# ROLLER CHAIN MECHANISMS PART II- INTERMITTENT MOTION MECHANISMS 

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## ABSTRACT

This work points out the design and constraint equations by which the combined flexible-planar mechanism can be an additional new intermittent motion mechanism. This mechanism can be used to realize finite intermittent motions. Moreover, such mechanism can be designed for a single or multiple dwells. Numerical examples are given to illustrate the simplicity of carrying out the design procedure and the potentiality of such contribution in mechanisms field.

## NOMENCLATURE

A Coefficient or vertical component of the position of joint B,
C Coefficient or chain length,
D Horizontal component of the position of joint B,
f Angular acceleration,
K Coefficient,
L Coupler length or distance between the pivot Q and center position O (Figures (1) and (2)),
m and n Numbers of dwells and sprockets respectively,
r Radius of circular path (Figure (1)) or of sprocket (Figure (2)) or distance between Q and joint B,
R Crank radius,
$0 \quad$ Center position of the circular path (Figure (1)) or of the sprocket (axle) (Figure (2)),
Q Position of the crank pivot which locates the original position of $\mathrm{X}-\mathrm{Y}$ axes,
$S \quad$ Input path (Figure (1)) or displacement of joint 3 (Figure (2)),
X and Y Horizontal and vertical components of O respectively,
$\beta \quad$ Contact angle between the chain and sprocket measured from $y$-axis of the sprocket axle in direction of the motion, Total angular displacement,
Angle between the axles of the two adjacent sprockets i and i1,
$\Psi \quad$ Inclination angle of the straight portion of the chain,
$\omega \quad$ Angular speed,
$\Phi \quad$ Angular displacement of the crank R or angle between the tangent point I and $x$-axis,
$\theta$ Angular displacement,
$v$ Dwell position,
$\Delta$ Interval,

## Subscripts

b Joint B,
c Circular portion,
d Driving,
e Ending position,
i Number of sprocket ( $\mathrm{i}=1,2, \ldots, \mathrm{n}$ ),
1 Coupler,
p For angular position $\phi$,
o Distance Z or X and Y components of position of joint B,
s Starting position or straight portion,
t Chain between the two adjacent sprockets,
II Second case,

## Superscripts

First derivatives with respect to time,
" Second derivatives with respect to time.

## INTRODUCTION

Intermittent motions can be easily realized by dwell mechanisms. These mechanisms are, generally, important for their many uses in machine tools , packing, indexing and textile machines. Usually, there are two groups of dwell mechanisms. The first group is used to accomplish a perfect or finite dwell, these finite-dwell mechanisms are the standard external or internal Geneva, Ratchet and intermittent gearing mechanisms [1-6]. The second group is formed by the combination of two or more simple
mechanisms and give imperfect or momentary dwells, such as six-bar planar mechanisms [7-10]. The fundamental difference between the two groups is in the generated motion within the duration of the dwell as will be explained later. Furthermore, the first group mechanisms have a locking device. On the other hand, in the second group, the locking is achieved kinematically i.e. there is no locking device to have dwell. The kinematic and dynamic analysis of the finite dwell mechanisms are presented in [1-6]. A derivation of the displacement equation and dwell characteristic of the spherical geared five-link mechanism have been developed in [5]. Also,a unified analysis formula and optimization procedure on design of external and internal parallel indexing cam mechanisms have been established in [6]. The design equation for single or multiple momentary dwells have been formulated in [7-10]. This survey papers showed that there are few number of dwell mechanisms which can be designed for multiple dwells and that the complexity involved in the analysis of these mechanisms often makes it difficult to obtain either an analytical or a simple numerical solution. The first part of this work [11] presented the general concept of a suggested combined mechanism (rigid crank-coupler links connected in series with roller chain sprocket system). That part showed that such combined mechanism can be designed to achieve different motions including intermittent motion. Consequently, the main purpose of this work is to carry out the design and constraint equations for roller-chain dwell mechmisms, with one or multiple dwells, as a first time.

## 2- GENERAL CONCEPT OF DWELL

Usually, dwell occurs due to the cancellation of the relative motion of one link of the mechanism with respect to the other. The following sections introduce the mathematical expressions for the criteria and characteristics of the dwell.

### 2.1 Dwell critcria

In Figure (1) , R and L are rigid kinematic links connected together by -joint A. L pulls R which rotates by $\phi$ about the pivot Q , while L -is rotated by $\theta \mathrm{l}$ about joint A due to the given path S . By the previous concept of the dwell, the physical meaning of the dwell in the motion of R states that "For fixing link R ( R remains stationary) i.e. $\phi=$ const., when joint A remains stationary,joint B moves on a circular path of radius $L$ with center at joint $A$."


