FORWARD AND BACKWARD WAVES IN TWO-AXIS RELUCTANCE SYNCHRONOUS MACHINE

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ABSTRACT

In this paper, Reluctance synchronous motor under hunting condition is studied in detail. Forward f and backward b wave components in the machine are observed and equivalent circuits in rotor and synchronous reference frames for both complete and true perturbation variables are developed. Electromechanical powers associated in the circuits are also studied, and the f, b components of torque coefficients are investigated.

1. INTRODUCTION

Reluctance synchronous machine is one of an important drives in modern industry. It is used in the field of variable speed; such diverse areas of application, covering a wide range of unit powers, include metered puping in the chemical industry, high speed cutting and grinding, and the production of glass-fibre yarn. It is also required for ganged multiple motors, as in spinning machine drives in the man-made-fibre industry, in table drives as part of a mill system or in the sheet-glass industry. This motor as part of variable speed drives when inverter fed, may exhibit lightly damped oscillations in speed around the average speed, following any sudden variation in the load condition [7], and in extreme cases it may exhibit sustained instability in the form of oscillations of limited or exponentially growing amplitudes.

The aim of this paper is to understand as clearly as possible the physical implications of the mathematics amount. It is believed that physical understanding is the best aid to appreciating the way in which instability is affected by machine parameters, load condition, operating speed and frequency. Concentration is upon the inherent stability characteristics without the complications of any further overall feedback control.

One important means for stability studies is to concentrate attention on the hunting condition and to study the forward and backward electromagnetic wave components within the machine [10] in this condition. Analysis has been developed [1 to 6] for finding rotating field components (f and b) of damping torque coefficient T_D and synchronising torque coefficient T_S in the hunting condition. This is based on the use of equivalent circuits, as given by [1,6] for the synchronous machine.

However, it can be seen by the use of k-operator [9] that the f and b components of the excited synchronous machine are easy to investigate in the rotor reference frame [1], but can not be directly observed in the synchronous flux wave reference frame. This is clearly appreciated through the transformation of the perturbation variables of the field winding, which exists on the rotor d-axis, as can be seen by the r-frame. These variables have two components on both d and q axes when viewed in the synchronous reference frame. This important characteristic is recognised in this paper, which does not appear to be previously appreciated.

The reluctance synchronous machine overcome this problem, since it has no field circuit; moreover, e-frame does not oscillate with respect to r-frame, because the steady-state d and q damper currents are zero (since the slip is zero in this machine). Additionally, under the effect of rotor shaft perturbation, the stator does not perturb; so that the relative perturbation velocity between stator and e-frame is zero (although the corresponding steady-state velocity is of course ω), which means that all stator perturbation variables are of frequency equals zero, as seen by the e-frame. By these facts, it should be noted that perturbation currents in both stator and rotor coils are true [10] in e-frame, rotor current is true in r-frame but stator current is complete in r-frame. Thus, e-frame is the 'true' frame, while r-frame is for the 'complete' frame.

In this paper, therefore, forward and backward components of currents are investigated and equivalent circuits in both e and r frames are developed. These are all new, simple and physically meaningful; they are obtained directly by the use of k-operator. Forward and backward components of torque coefficients are also given in terms of the power associated in the network of true variables (e-frame). Analytical forms are investigated for the individual components of torque coefficients, which must satisfy the physical appreciation of the forward and backward waves in the machine, in consistant with the power associated in the equivalent circuit derived in a physical terms. The power and the corresponding components of torque under hunting condition are generated from the power oscillation restored on the rotor shaft, which may be obtained by perturbing the shaft with small oscillation of constant amplitude A at frequency β. It is, therefore, expecting to see the spectrum of the components of torque coefficients in frequency domain, which relate to the real and reactive powers in the circuit. Thus, analysis for developing these characteristics in time domain are not observed in this paper (although it may be valuable), since its calculations depend on the total coefficients, T_D and T_S plus the inertial torque, not the f and b components, and its behaviour should be studied at a particular operating condition (may be by using Laplace or Fourier transform).

2 VOLTAGE/CURRENT RELATION AND EQUIVALENT CIRCUIT FOR COMPLETE VARIABLES IN r-frame

The voltage/current equation for 2-axis reluctance machine and the steady-state variables are given by [5,7], which are expressed in the rotor reference frame. These can be written in the general matrix form as follows:

$$\underline{v}^r_{dq} = [\underline{R} + \underline{L}p + \underline{G}_s\omega]\underline{i}^r_{dq}$$

where

$$\underline{v}^r_{dq} = \begin{bmatrix} v^r_{ds} \\ v^r_{qs} \\ v^r_{kd} \\ v^r_{kq} \end{bmatrix} = \begin{bmatrix} -v\sin\delta \\ v\cos\delta \\ 0 \\ 0 \end{bmatrix}, \quad \underline{i}^r_{dq} = \begin{bmatrix} i^r_{ds} \\ i^r_{qs} \\ i^r_{kd} \\ i^r_{kq} \end{bmatrix}$$

$$= \begin{bmatrix} v(L_q\omega\cos\delta - R_s\sin\delta)/(R_s^2 + L_dL_q\omega^2) \\ v(R_s\cos\delta + L_d\omega\sin\delta)/(R_s^2 + L_dL_q\omega^2) \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{R} = \begin{bmatrix} R_s & & & \\ & R_s & & \\ & & R_{kd} & \\ & & & R_{kq} \end{bmatrix} \begin{matrix} ds \\ qs \\ kd \\ kq \end{matrix},$$

$$\underline{L} = \begin{bmatrix} L_d & & M_d & \\ & L_q & & M_q \\ M_d & & L_{kd} & \\ & M_q & & L_{kq} \end{bmatrix} \begin{matrix} ds \\ qs \\ kd \\ kq \end{matrix}$$

$$\underline{G}_s = \begin{bmatrix} & -L_q & & -M_q \\ L_d & & M_d & \\ & & & \\ & & & \end{bmatrix} \begin{matrix} ds \\ qs \\ kd \\ kq \end{matrix}, \quad p = \frac{d}{d\,\omega_o t}$$

