OPTIMIZATION ALGORITHM FOR PLANNING OF URBAN BUS ROUTES

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ABSTRACT

There are real risks that the modification of bus route configurations in an urban area will result in poorer performance, however measured, if not carefully thought through. This paper presents an approach to the planning of bus routes taking account of both passenger and operator interests. The proposed approach is based on constrained optimization algorithm in which several operating parameters are selected to maximize an objective function reflecting passenger benefits. The paper describes the key features of the problem, the formulation and the solution of the algorithm. An interactive planning system developed to facilitate the application is then demonstrated, and an example to illustrate the proposed algorithm, as applied to the bus network of Alexandria, is finally provided.

INTRODUCTION

According to the continuous development in urban areas worldwide, bus operators often tend to modify the existing routes randomly without any attempt to optimize the whole bus system. This may negatively affect, for instance, average travel time, number of transfers, capital and operating costs.

The main purpose of this paper is to propose a bus system optimization algorithm that can be used for planning of the routes of a new bus network or reorganizing the routes of an existing one. Optimization means here that, maximum user benefits (minimum travel time and number of transfers) are combined with minimum operator expenses (minimum capital and operating costs). The user demand is usually different from the requirements of the operator [9]. For example, realization of quickest direct trips (without transfer) may maximize operator costs.

The proposed algorithm can also be applied, within a planning process, to evaluate route modifications which may be projected intuitively by the planner. The share of the planner with his experience in the planning process aims to produce an acceptable practicable solution which satisfies the requirements of both user and operator. In practice, fundamental changes of existing bus routes are lways *undesirable [11]*.

The paper also presents an interactive man-computer lanning system which is developed to facilitate the generation and evaluation of alternative plans, and to deal quickly and accurately with great bus networks.

The application of the proposed algorithm is then carried out to optimize the bus network of Alexandria. The goal of the application is to examine the efficiency and practical capability of the algorithm, and to investigate the effects of changing the operating conditions on the planning outputs.

FORMULATION OF OPTIMIZATION PROBLEM

An optimization algorithm is developed here to perform the optimum demand-oriented routes on the basis of an existing transportation bus network. It includes an objective function that can be solved within certain constraints.

The objective function is formulated to minimize the travel time as well as the number of transfers between the various bus stops (user benefit). The optimization constraints, on the contrary, are expressed to fulfill the requirements of the bus operator.

To simplify the formulation of the optimization problem, the following basic assumptions are firstly considered:

- The travel demand is supposed to be constant during each operating time period.
- The nodes of the network may be a terminal or an intermediate stop.

- Each node is taken as a centroid of a traffic zone.
- The links of the bus network are at the same time links of the road network.
- The total number of assigned bus vehicles on any link affects passively its service level.

Objective Function

Since the optimization of a bus network can mainly be achieved through minimizing travel time and number of transfers, the ratio between the actual and shortest travel time should be determined. This ratio is called the "Detour Factor" [10]. It is an indicator of the service level offered by the bus transport system. A smaller detour factor means a better service level, i.e. less travel time and number of transfers.

The so-called "Average route Detour Factor" is the average value of the detour factors between the nodes located along a certain route. This factor can also be defined as follows :

$$D(l) = \{ \sum_{i,j \in N} [F_{(i,j,l)} * t_{(i,j,l)} / ts_{(i,j)}]\} \;/ \sum_{i,j \in N} F_{(i,j,l)}$$

where
$D(l)$ = Average detour factor on bus route l
l = route number; $l = 1,2, \ldots, nl$
nl = number of routes
N = number of nodes
i = set of nodes as origins; $i \in N$
j = set of nodes as destinations; $j \in N$
$F_{(i,j,l)}$ = number of passenger trips from i to j on route l
$ts_{(i,j)}$ = short path travel time between i and j
$t_{(i,j,l)}$ = actual travel time between i and j on route l

The number of passenger trips between two nodes, divided by its detour factor is called the "Equivalent productivity". It achieves its maximum value, when the detour factor equals to 1.0, i.e. all passengers travel on the short time path. Thus, the summation of the equivalent productivities between the nodes on a certain route is the "Equivalent Route Productivity". It can be written in the form:

$$P(l) = \sum_{i,j \in N} [\, F_{(i,j,l)} * ts_{(i,j)} / t_{(i,j,l)} \,]$$

where
$P(l)$ = Equivalent productivity of route l

A greater "Equivalent Route Productivity" means tha the route has more trips with short path travel times, i.e more direct trips (without transfer) than indirect trips Thus, the maximization of P(l) satisfies the optimizatio objective function, which can, therefore, be formulated a follows :

$$\sum_{l=1}^{nl} P(l) \;==> \; \text{Max.}$$

The number of transfers is not explicitly involved in the objective function. However, the maximization of direc trips means at the same time minimization of transfers.