Figure 1. Dwell Representation.
Therefore, the input path S is assumed to be circular arc of radius $r_{s}$ with fixed center $O_{s} . L_{s}$ is the distance between the pivot Q and $\mathrm{O}_{\mathrm{s}}$ and inclined by an angle $\alpha_{\mathrm{s}}$ with the X -axis as shown.
The solid lines $\mathrm{QA}(\mathrm{R})$ and $\mathrm{AB}(\mathrm{L})$ represent the starting position of the circular path $S$ at which $r_{s}$ has an angular position $\theta_{s}$, while the dashed lines $Q A_{1}(R)$, and $A_{1} B_{1}(L)$ represent the ending position of the circular path S at which $\phi$ becomes $\phi_{1}$ and both $\theta_{\mathrm{i}}$ and $\theta_{\mathrm{s}}$ increase to $\theta_{\mathrm{n} 1}$ and $\phi_{\mathrm{s}}$ respectively. Hence, we have two dyads, one is represented by $\mathrm{R}, \mathrm{L}$ and S and the other is constructed by $L, r_{s}$ and $S$. These dyads reveal the following mathematical expressions.

## -At starting position

$$
\begin{equation*}
R e^{j \Phi}+L e^{j \theta t}=L_{s} e^{j \alpha_{s}}+r_{s} e^{j \theta s} \tag{1}
\end{equation*}
$$

-At ending position

$$
\begin{equation*}
R e^{j \not \phi_{1}}+L e^{j \theta}=L_{s} e^{j \alpha_{s}}+r_{s} e^{j \theta s} \tag{2}
\end{equation*}
$$

Where the right hand parts describe the finite circular arc path which exists if and only if the following necessary conditions are satified;

$$
\left.\begin{array}{l}
\mathrm{L}^{2}=\left(\mathrm{X}_{\mathrm{s}}-\mathrm{X}_{\mathrm{A}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{B}}-\mathrm{Y}_{\mathrm{A}}\right)^{2}=\left(\mathrm{X}_{\mathrm{B} 1}-\mathrm{X}_{\mathrm{A} 1}\right)^{2}+\left(\mathrm{Y}_{\mathrm{B} 1}-\mathrm{Y}_{\mathrm{A} 1}\right)^{2} \\
\text { and }  \tag{3}\\
\mathrm{r}_{\mathrm{s}}^{2}
\end{array}=\left(\mathrm{X}_{\mathrm{s}}-\mathrm{X}_{\mathrm{OS}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{B}}-\mathrm{Y}_{\mathrm{OS}}\right)^{2}=\left(\mathrm{X}_{\mathrm{B} 1}-\mathrm{X}_{\mathrm{OS}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{B} 1}-\mathrm{Y}_{\mathrm{OS}}\right)^{2}\right) ~ \$
$$

By inspection, we can find out the criteria of dwell as; For link R remains stationary (fixed) i.e. $\phi=\boldsymbol{\phi}_{1}=$ Const., if and only if the following conditions are satisfied.
$\mathrm{R}=\mathrm{L}_{\mathrm{s}}, \mathrm{L}=\mathrm{r}_{\mathrm{s}}$ and $\phi_{1}=\phi=\alpha_{\mathrm{s}}$
Therefore,

$$
\theta_{t}=\theta_{s}, \theta_{\imath 1}=\phi_{s}
$$

If the conditions of Equation. (4) are satisfied, the positions of joint A and $\mathrm{O}_{\mathrm{s}}$ coincide, also the conditions of Eq. (3) are verified. This means that joint $A$ is stationary. In the case of approximate verification of conditions of Eq. (4) , the finite dwell could not be existed but a momentary dwell may be occurred where small variation in $\phi$ is existed. During the finite dwell period, the angular velocity $\omega$ and acceleration $f$ of the crank $R$ should be zeros.

### 2.2. Dwell Characteristics

These characteristics are the positions, period and frequency of the dwell.
i- The dwell duration $\Delta \theta$ can be easily estimated by;

$$
\begin{equation*}
\Delta \theta=v_{e}-v_{s} \tag{5}
\end{equation*}
$$

Where;
$\mathbf{v}_{\mathrm{e}}$ Ending angular position of the dwell at which one or more conditions of equation (4) are violated.
$v_{s}$ Starting angular position of the dwell at which the conditions of equation (4) are satisfied.

Referring to Figure (1), the following relation can be pointed out;
$1-v_{\mathrm{s}}$ and $v_{\mathrm{s}}$ are existed and can be estimated by;

$$
\begin{equation*}
v_{\mathrm{s}}=\theta_{\text {in }} \tag{6}
\end{equation*}
$$

Where $\theta_{\text {in }}$ at which the following equalities are occurred;

$$
\begin{align*}
& \phi=\theta_{t}=\alpha_{S} \\
& \text { and }  \tag{7}\\
& r_{b}=R+L=L_{S}+r_{S} \\
& v_{S}=\theta_{i n} \tag{8}
\end{align*}
$$

