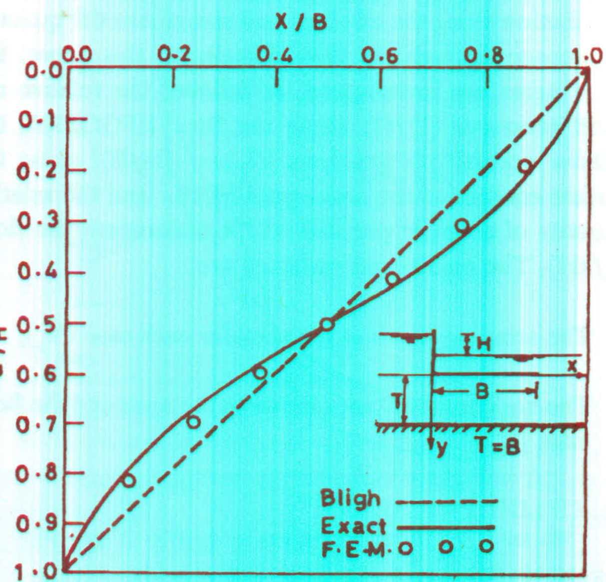

Figure 2. Comparison between the F.E.M. and the exact (mathematical) method to determine the uplift pressures for the case of simple floor.

COMPUTATIONS

In order to carry out the previous finite element computation steps, a finite element program FEM2D [6] has been utilised. The output of this program are: the potential (ϕ) and the stream function (ψ) at the nodes and the velocity components at the centroid of elements. Figure (3) shows the finite element mesh for the investigated problem using triangular elements. The vertical dimensions for the mesh at the downstream (exit face) are varied to give the required dimensions for the scour pool. The element sizes are decreased at the critical regions to give a good accuracy of the results.

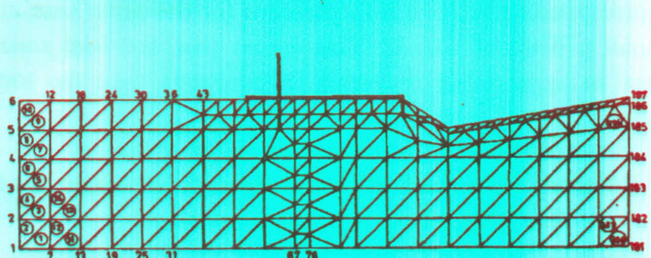

Figure 3. Finite element mesh.

RESULTS AND ANALYSIS

The most important factors for the design of the hydraulic structure floors are: the uplift pressures distribution along the floor, the exit pressure gradients distribution along the exit face and sometimes the quantity of seepage through the flow domain. In this paper, the three items are investigated, as follows: the relative net uplift pressures (U/H) along the floor EFGOPJM, the relative normal exit gradients $[(I_e)n\ /\ (H/B)]$ along the surface contour of the scour pool MNB, and the relative quantity of seepage per unit width underneath the floor (q/kH). The considered variables are:

1. The scouring depth at the apex for each case (Y_a), and

2. The horizontal distance between the apex and the floor edge (x_o).

I. UPLIFT PRESSURES

Effect Of Scouring Depth at the Apex (Y_a)

For constant values of the relative depths of cut-offs; $d_1/B = 0.20$ & $d_2/B = 0.10$, the relative length of upstream bed $L_1/B = 1.50$; the relative extension of the floor at the upstream side $b/B = 0.20$, the relative thickness of pervious layer $T/B = 1.0$, the relative length of scour pool $L/B = 1.50$, the relative effective head $H/B = 0.20$, the relative horizontal distance between the apex and the floor edge $X_a/B = 0.10$ and the relative depth of apex, Y_a/d_2, is varied from 0.0 to 4.0. As shown in Figure (4), the relative net uplift pressures (U/H)along the floor EFGOPJM are affected by such a variation. The figure indicates that the net uplift pressures are decreased when the depth of local scour pool is increased. The reduction in pressures along the floor is nonuniform. Increasing the relative scouring depth (Y_a/d_2) from 0.0 to 4.0 causes a decrease of pressure at point 0 about 0.08 H, while the corresponding reduction in pressure for the same case at point p is about 0.15 H. This means that, the heel zone has an appreciable reduction in pressures than the toe zone.