Optimization Constraints

The objective function should be solved within th following operating conditions as constraints:

a) $D(l) \leq D_{max}$
b) $H_{min} \leq H(l) \leq H_{max}$
c) $Oc_{min} \leq Oc(l) \leq Oc_{max}$
d) $L_{min} \leq L(l) \leq L_{max}$

where
$D(l)$ = average detour factor of route l
$H(l)$ = headway on route l
$L(l)$ = length of route l
$Oc(l)$ = maximum vehicle occupancy on route l
D_{max} = maximum permissible value o "Average Route Detour Factor"
H_{min} & H_{max} = lower and upper limits of permissibl headway
L_{min} & L_{max} = lower and upper limits of permissible route length
Oc_{min} & Oc_{max} = lower and upper limits of vehicle occupancy

The average route detour factor should be limited to a maximum value. This means that the actual travel time between each pair of nodes should not exceed the short travel time by more than a given permissible percentage The aim is to guarantee that the actual travel time will no dramatically exceed the shortest travel time.

To maintain an adequate service quality of the bus system, the projected headway should not go beyond a maximum allowed value (policy headway). Also, the headway should not be less than a minimum value which reflects certain specific operational constraints such as terminal capacity [6].

Vehicle occupancy should also be restricted between a maximum and minimum permissible value. From the economical point of view, processing a low vehicle occupancy means increasing the required number of buses. On the other side, a high vehicle occupancy indicates a lower level of service. However, a high occupancy value can be recommended during the peak-hour operating periods to reduce the needed fleet size [3].

The route length should be located within a permissible range (maximum and minimum values). The maximum value can be appointed to be compatible with the length of the majority of bus trips in the concerned urban area, and the minimum value with the average bus trip length [1]. A longer route may negatively influence the vehicle occupancy, and a shorter route may increase the number of transfers.

SOLUTION OF THE OPTIMIZATION PROBLEM

The method needed for the planning of demand-oriented routes should recognize the objective function and its constraints. Within the proposed algorithm, route configurations can progressively be determined according to two opportunities:

- optimum mathematical solution, and
- optimized solution based on planner experience.

Optimum Mathematical Solution

The short path between the pair of nodes with the biggest sum of trips (as origin and/or destination) is firstly determined and defined as a base route. This route can be expanded by inserting additional nodes, taking into consideration the permissible operating constraints. The O-D matrix is then assigned to the different links of the route due to their travel resistance. After erasing the processed trips from the travel demand matrix, the procedure is repeated until all trips is served. According to the resulting loads, the performance characteristics of each route can be calculated. Comparing the calculated characteristics with the operating constraints, additional routes may be added and/or existing routes may be canceled.

This solution can be carried out according to the following steps :

0. Define the operating constraints and set route number $l = 1$.
1. Select the couple of nodes with the biggest sum of trips as origin and/or destination. These nodes would be the terminals of a base route.
2. Determine the short time path connecting the two terminals selected in 1 and define it as the base route (l).
3. Calculate the equivalent productivity $P(l)$ as the initial value of objective function; set $S = P(l)$.
4. If the following two operating constraints are satisfied :
   $L(l) \leq L_{max}$, and
   $H(l) \geq H_{min}$, go to 5;
   otherwise reduce the route length by excluding terminus node and return to 3.
5. If an intermediate node can be inserted to the base route, causing no loops go to 6; otherwise go to 8.
6. Insert this intermediate node and calculate the new value of $P(l)$. If the following four conditions are satisfied :
   $H(l) \geq H_{min}$ $\quad D(l) \leq D_{max}$
   $L(l) \leq L_{max}$ $\quad S \leq P(l)$
   go to 7; otherwise exclude the inserted node and return to 5.
7. Replace the existing value of S by the new value of $P(l)$; $S = P(l)$, and return to 5.
8. If the following two conditions are satisfied :
   $H(l) \leq H_{max}$
   $L(l) \geq L_{min}$, go to 10; otherwise continue.
9. If all terminals are examined: Stop, otherwise select another couple of nodes, having the next biggest sum of trips as origin and/or destination as a base route and return to 2.
10. Erase served trips from the O/D matrix. If there are remaining trips, set $l = l + 1$ and return to 1; otherwise go to 11.
11. Derive the load profile on all routes.
12. If $H(l) \leq H_{max}$ go to 13; otherwise cancel the unsatisfactory routes and return to 11.
13. If $H(l) < H_{min}$ or there are remaining passengers set $l = l + 1$ and return to 1; otherwise Stop.

Optimized Solution based on the Planner Experience

The route configurations produced from the previous presented optimum mathematical solution may sometimes be unrealistic and often can not be accepted by the user [2]. An overall changes of the existing routes can not easily be verified, from both planning and economical viewpoints. Therefore, the previous solution is modified to enable the planner to give his experience by the planning of practicable routes. In this case, the method can be described as follows:

1. Modify the layout of the base route (l) previously determined.
2. If the following three conditions are satisfied :
   $L(l) \leq L_{max}$
   $L(l) \geq L_{min}$
   $D(l) \leq D_{max}$, go to 3; otherwise return to 1.
3. If the following condition is satisfied :
   $H(l) \geq H_{min}$, go to 4; otherwise go to 5.
4. If the following condition is satisfied :
   $H(l) \leq H_{max}$, go to 5;
   otherwise return to step 1.
5. If the decision that, either the route (l) is to be canceled or its layout to be changed, is taken (by planner), return to 1; otherwise go to 6.
6. Erase the served trips from the O/D matrix. If there are remaining passengers set l = l + 1 , select new base route (l), and return to 1; otherwise go to 7.
7. If all passenger are directly served go to 9; otherwise go to 8.
8. If it is desired to add more routes (decision is made by planner), set l = l + 1 , select new route (l), and return to 1; otherwise go to 9.
9. Derive the load profile on all routes.
10. If $H(l) \leq H_{max}$ go to 11; otherwise cancel the unsatisfactory routes and return to 9.
11. If $H(l) < H_{min}$ or there are remaining passengers set l = l + 1, select new base route (l), and return to 1; otherwise Stop.