Where $\theta_{\text {in }}$ at which the following equalities are satisfied;

$$
\begin{equation*}
\theta_{\mathrm{i}}=\alpha_{\mathrm{s}}+\phi_{\mathrm{s}} \tag{9}
\end{equation*}
$$

and

Where $\phi, \theta_{\mathrm{a}}$ and $\mathrm{r}_{\mathrm{b}}$ : Parameters are function of the input angular displacement $\theta_{\text {in }}$

2-The dwell period $\Delta \theta$ can be obtained by;

$$
\begin{equation*}
\Delta \theta=\phi_{\mathrm{s}}-\alpha_{\mathrm{s}} \tag{10}
\end{equation*}
$$

ii-The dwell frequency $M$ which is the number of dwells per operating cycle, may be obtained by;

$$
\begin{equation*}
\mathrm{M}=\sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{~N}_{\mathrm{ci}} \mathrm{~m}=1,2, \ldots, \mathrm{n} \tag{11}
\end{equation*}
$$

Where;
$\mathrm{N}_{\mathrm{ci}}$ is the number of circular arc paths which satisfy conditions of equation (4)

## 3. DESIGN APPROACH

A general approach for the design of the intermittent motion mechanisms is presented. The approach involves the following steps;

### 3.1 Suggested Mechanism

The suggested mechanism consistes of a roller chain with two or more sprockets placed in a general orientation. One of the sprockets, called driving sprocket,provides the input angular displcement $\theta_{1}$ in the counterclockwise direction. The motion of the roller chain is transmitted to a crank $\mathbf{R}$ through a coupler $L$. This motion has either a straight or circular path $S$ as stated in [11]. In order to obtain the relation between $\phi$ and $\theta_{1}$ within an operating cycle, consider a portion of the system shown in Figure (2).


Figure 2. Suggested system, - first case, --- second case.
adjacent sprockets i and $\mathrm{i}+1$, the chain is connected with the crank R through the coupler L . The sprocket center (axle) $\mathrm{O}_{\mathrm{i}}$ is located at $\mathrm{X}_{\mathrm{i}}$ and $\mathrm{Y}_{\mathrm{i}}$ and displaced by $\mathrm{L}_{\mathrm{i}}$ from the crank pivot Q . $\mathrm{L}_{\mathrm{i}}$ is inclined by an angle $\alpha_{\mathrm{i}}$ with X -axis. The $\phi-\theta_{1}$ relation for this part of the system is carried out as follows;

### 3.2 Geometric Analysis

The geometric analysis of the suggested mechanism, Figure (2) gives;

$$
\begin{align*}
& \mathrm{L}_{\mathrm{i}}=\sqrt{\mathrm{X}_{\mathrm{i}}^{2}+\mathrm{Y}_{\mathrm{i}}^{2}} \text { and } \alpha_{\mathrm{i}}=\tan ^{-1}\left(\mathrm{Y}_{\mathrm{i}} / \mathrm{X}_{\mathrm{i}}\right)  \tag{12}\\
& \mathrm{Z}_{\mathrm{i}}=\sqrt{\Delta \mathrm{X}^{2}+\Delta \mathrm{Y}^{2}}, \mathrm{C}_{\mathrm{i}}=\sqrt{\mathrm{Z}_{\mathrm{i}}^{2}-\Delta \mathrm{r}^{2}}  \tag{13}\\
& \zeta_{\mathrm{i}}=\tan ^{-1}\left(\mathrm{Y}_{\mathrm{s}} / \mathrm{X}_{\mathrm{s}}\right), \theta_{\mathrm{oi}}=\tan ^{-1}(\Delta \mathrm{Y} / \Delta \mathrm{X})  \tag{14}\\
& \alpha_{\mathrm{ti}}=\tan ^{-1}\left(\Delta \mathrm{r} / \mathrm{C}_{\mathrm{ti}}\right), \mathrm{i}=1,2, \ldots \ldots . \mathrm{n} \tag{15}
\end{align*}
$$

where $\mathrm{X}_{\mathrm{s}}=\mathrm{L}_{\mathrm{i}}{ }^{2}+\mathrm{L}_{\mathrm{i}+1}{ }^{2}-\mathrm{Z}_{\mathrm{i}}^{2}, \mathrm{Y}_{\mathrm{s}}=\sqrt{\left(2 \mathrm{~L}_{\mathrm{i}} \mathrm{L}_{\mathrm{i}+1}\right)^{2}-\mathrm{X}_{\mathrm{s}}{ }^{2}}$

$$
\Delta \mathrm{X}=\mathrm{X}_{\mathrm{i}}-\mathrm{X}_{\mathrm{i}+1}, \Delta \mathrm{Y}=\mathrm{Y}_{\mathrm{i}}-\mathrm{Y}_{\mathrm{i}+1}, \Delta \mathrm{r}=\mathrm{r}_{\mathrm{i}}-\mathrm{r}_{\mathrm{i}+1}
$$

n :Total number of the sprockets.
For achieving the actual position of the two adjacent sprockets, the following equations should be considered;

$$
\left.\begin{array}{rl}
\beta_{\mathrm{i}} & =\theta_{\mathrm{oi}}+\alpha_{\mathrm{ti}}  \tag{16}\\
\Phi_{\mathrm{i}} & =\mathrm{C}_{\mathrm{i}}+\beta_{\mathrm{i}} \\
\text { and } & \\
\Psi_{\mathrm{i}} & =\mathrm{A}_{\mathrm{i}}+\beta_{\mathrm{i}}
\end{array}\right\}
$$