Effect of Horizontal Distance Between the Apex and Floor End (X_a).

To study the effect of horizontal distance of apex (lowest point of the scour pool) from the floor, the relative dimensions of the structure are kept constant as before. The relative maximum depth of scour (Y_a/d_2) is considered 2.0, while the relative horizontal distance (X_a/B) is gradually increased from 0.10 to 2.0. The case of $X_a/B = \infty$, which means the case without scour is also considered.

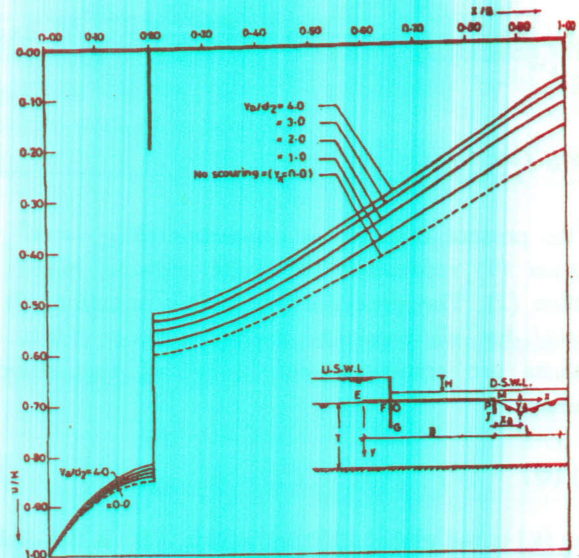

Figure 4. Effect of scouring depth (Y_a) on the relative uplift pressures along the floor, ($X_a/B=0.10$).

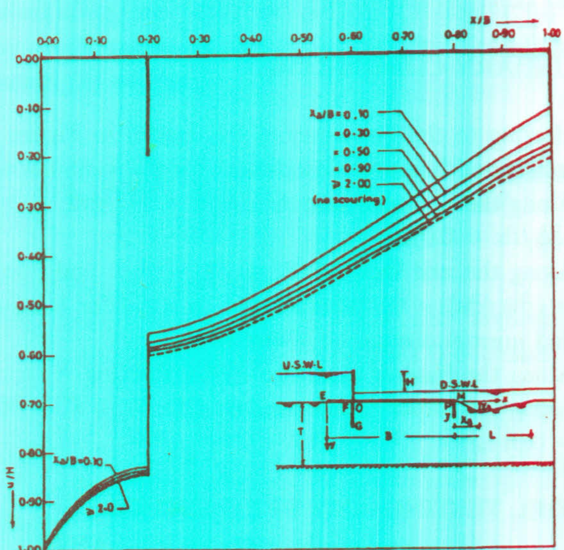

Figure 5. Effect of horizontal distance of apex from the floor end (X_a) on the relative uplift pressures along the floor, ($Y_a/d_2=2.0$).

Figure (5) shows that the net uplift pressures along the floor are affected by the variation of X_a/B. The figure indicates that, the net uplift pressures along the floor are increased when the relative distance X_a/B is increased. This means that these pressures are always less than the net uplift pressures for the case of fixed bed (no scour) i.e. the uplift pressures are decreased when the scouring apex is approached to the floor. The rate of reduction in pressures is increased as the scour pool becomes closer to the floor end.

II . EXIT GRADIENTS

In addition to the uplift pressures under the structure floor, it is also important to study the effect of the scour pool dimensions on the hydraulic gradients along the scoured downstream bed. These gradients are always at the normal direction to the bed surface. They should not exceed than the critical values of gradients which depend on the soil characteristics. The suitable factor of safety should be also taken into consideration. In this paper, the gradients are determined in the dimensionless form $(I_e)_n/(H/B)$.