PLANNING SYSTEM

System Structure

Route optimization requires a large amount of data and long computing time to deal with great bus networks [7]. A man-computer planning system, is developed to facilitate the entering of data, generation, modification, and evaluation of alternatives. It can also be used to display graphically the output results. The structure of this planning system (called IPS) is illustrated in Figure (1).

The planning system consists of three main modules. The first (Input Data Management) is a data editor to introduce the elements of bus networks (nodes and links) and travel demand matrices. It can either enter new data set or modify existing one. The input data is stored automatically on a disk, from which it can be displayed on the screen or printed.

The second module (Network Optimization) receives the input data stored by the first module. It is also provided with editing facilities to introduce the operating conditions to the computer. It carries out the optimization procedure previously described, and evaluates the planning alternatives. The performance characteristics of each alternative will be stored on a computer disk, and can optionally be displayed or printed. This module consists of a main routine and nine subroutines (described later). The main routine connects these subroutines, and helps the planner to modify route configurations.

The third module (Graphical Representation) is designed for the colour graphic display of the outputs, such as the bus network, the route layouts, and the evaluation results of each alternative. The colour display of results can automatically be achieved without the need of any additional graphic system.

The first and second modules are written in FORTRAN 77 and the last in TURBO-C for the use of personal computers (IBM or compatible).

The nine subroutines of the second module (Network Optimization) can briefly be described as follows:

1. Subroutine CHOOSE: It identifies the pair of nodes with the biggest sum of trips as origin and/or destination, and defines the base route by connecting these nodes with the short path (using the second subroutine SHORT).
2. Subroutine SHORT: This subroutine is a modified form of the "Matrix Short Path Algorithm" [5]. It can be used to find both the short time path between each pair of nodes and the actual travel time on the different links of a certain route.
3. Subroutine ASSIGN: It is an assignment model based on "all-or-nothing method", which is applied here to assign O-D travel demand matrices on bus networks for each operating time period.
4. Subroutine HEADWAY: According to the route loads