Where $\mathrm{C}_{\mathrm{i}}$ and $\mathrm{A}_{\mathrm{i}}$ are values depending on the quadrant at which the position of the two sprockets (i and $\mathrm{i}+1$ ) are located. Hence, $\mathrm{C}_{\mathrm{i}}$ and $\mathrm{A}_{\mathrm{i}}$ are estimated according to the following restrictions;
$\left.\begin{array}{lll}C_{i}=0.5 \pi & \text { and } A_{i}=\pi & \text { For } 0 \leq \beta_{i} \leq \pi \\ C_{i}=0.5 \pi & \text { and } A_{i}=-\pi & \text { For } \pi \leq \beta_{i} \leq 1.5 \pi \\ C_{i}=-1.5 \pi & \text { and } A_{i}=-\pi & \text { For } 1.5 \pi \leq \beta_{i} \leq 2 \pi\end{array}\right\}(17)$
These three conditions verify any possible positions of two adjacent sprockets. The total angular displacements of
the driving sprocket $1, \tau_{\text {si }}$ and $\tau_{\text {ci }}$ which are corresponding to the straight and circular portions ,respectively, are estimated by;

$$
\left.\begin{array}{rl}
\tau_{\mathrm{si}} & =\left(\mathrm{C}_{\mathrm{ti}} / \mathrm{r}_{1}\right) \cdot 180 / \pi  \tag{18}\\
\text { and } \\
\tau_{\mathrm{ci}} & =\left(\Phi_{\mathrm{i}+1}-\Phi_{\mathrm{i}}\right) \cdot \mathrm{r}_{\mathrm{i}+1} / \mathrm{r}_{1}
\end{array}\right\}
$$

Where the condition of $i=n$ reveals $\tau_{c n}$ as;

$$
\begin{equation*}
\tau_{\mathrm{cn}}=\mathrm{K}_{\mathrm{t}}-\left(\Phi_{\mathrm{i}}-\Phi_{1}\right) \tag{19}
\end{equation*}
$$

where;

$$
K_{\mathrm{t}}= \begin{cases}2 \pi & \text { for } \phi_{\mathrm{n}}>\phi_{1} \\ 0 & \text { for } \phi_{\mathrm{n}}<\phi_{1}\end{cases}
$$

### 3.2 Kinematic Analysis

These analysis are carried out as;

### 3.2.1 Position Analysis

As stated in [11] and referring to Figure (2), the governing equations of the crank $R$ and coupler $L$ are given by;

$$
\left.\begin{array}{l}
\mathrm{R} \sin \Phi+\mathrm{L} \sin \theta_{\mathrm{i}}=\mathrm{A}_{\mathrm{o}}  \tag{20}\\
\text { and } \\
\mathrm{R} \cos \Phi+\mathrm{L} \cos \theta_{\mathrm{i}}=\mathrm{D}_{\mathrm{o}}
\end{array}\right\}
$$

Where

$$
\left.\begin{array}{l}
\mathrm{A}_{\mathrm{o}}=\mathrm{L}_{\mathrm{i}} \sin \alpha_{\mathrm{i}}+\mathrm{r}_{\mathrm{i}} \sin \Phi_{\mathrm{i}}+\mathrm{S} \sin \psi_{\mathrm{i}}  \tag{21}\\
\mathrm{D}_{\mathrm{o}}=\mathrm{L}_{\mathrm{i}} \cos \alpha_{\mathrm{i}}+\mathrm{r}_{\mathrm{i}} \cos \Phi_{\mathrm{i}}+\mathrm{S} \cos \psi_{\mathrm{i}}
\end{array}\right\}
$$

For $0 \leq \theta_{\mathrm{i}} \leq \tau_{\mathrm{si}}$ which corresponds to the straight portion $\mathrm{C}_{\mathrm{t}}$.
And

$$
\left.\begin{array}{l}
A_{o}=L_{i+1} \sin \alpha_{i+1}+r_{i+1} \sin \varepsilon_{i+1} \\
D_{o}=L_{i+1} \cos \alpha_{i+1}+r_{i+1} \cos \varepsilon_{i+1}  \tag{22}\\
\varepsilon_{i+1}=\boldsymbol{\Phi}_{i}+\theta_{i+1}, \theta_{i+1}=S / r_{i+1}, S=r_{1} \theta_{1}
\end{array}\right\}
$$