Effect of Scouring Depth at the Apex (Y_a)

To study the effect of scouring depth at the apex, the relative dimensions of the structure floor and the scouring pool length are kept constant as before. The relative distance between the apex and the floor end, X_a/L is considered 0.33, while the relative scouring depth at the apex is varied from 0.0 (case without scour) to 4.0.

Inspection of Figure (6) reveals that the exit velocity gradients are affected by the variation of the scouring depth. When the relative scouring depth is increased, the exit gradients are increased along whole the downward slope and part of the upward slope. The maximum increase in gradients occurs at the apex point, N and then the differences decrease to a some point along the upward slope (turning point). This is because, the apex point represents the end of a stream line which has a minimum length. After the turning point the exit gradients are decreased when the scouring depth at apex is increased. This means that the values of exit gradients at the end zone (from the turning point to the end of scour) for all scouring cases are always less than the corresponding gradients for the case without scour($Y_a = 0.0$). In other words this area does not needs any new protection.

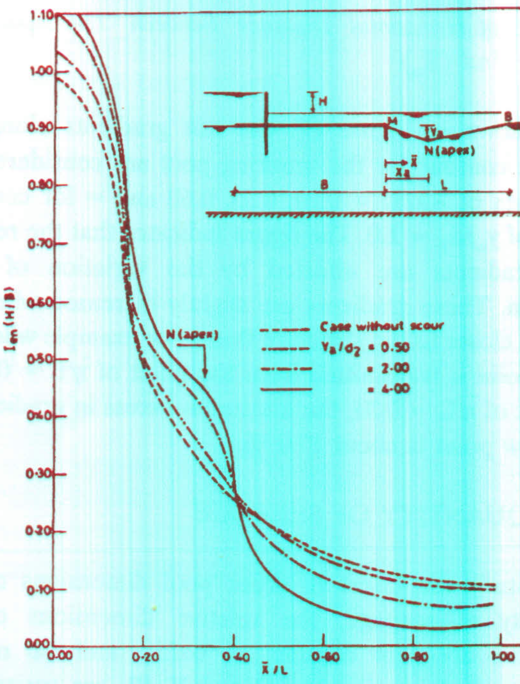

Figure 6. Effect of scouring depth (Y_a) on the relative normal exit gradients along the scour pool surface, ($X_a/L = 0.33$).

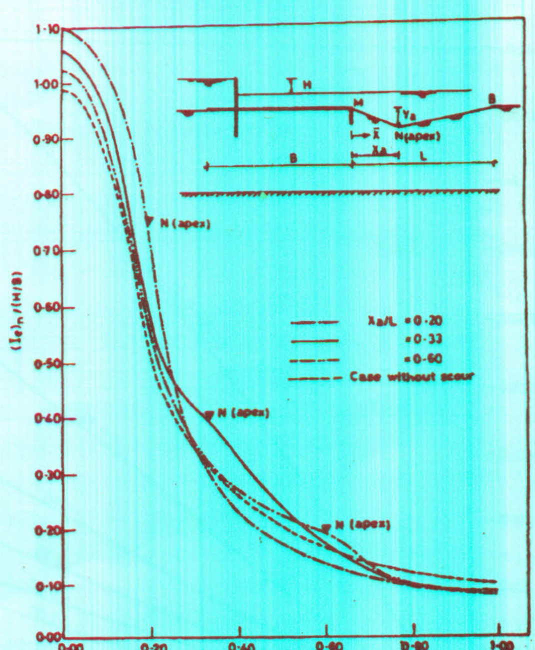

Figure 7. Effect of horizontal distance of apex from the floor end (X_a) on the relative normal exit gradients along the scour pool sueface, ($Y_a/d_2 = 1.0$).

Effect of Horizontal Distance Between The Apex and Floor End (X_a)

As shown in Figure (7) the exit gradients along the surface contour for the scouring pool are considered for the cases of Xa/L = 0.20; 0.33, 0.60 and ∞ for constant value of y_a/d_2 = 1.0. The figure indicates that the relative exit gradients are affected by the variation of apex position. These gradients are slightly increased when the apex is closer to the structure floor. For example when the apex point is transmitted from the point of $\bar{x}/L$ = 0.60 to a point of $\bar{x}/L$ = 0.33, the maximum excess in gradients at the new point is about 17.0 %.