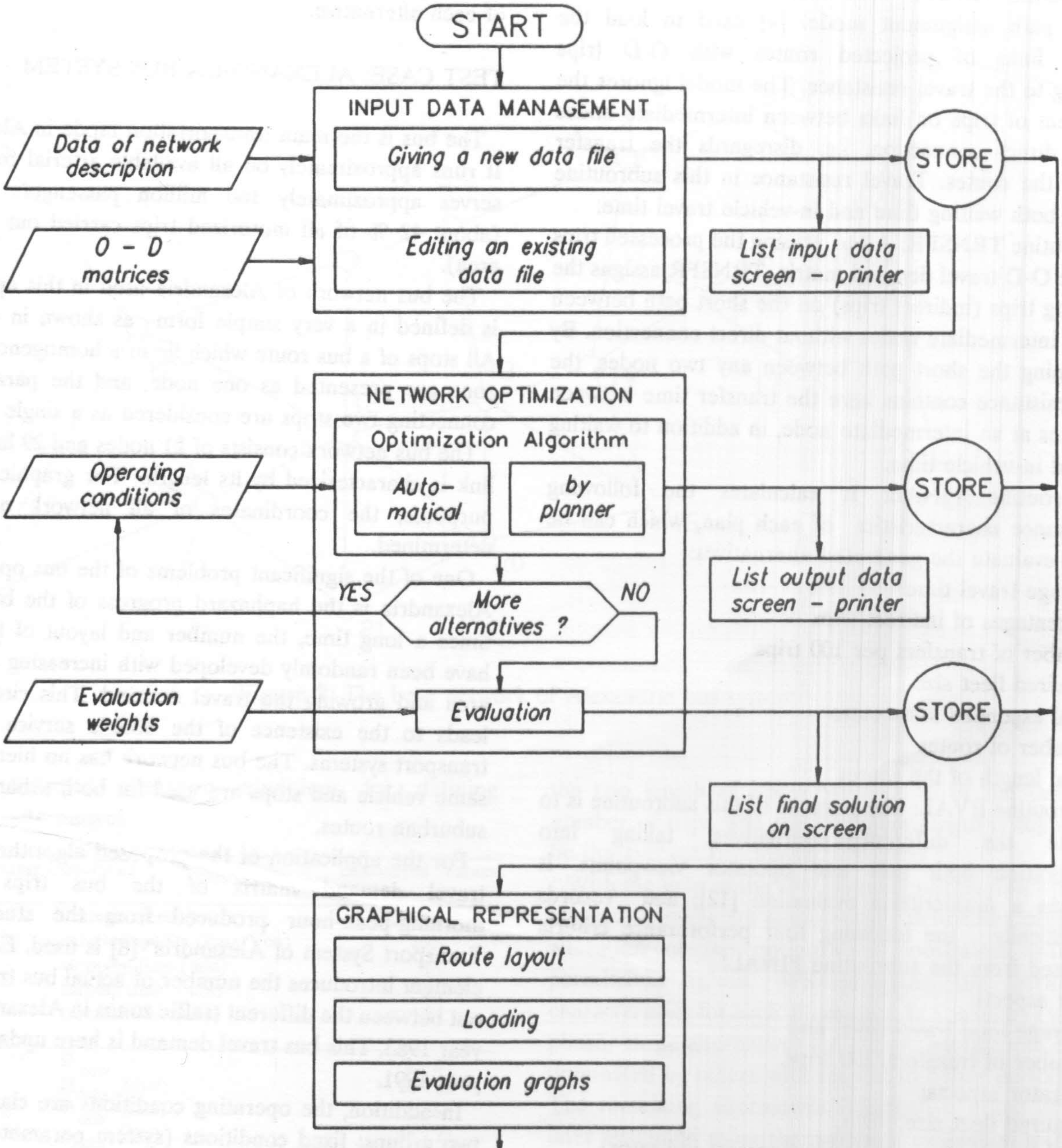

Figure 1. Structure of the planning system

produced from ASSIGN, this subroutine calculates the headways, the required number of vehicles and the occupancy rate for each route.

5. Subroutine OPTIMUM: It is used to modify the layout of a base route by inserting intermediate nodes. Each node adjacent to a link and does not belong to the route layout is examined to be inserted. If node inserting increases the value of the objective function and satisfies the constraints, this node has to be accepted as a supplementary intermediate node.

6. Subroutine MULTI: This subroutine is based on a multiple path assignment model [4] used to load the different links of projected routes with O-D trips according to the travel resistance. The model ignores the assignment of trips on links between intermediate nodes without direct connection, i.e. disregards the transfer between the routes. Travel resistance in this subroutine includes both waiting time and in-vehicle travel time.

7. Subroutine TRNSFR: After erasing the processed trips from the O-D travel demand matrix, TRNSFR assigns the remaining trips (indirect trips) on the short path between any two intermediate nodes without direct connection. By determining the short path between any two nodes, the travel resistance contains here the transfer time between the routes at an intermediate node, in addition to waiting time and in-vehicle time.

8. Subroutine FINAL: It calculates the following performance characteristics of each plan, which can be used to evaluate the generated alternatives:

- average travel time
- percentages of indirect trips
- number of transfers per 100 trips
- required fleet size
- total expended kilometers
- number of routes
- total length of the routes

9. Subroutine EVAL: The purpose of this subroutine is to evaluate the different alternatives, taking into consideration both user and operator viewpoints. It performs a multicriteria evaluation [12], and records automatically the following four performance criteria calculated from the subroutine FINAL:

a) user aspects:
  - average total travel time, and
  - number of transfers/100 trips.

b) operator aspects:
  - required fleet size,
  - total length of routes (operating cost indicator).

Each effective criterion yields a range of performance values, one for each alternative. These values are placed linear on such scale that sets the worst value to equal 0 and the best to 10. Thus, the scale is related only to the difference between best and worst.

Weights, which present the relative importance of each evaluation criterion from both user and operator perspectives are introduced. The model ranks then the different alternatives according to the total weighted score of each alternative.

TEST CASE: ALEXANDRIA BUS SYSTEM

The bus is the main transportation mode in Alexandria. It runs approximately on all available arterial roads, and serves approximately 160 million passengers annually (about 42 % of all motorized trips carried out in urban area).

The bus network of Alexandria used in this application is defined in a very simple form as shown in Figure 2. All stops of a bus route which lie in a homogenous traffic zone are presented as one node, and the parallel links connecting two stops are considered as a single link.

The bus network consists of 21 nodes and 29 links. Each link is characterized by its length. For graphical display purposes, the coordinates of all network nodes are determined.

One of the significant problems of the bus operation in Alexandria is the haphazard progress of the bus routes. Since a long time, the number and layout of the routes have been randomly developed with increasing the urban area and growing the travel demand. This circumstance leads to the existence of the double service of public transport systems. The bus network has no hierarchy; the same vehicle and stops are used for both urban and long suburban routes.

For the application of the proposed algorithm, an O-D travel demand matrix of the bus trips for the morning-peak-hour produced from the study "Public Transport System of Alexandria" [8] is used. Each matrix element introduces the number of actual bus trips carried out between the different traffic zones in Alexandria in the year 1983. This bus travel demand is here updated for the year 1991.

In addition, the operating conditions are classified into two groups: fixed conditions (system parameters), which are assumed to be constant during each operating period, and variable conditions (optimization constraints), which can be varied during the construction of alternative plans.