For $0 \leq \theta_{1} \leq \tau_{\mathrm{ci}}$ which corresponds to the circular portion of sprocket $\mathrm{i}\left(\tau_{\mathrm{ci}}\right)$.
After performing some mathematical manipulations, equations (20) can be transformed to one equation as;

$$
\begin{equation*}
\mathrm{A}_{\mathrm{o}} \sin \Phi+\mathrm{D}_{\mathrm{o}} \cos \Phi=\mathrm{Y}_{\mathrm{p}} \tag{23}
\end{equation*}
$$

Solving for $\phi$ [12], we get

$$
\begin{equation*}
\Phi=\tan ^{-1}\left(\mathrm{Y}_{\mathrm{p}} / \mathrm{X}_{\mathrm{p}}\right)-\lambda \tag{24}
\end{equation*}
$$

Where

$$
\begin{align*}
& Y_{p}=\left(r_{b}^{2}+R^{2}-L^{2}\right) / 2 R \\
& X_{p}=\sqrt{r_{b}^{2}-y_{p}^{2}}, r_{b}=\sqrt{A_{o}^{2}+D_{o}^{2}}  \tag{25}\\
& \lambda=\tan ^{-1}\left(D_{o} / A_{o}\right)
\end{align*}
$$

Equation (24) yields two values for $\phi$ since $\mathrm{X}_{\mathrm{p}}$ has two values as shown in Figure (2) by $\phi$ and $\phi_{\text {II }}$. In the first case joint B leads or pulls the crank R, named leading case. While, in the second case joint B lags or pushes the crank R, termed as lagging case. The analysis of the second case, $\phi_{\mathrm{II}}$, will be given later on. From the geometry of Figure (2) the coupler angular position $\theta_{\mathbf{t}}$ is expressed by;

$$
\begin{equation*}
\theta_{\mathrm{t}}=\Phi+\beta_{r}+\beta_{\mathrm{t}} \tag{26}
\end{equation*}
$$

Where $\beta_{t}$ and $\beta_{r}$ are the angles between the coupler $L$, the crank $R$ and the direction of $r_{b}$ respectively, therefore these angles are determined by;

$$
\begin{equation*}
\beta_{\mathrm{r}}=\tan ^{-1}\left(\mathrm{X}_{\mathrm{p}} / Y_{\mathrm{p}}\right), \beta_{\mathrm{t}}=\tan ^{-1}\left(\mathrm{X}_{\mathrm{t}} / Y_{\mathrm{v}}\right) \tag{27}
\end{equation*}
$$

Where

$$
Y_{t}=\left(r_{b}^{2}+L^{2}-R^{2}\right) / 2 L, X_{t}=\sqrt{r_{b}^{2}-Y_{t}^{2}}
$$

The design constrains which control the real values of $\phi$ and $\theta_{\mathrm{t}}$ and give positive values to $X_{p}$, equation (25), and $X_{v}$, equation (27), should be considered.
The $\phi-\theta_{1}$ relation, during one cycle, can be obtained by the previous analysis in addition to the response of other successive two adjacent sprockets, until it reaches to the last two adjacent sprockets n and $\mathrm{n}+1$. Where the sprocket $\mathrm{n}+1$ is the driving sprocket 1 .

### 3.2.2 Velocity Analysis

Differentiating Equations (20) with respect to time gives two equations which could be solved for crank and coupler speeds $\omega$ and $\omega_{\mathrm{t}}$ respectively as;

$$
\begin{equation*}
\omega=\left(\mathrm{A}_{1} \mathrm{a}_{22}-\mathrm{D}_{1} \mathrm{a}_{12}\right) / \mathrm{D}_{\mathrm{t}}, \omega_{1}=\left(\mathrm{D}_{1} \mathrm{a}_{11}-\mathrm{A}_{1} \mathrm{a}_{21}\right) / \mathrm{D}_{\mathrm{t}} \tag{28}
\end{equation*}
$$

Where;
$\mathrm{a}_{11}=\mathrm{R} \cos \Phi, \mathrm{a}_{12}=\mathrm{L} \cos \theta_{\mathrm{r}}$
$\mathrm{a}_{21}=\mathrm{R} \sin \Phi, \mathrm{D}_{1}=\mathrm{L} \sin \theta_{\mathrm{t}}$

$$
A_{1}=A_{o}^{\prime}, D_{1}=-D_{o}^{\prime}, D_{t}=a_{11} a_{22}-a_{21} a_{12}
$$

( )' denotes the derivative with respect to time.