III. QUANTITY OF SEEPAGE

To study the effect of scour pool dimensions on the quantity of seepage, the relative dimensions of the structure are kept constant as before and the relative scour pool dimension $Y_a/d2$ and X_a/B, are varied. The quantity of seepage through the flow domain per unit width was considered and studied on the relative form, q/kH.

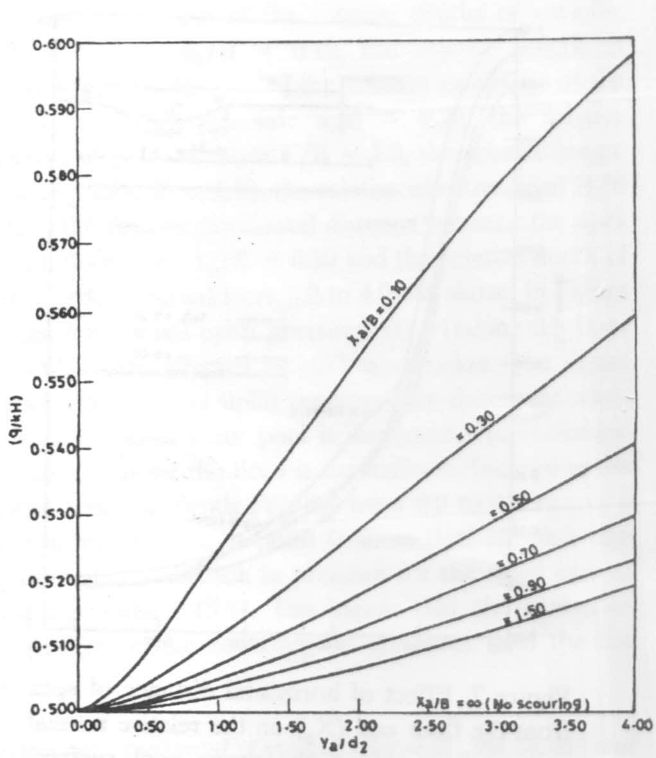

Figure 8. Effect of local scour pool dimension on quantity of seepage beneath structure.

Figure (8) shows that the relative quantity of s underneath the structure is slightly affected by the in the relative dimensions of the scour pool. The indicates that the increase in the value of X_a/B c decrease in the value of q/kH, while the increase value $Y_a/d2$ causes an increase in the value of The figure also indicates that the value of relative c of seepage for any scour case is always greater tl value for the case with fixed bed. This means tl quantity of seepage beneath any hydraulic struc increased when a local scour is created al downstream bed.

CONCLUSIONS

A finite element model was designed for the pro local scour created downstream a hydraulic structu floor of the structure has two cut-offs. The boundary conditions were assumed in the mod finite element model was tested and verified for t of simple flat floor without downstream scourir scour pool shape was assumed as a triangle. Th parameters for the problem are: the maximum s depth (at apex) and; the horizontal distance betw apex and the floor edge. The finite element mo used for computing the seepage characteristics du variation of the parameters. The results have been in the form of curves. Effects of each of these par on the net uplift pressures distribution along the fl exit gradients and on the quantity of seepage und the floor are analysed. The values of net uplift p along the floor are decreased when the depth scour is increased. These pressures are also de when the scour apex becomes closer to the floor the local scour is created, the exit gradients are in along the downward slope and part of the upwar They are always less than their initial values (scour) along the remained part of the upward slop quantity of seepage is slightly increased than it value. Hence, the change of bed levels downstre hydraulic structure causes an appreciable variation seepage characteristics.

ACKNOWLEDGMENT

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carried out by using the FEM2D program at the University of Mississippi computer center, U.S.A. (Oct. 1989 to March 1990).

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