Table 1 presents the values of both system parameters and optimization constraints which are used in this exercise for the initial generation of various alternative plans.

It should be noted that by projecting the maximum detour factor, it is assumed that the actual travel time is

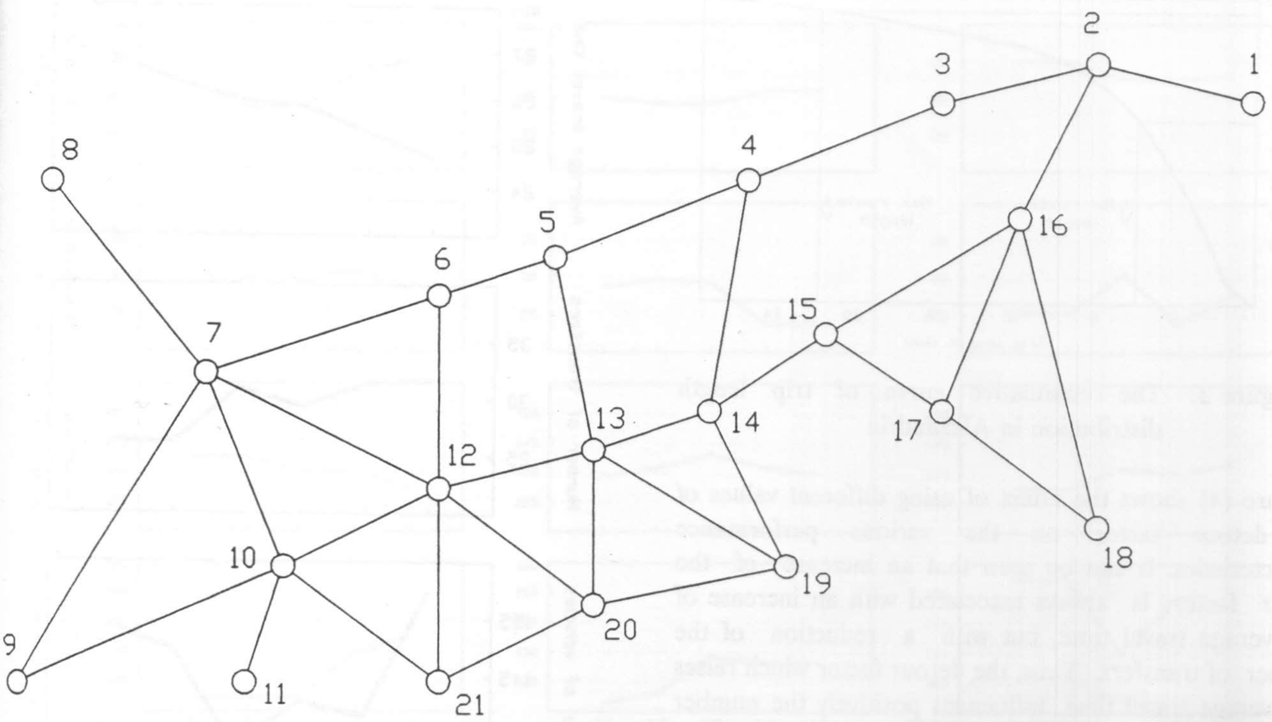

Figure 2. The base network of Alexandria bus system

Table 1 : Selected operating conditions for defining alternatives

	Operating Conditions	considered Values
System parameters	average speed (km/h)	20
	vehicle capacity (Pass/Veh)	100
	stopping time (min)	1
	turning time (min)	3
	transfer time (min)	2
Optimization constraints	D_{max}	1 - 2
	H_{max} (min)	20 - 35
	H_{min} (min)	2 - 6
	L_{max} (km)	16 - 22
	L_{min} (km)	6 - 11
	OC_{max} (%)	120 - 150
	OC_{min} (%)	60 - 110

not permitted to be more than twice the shortest travel time. Of course, it must not be less than 1.0. In addition, the cumulative curve of trip length distribution is used for determining the maximum and minimum bus route length (Figure 3). The maximum route length is assumed to be the trip length of about 90 - 95 % of the cumulative number of trips, and the minimum length is supposed to be the average trip length.

With the help of the IPS planning system, five optimized alternative plans are automatically defined by purposeless alteration of the optimization constraints within the permissible values. IPS produces the performance characteristics for each alternative. In a primary evaluation phase, three alternatives are eliminated that can easily be dominated by others with respect to better performances. The remaining alternatives (alternative 1 and 2) will be later evaluated using the proposed evaluation method.

Besides the automatic generation of alternatives, the IPS planning system is also tested to create manually a supplementary alternative plan (alternative 3) and to predict its performance characteristics. This alternative contains the same alignment structure of the existing routes in Alexandria with some proposed modifications.

The IPS planning system is then applied to investigate the effect of changing the optimization constraints on the resulting performance characteristics.