### 3.2.3 Acceleration Analysis

The differentiation of Equations (28) gives the following angular accelerations of the crank $f$ and of the coupler $f_{i}$ as;

$$
\begin{equation*}
\mathrm{f}=\left(\mathrm{A}_{2} \mathrm{a}_{22}-\mathrm{D}_{2} \mathrm{a}_{12}\right) / \mathrm{D}, \mathrm{f}_{\mathrm{t}}=\left(\mathrm{D}_{2} \mathrm{a}_{11}-\mathrm{A}_{2} \mathrm{a}_{21}\right) / \mathrm{D}_{\mathrm{t}} \tag{29}
\end{equation*}
$$

where
$A_{2}=\left(A_{o}^{\prime \prime}+a_{21} \omega^{2}+a_{22} \omega_{1}^{2}\right), D_{2}=\left(D_{o}^{\prime \prime}+a_{11} \omega^{2}+a_{12} \omega^{2}{ }_{\imath}\right)$
( )" denotes the second derivative with respect to time.

### 3.3 Analysis of the Lagging Case

Referring to Figure (2), the second case is represented by dashed lines. The angular position of the crank $\phi_{\text {II }}$ and the coupler $\theta_{\text {ıII }}$ can be derived as;

$$
\begin{equation*}
\Phi_{\mathrm{II}}=\Phi+2 \beta_{\mathrm{r}} \tag{30}
\end{equation*}
$$

and

$$
\theta_{\mathrm{III}}=\mathrm{K}_{\mathrm{p}}+\Phi+\beta_{\mathrm{r}}-\beta_{\mathrm{t}}
$$

where;

$$
K_{p}= \begin{cases}2 \pi & \text { for } \beta_{\mathrm{t}}>\pi / 2 \\ 0 & \text { for } \beta_{\mathrm{t}}<\pi / 2\end{cases}
$$

Replacing $\phi_{\text {II }}$ and $\theta_{\text {III }}$ instead of $\Phi$ and $\theta_{1}$ in equations (28) and(29) the velocity and acceleration of both crank R and coupler $L$, in this case, can be estimated respectively. The total driving angular displacement $\boldsymbol{\theta}_{\mathrm{d}}$, which corresponds to $\theta_{\text {in }}$ of Eqs. (6) and (8), is computed by;

$$
\begin{equation*}
\theta d=\sum_{j=1}^{2 n} \theta_{1 j}=\left(S_{t} / r_{1}\right) 180 / \pi \tag{31}
\end{equation*}
$$

Where;
$\theta_{1 \mathrm{j}}$ Total angular position of the driving sprocket within a straight or circular portion i.
$S_{t}$ Total input displacement of joint $B$ within one operating cycle and is given by;

$$
\begin{equation*}
S_{t}=\sum_{i=1}^{n} C_{t i}+\tau_{c i} r_{i+1} \tag{32}
\end{equation*}
$$

It should be noted that $\mathrm{r}_{\mathrm{n}+1}=\mathrm{r}_{1}$ and that $\mathrm{c}_{\mathrm{ti}}$ and $\tau_{\mathrm{ci}}$ are given previously.

### 3.4. Design Constraints

The necessary constraints for insuring continuous operation are stated in the following sections.

### 3.4. General Constraints

To avoid locking of the system and to eliminate jamming during operation,the following constraints and limitations must be taken into account;
I- For working domain

$$
\begin{align*}
& r_{i t} \leq r_{i} \leq r_{i u}, L_{i}>r_{i} \\
& r_{i}+r_{i+1} \leq Z_{i} \leq L_{i}+L_{i+1}  \tag{33}\\
& R+L \leq\left(L_{i}+r_{i}\right)_{\max }
\end{align*}
$$

$\mathrm{R}-\mathrm{L}<\mathrm{r}_{\mathrm{b} \min }$ and $\sum_{\mathrm{i}=1}^{\mathrm{n}} \zeta_{\mathrm{i}}=2 \pi$
Where subscripts $\mathfrak{i}$ and $u$ denote lower and upper limits of $\mathrm{r}_{\mathrm{i}}$.

II- For controlling the values of $\phi$ and $\theta_{\mathrm{t}}$ [Eqs.(24) and (26)]
$\mathrm{L}>0.5 \mathrm{r}_{\mathrm{b}}+(\mathrm{R}+\mathrm{L})(\mathrm{L}-\mathrm{R}) / 2 \mathrm{r}_{\mathrm{b}}$
and
$\mathrm{R}>0.5 \mathrm{r}_{\mathrm{b}}+(\mathrm{R}+\mathrm{L})(\mathrm{R}-\mathrm{L}) / 2 \mathrm{r}_{\mathrm{b}}$
Where $r_{b}$ is given by Eq. (25)

### 3.4.2 Constraints For Dwell Occurrence

The occurence of the dwell by the mechanism requires verification of Eq. (4)., this can be achieved if and only if the crank R and the coupler L are designed to be as;

$$
\begin{equation*}
\mathrm{R}=\mathrm{L}_{\mathrm{i}} \text { and } \mathrm{L}=\mathrm{r}_{\mathrm{i}} \quad \mathrm{i}=1,2, \ldots \mathrm{~m} \tag{35}
\end{equation*}
$$