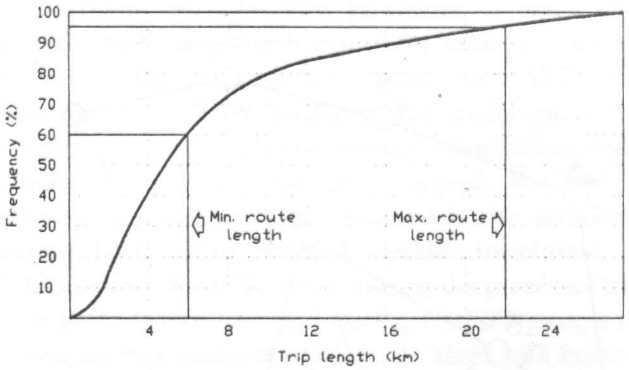

Figure 3. The cumulative curve of trip length distribution in Alexandria

Figure (4) shows the effect of using different values of the detour factor on the various performance characteristics. It can be seen that an increase of the detour factor is always associated with an increase of the average travel time, but with a reduction of the number of transfers. Thus, the detour factor which raises the average travel time, influences positively the number of transfers, i.e. any employed detour factor has a positive and negative effect on the user. Furthermore, the values of both number of required vehicles and total length of routes decrease simultaneously with increasing the detour factor up to a certain limit (about 1.4) then the values begin to increase again. In this manner, these relationships clearly illustrates that it is very difficult to find a value of the detour factor which satisfies both requirements of user and operator. Likewise consequences, however, are also achieved by studying the effects of changing the other optimization constraints on the performance characteristics (Figure 5).

Based on this investigation, the following optimization constraints are identified and defined as the most convenient operating conditions:

D_{max} = 1.4
H_{max} = 25 min H_{min} = 5 min
L_{max} = 18 km L_{min} = 8 km
OC_{max} = 1.5 OC_{min} = 1.1

Using these operating conditions, two additional alternatives (alternative 4 and 5) are created. Alternative 4 is automatically generated, and alternative 5 includes the modified routes of alternative 3 (existing situation) operated with the most convenient conditions.

Figure 6 shows the comparison of the five alternatives regarding their performance characteristics, as a sample of the evaluation graph that can be produced by the IPS planning system.

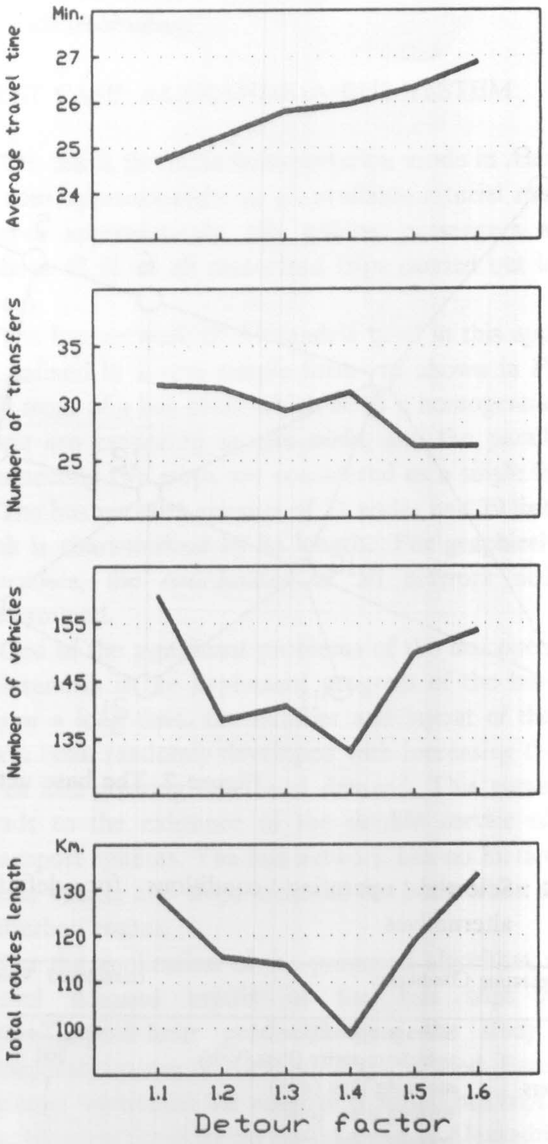

Figure 4. Effect of detour factor on performance characteristics

The evaluation process of these alternatives depends on the weights of both user and operator for each performance characteristic. By setting some weights (test values), the computer calculates the scaling and the scoring of each alternative. Alternative 1 is the best solution from the user perspective, while alternative 2 is the best one from the operator viewpoint. Alternative 4 is

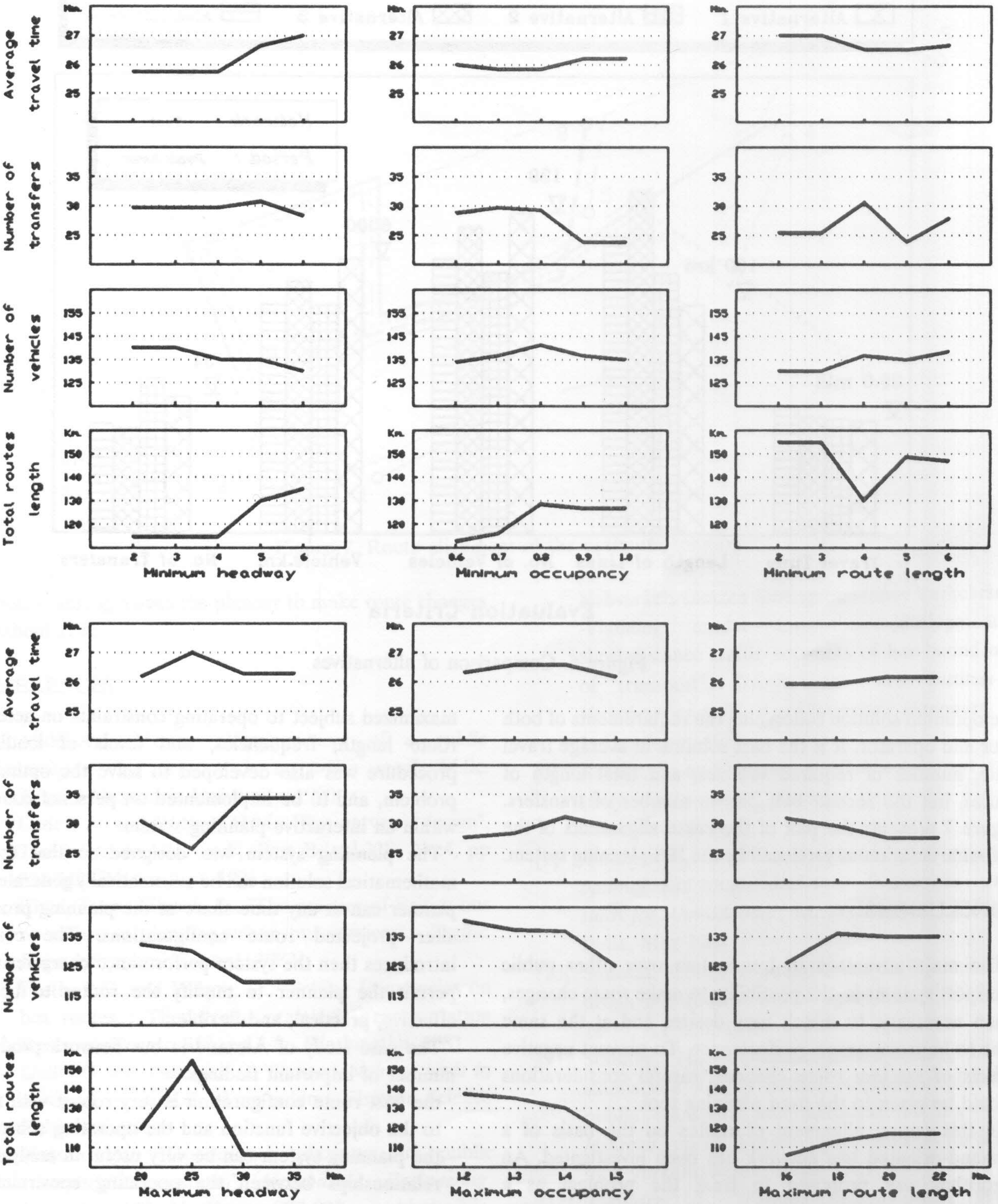

Figure 5. Effects of optimization constraints on performance characteristics

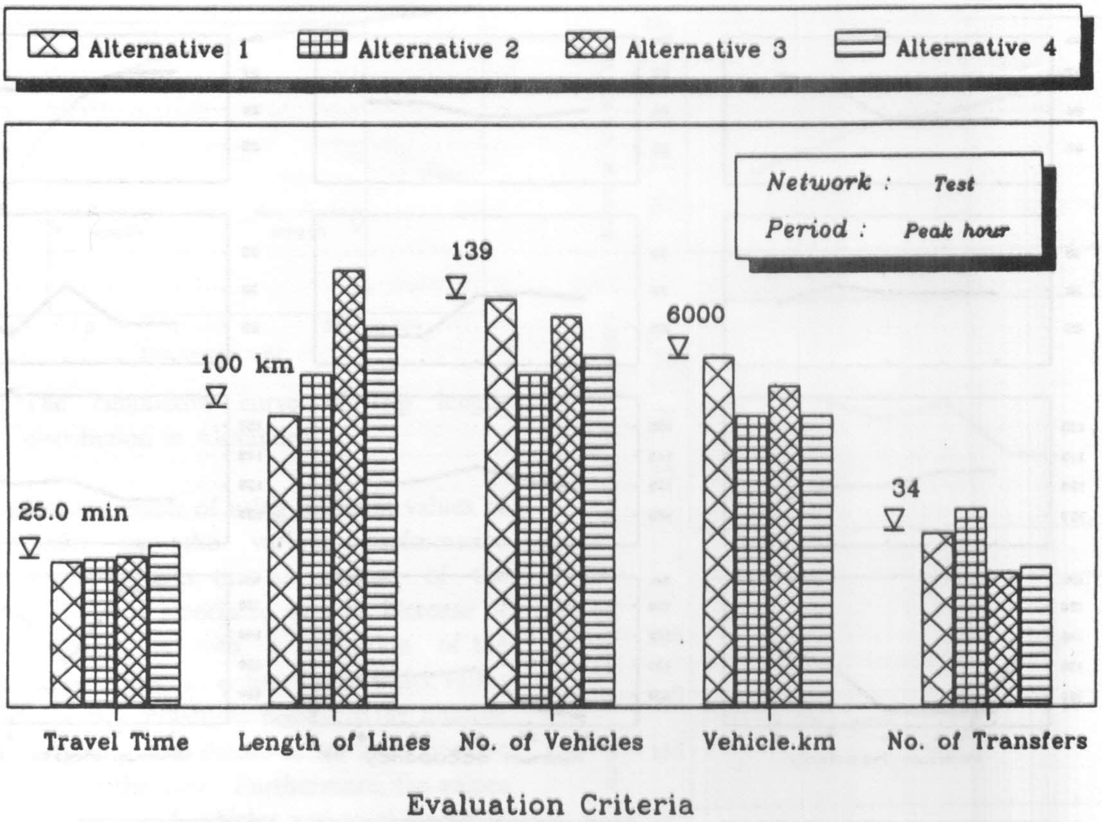

Figure 6. Comparison of alternatives

the optimum solution concerning the requirements of both user and operator. It is the best solution in average travel time, number of required vehicles, and total length of routes, but the second best one in number of transfers. Figure 7 presents the plot of the route alignments of the optimum solution as produced by the IPS planning system.

CONCLUSIONS

The main advantage of bus system over other public transport systems is the possibility to make route changes, when necessary, to match user desires and at the same time to improve system performance. To prevent negative effects of random route changes, careful considerations should be given to the used planning tool.

In this paper, alignment of routes on the basis of a demand-oriented bus network has been investigated. An algorithm was proposed to treat the problem as a constrained optimization problem. The objective was that user benefit, which consists of travel time saving, be maximized subject to operating constraints