Where m is the number of dwells ( $\mathrm{m} \leq \mathrm{n}$ ). The starting and ending positions of the dwell, $v_{s}$ and $v_{e}$ respectively, are determined if and only if the following equalities are achieved,

$$
\begin{align*}
& v_{s}=\theta_{d} \\
& \text { if } r_{b}=R+L=L_{i}+r_{i} \text { and } \phi=\theta_{i}=\alpha_{i}  \tag{36}\\
& \text { and } \\
& v_{e}=\theta_{d} \\
& \text { if }  \tag{37}\\
& r_{b}=\sqrt{L_{i}^{2}+r_{i}^{2}-2 L_{i} r_{i} \cos \left(\pi+\alpha_{i}-\phi_{i}\right)} \text { and } \\
& \theta_{i}=\phi_{i}+\alpha_{i}
\end{align*}
$$

Where;

$$
\begin{array}{ll}
\boldsymbol{\phi}, \theta_{\mathrm{l}} \text { and } \theta_{\mathrm{d}} \quad \begin{array}{l}
\text { Given by Eqs. (24), (26) and (31) } \\
\text { respectively. } \\
\text { Given by Eq.(25)., }
\end{array} \\
\mathrm{r}_{\mathrm{b}} & \text { 3.4.3 Rotatabilty And Transmission Angle Conditions }
\end{array}
$$

The rotatability of the crank R should be examined by the use of Grashof criterion. Also,the transmission-angles of the slider-crank mechanisms (within straight portions) and of the four-bar linkages (within circular portions) should be restricted in condition that the maximum variation of this angle from the right angle on the entire operating cycle should be minmized [13], if the optimization of such system is required.

## 4.IMPLEMENTATION

The following mechanisms data which are considered to illustrate the results of the presented analysis, are some of several examined data;

| Mech No. | i | $\mathrm{r}_{\mathrm{i}} \mathrm{cm}$ | $\mathrm{X}_{\mathrm{i}} \mathrm{cm}$ | $\mathrm{Y}_{\mathrm{i}} \mathrm{cm}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 | 3 | 5 | 0 |
| 1 | 2 | 3 | 0 | 5 |
|  | 3 | 3 | -5 | 0 |

$$
\omega_{1}=1 \mathrm{ra} \mathrm{~d}_{\mathrm{d}} / \mathrm{s}, \mathrm{R}=\mathrm{L}_{\mathrm{i}}=5, \mathrm{~L}=\mathrm{r}_{\mathrm{i}}=3 \mathrm{~cm}
$$

| Mech No. | i | $\mathrm{r}_{\mathrm{i}} \mathrm{cm}$ | $\mathrm{X}_{\mathrm{i}} \mathrm{cm}$ | $\mathrm{Y}_{\mathrm{i}} \mathrm{cm}$ |
| :---: | :---: | :---: | :---: | :---: |
| 2 | 1 | 3 | 5 | 0 |
|  | 2 | 3 | 0 | 5 |
|  | 3 | 3 | -5 | 0 |
|  | 4 | 3 | 0 | -5 |

Table 1.

| $n$ | i | $r_{i}$ | Li | $p_{i}$ | $\alpha_{i}$ | $\Delta \theta$ | m | Configuration figure \& Constraints |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 1 2 | $\begin{aligned} & L \\ & <L \end{aligned}$ | $\begin{aligned} & R \\ & R \end{aligned}$ | $\begin{aligned} & \pi / 2+\beta \\ & 1.5 \pi-\beta \end{aligned}$ | $0$ | $\begin{gathered} \pi / 2+\beta \\ \end{gathered}$ | $\begin{array}{r} 1 \\ - \end{array}$ | $\begin{aligned} & r_{1}=L, r_{2}<r_{1} \\ & L_{1}=L_{2}=R \\ & n=2, m=1 \end{aligned}$ |
| 2 | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | L | $\begin{array}{r} R \\ <R \end{array}$ | $\begin{array}{r} \pi / 2 \\ 312 \pi \end{array}$ | 0 <br> $\pi$ | $\pi 12$ | $\begin{gathered} 1 \\ \\ \hline \end{gathered}$ | $\begin{aligned} & r_{1}=r_{2}=L \\ & L_{1}=R, L_{2}<L_{1} \end{aligned}$ |
| 2 | 1 2 | $\mathrm{L}$ <br> L | $R,$ <br> R | $\begin{gathered} \pi / 2 \\ 3 / 2 \pi \end{gathered}$ | 0 <br> $\pi$ | $\pi / 2$ $\pi / 2$ | 2 | $\begin{aligned} & L_{1}=L_{2}=R \\ & m=n=2 \end{aligned}$ |
| 3 | $\begin{aligned} & 1 \\ & 2 \\ & 3 \end{aligned}$ | $\begin{aligned} & \mathrm{L} \\ & \mathrm{~L} \\ & \mathrm{~L} \end{aligned}$ | R <br> R <br> R | $\begin{aligned} & \pi 14 \\ & 314 \pi \\ & 312 \pi \end{aligned}$ | 0 <br> $\pi 12$ <br> $\pi$ | $\pi / 4$ <br> $\pi / 4$ <br> $\pi / 2$ | 3 |  |
| 4 | $\begin{aligned} & 1 \\ & 2 \\ & 3 \\ & 4 \end{aligned}$ | $\begin{aligned} & \mathrm{L} \\ & \mathrm{~L} \\ & \mathrm{~L} \\ & \mathrm{~L} \end{aligned}$ | $R$ | $\pi / 4$ $3 / 4 \pi$ $514 \pi$ $3 / 4 \pi$ | 0 $\pi / 2$ $\begin{gathered} \pi \\ 1.5 \pi \end{gathered}$ | $\pi 14$ <br> $\pi / 4$ <br> $\pi / 4$ <br> $\pi / 4$ | 4 |  |