on acceptable route length, frequencies, and levels of loading. A procedure was also developed to solve the optimization problem, and to be implemented on personal computers within an interactive planning system.

The planning system was designed so that the best mathematical solution can be automatically generated. The planner can at any time share at the planning process to alter projected route configurations. The computer introduces then the system performance characteristics to permit the planner to modify the routes to be more effective, practical, and flexible.

The case study of Alexandria bus network produced a number of important findings:

- the best route configuration is very robust with respect to the objective function and the operating constraints,
- the planning system can be very useful in analyzing the relationships between the operating constraints and system performances, and
- the better understanding of the factors affecting bus

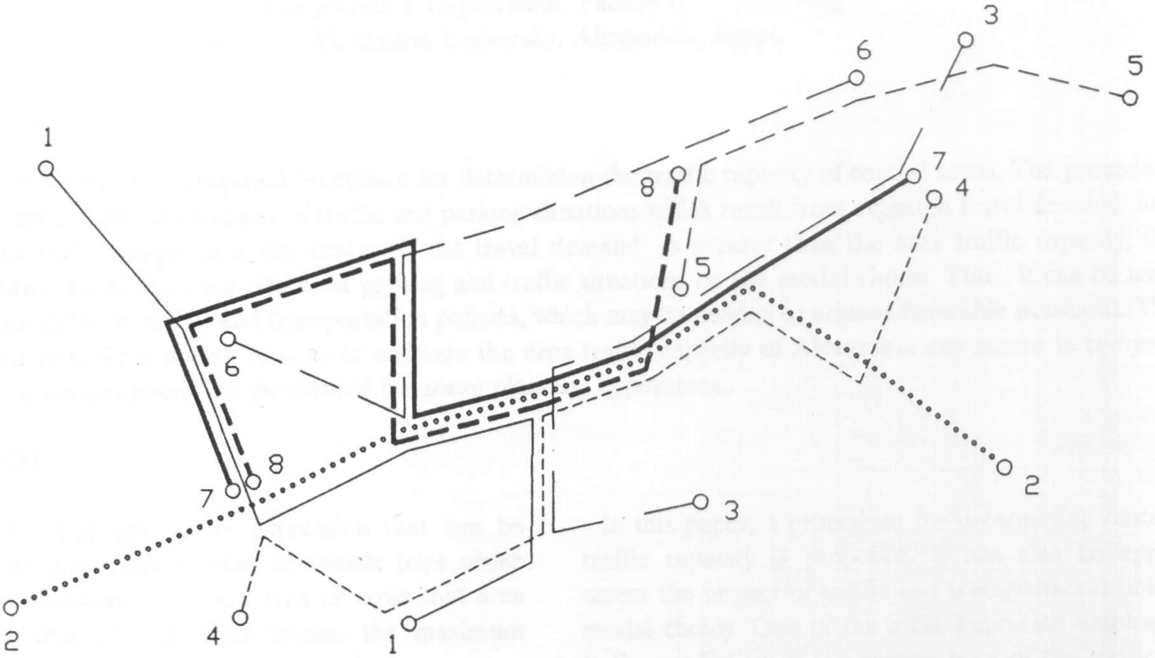

Figure 7. Route alignment of the optimum solution

route planning assists the planner to make route changes without risks.

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