## 5. RESULTS AND DISCUSSION

### 5.1. Geometric Analysis And Constraints Results

Some results of the geometric analysis and constraint equations are listed in Table (1). This table indicates that the dwell characteristics $v_{s}, v_{e}, \Delta \theta$ and $m$ can be easily determined as the constraints are known. The shaded areas represent the dwell period and frequency. The dwell duration $\Delta \theta$ increases either as the inclination angle $\alpha_{i}$ decreases or as $\phi_{\mathrm{i}}$ increases

### 5.2. Kinematic Analysis Results

Some results of $\phi, \theta_{1}, \omega, \omega_{1}, f$ and $f_{t}$ of the first case and $\phi_{\text {II }}, \theta_{\text {III }}, \omega_{\text {II }}, \omega_{\text {iII }}, f_{\text {II }}$ and $f_{\text {III }}$ of the second case are plotted versus $\theta_{\mathrm{d}}$ in Figures (3-8). These figures
indicate the following observations;
1 For the first case, $v_{s}$ occurs when $\theta_{i}=\phi_{i}=\alpha_{i}$. This verifies conditions of equations (7) and (36).
$2 \phi$ delays $\phi_{\mathrm{II}}$ while they are equal during the dwell periods
$3 \phi_{\text {II }}$ lags $\theta_{\mathrm{t}}$, while $\phi_{\text {II }}$ leads $\theta_{\text {tII }}$.
$4 \phi_{\text {II }}$ equals to $\theta_{\text {III }}$ at the end position of each dwell period, this means that $v_{e}$ occurs when $\theta_{\text {tII }}=\theta_{\text {II }}=$ $\alpha_{I}$ for the second case
$5 \omega$ and ( $\omega_{\mathrm{II}}$ ) are suddenly decreased and (increased) at $v_{\mathrm{s}}$ and ( $v_{\mathrm{e}}$ ) of each dwell respectively, and,
$6 f_{\text {II }}$ and $f_{\text {iII }}$ are much greater in magnitude than $f$ and $f_{t}$ respectively.
In addition, Figures. (3-5) show the results of mech. 1
where $\mathrm{n}=3$ sprockets, three dwells $(\mathrm{m}=3)$ periods of $45^{\circ}$, $45^{\circ}$ and $90^{\circ}$,respectively, are occurred at $\phi=0^{\circ}, 90^{\circ}$ and $180^{\circ}$. Also, Figures. (6-8) illustrate the results of mech. 2 where $n=4$ sprockets for dwells $(m=4)$ with equal four periods of $45^{\circ}$ are occurred at $\phi=0^{\circ}, 90^{\circ}, 180^{\circ}$ and $270^{\circ}$ respectively. These results are in agreement with the -corresponding tabulated one. Then, in point of view of the dynamic effects, it is worth noting that such mechanisms should be operated according to the first case.


Figure 3. Mech. 1-Displacements.


Figure 4-A. Velocity


Figure 4-B. Velocity.


Figure 5. Mech. 1-Accelerations.



Figure 6. Mech. 2-Displacements.


Figure 8. Mech. 2-Accelerations.

## 6. CONCLUSIONS

As a result of the presented analysis, one easily concludes the following,
1 The combination of the flexible system (roller chain) with crank coupler links gives useful and an additional new intermittent motion mechanisms. These mechanisms can be designed to accomplish a single or multiple dwells without the need of locking device.
2 The analysis and constraint equations could easily be modified to optimize or synthesize such system for specifying a desired motion characteristics. The desired motion would be either reversing or
nonreversing one with or without dwells.
3 The main advantages of the mechanism presented are., The structure of the system is simple.
ii The nature of the analysis procedure is rational.
iii The formulation don't lead to tedious algabraic manipulation in finding the solution for multiple dwell requirements. and,
iv The mechanism can provide dwells with various characteristics (positions, periodes and frequences) within, one cycle. This may be useful in certain industrial applications.

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