The perturbation equation of the voltage/current in the r-frame is:

$$\underline{\Delta v}^r_{dq} = [\underline{R} + \underline{L}p + \underline{G}_s\omega]\underline{\Delta i}^r_{dq} + \underline{G}_s\underline{i}^r_{dq}A\cos\beta t \qquad (1)$$

Equation (1) is obtained by perturbing the shaft under forced oscillation of constant amplitude A at frequency β. In this condition the flux in the stator coil oscillates with respect to r-frame, which is represented by the term $\underline{G_s i_{dq}}A\cos\beta t$. Moreover the steady-state velocity of the rotor with respect to the stator is ω in the direction from d to q axis, which when viewed with respect to r-frame it will be seen that the coil rotates by frequency w in the direction from q to d, and hence the motional term $\underline{G_s}\omega\underline{\Delta i}_{dq}$ represents the voltage induced in the stator coil due to the motion. The angular displacement of the stator as seen by the rotor reference frame is: $\theta^r_s = \omega t - \delta$ for steady-state variables or $\underline{\Delta\theta}^r_s = A\sin\beta t/\beta$ for perturbation variables, both in the direction from q to d.

The perturbation voltage $\underline{\Delta v}^r_{dq}$ is usually determined as follows: [7]

$$\underline{\Delta v}^r_{dq} = \frac{\partial \underline{v}^r_{dq}}{\partial \delta}\frac{\partial \delta}{\partial \theta^r_s}\Delta\theta^r_s = \frac{\partial \underline{v}^r_{dq}}{\partial \delta}(-1)A\frac{\sin\beta t}{\beta}$$

$$= \begin{bmatrix} v\cos\delta \\ v\sin\delta \\ 0 \\ 0 \end{bmatrix} A\frac{\sin\beta t}{\beta}$$

Equation (1) can be therefore rearranged as follows

$$[\underline{R}+\underline{L}p+\underline{G}_s\,\omega]\underline{\Delta i}^r_{dq} = A[\begin{bmatrix} v\cos\delta \\ v\sin\delta \\ 0 \\ 0 \end{bmatrix}\frac{\sin\beta t}{\beta} + \begin{bmatrix} L_q i^r_{qs} \\ -L_d i^r_{ds} \\ 0 \\ 0 \end{bmatrix}\cos\beta t\,] \qquad (2)$$

where $\underline{\Delta i}^r_{dq} = \underline{K\Delta i}^r_{fb}$

Multiplying both sides of (2) by $\underline{K}^{-1}$ [9], we get:

$$\underline{K}^{-1}[\underline{R} + \underline{L}p + \underline{G}_s\,\omega]\underline{K\Delta i}^r_{fb}$$

$$= A\underline{K}^{-1}[\begin{bmatrix} v\cos\delta \\ v\sin\delta \\ 0 \\ 0 \end{bmatrix}\frac{\sin\beta t}{\beta} + \begin{bmatrix} L_q i^r_{qs} \\ -L_d i^r_{ds} \\ 0 \\ 0 \end{bmatrix}\cos\beta t\,] \qquad (3)$$

By multiplying the first row in (3) by $\beta/(\beta+\omega)$ and the second by $\beta/(\beta-\omega)$ we can easily derived the equivalent circuit of complete variables in r-frame, as shown in Figure (1).

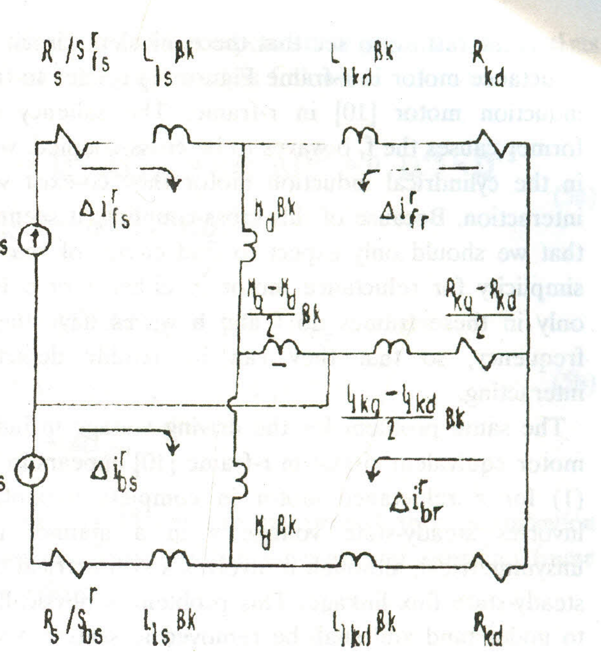

Figure 1. Perturbation network for reluctance motor in r-frame.

The effective slips (shown in Figure (1)) in the r-frame are:

$$S^r_{fs} = \frac{\beta+\omega}{\beta},\; S^r_{bs} = \frac{\beta-\omega}{\beta},\; S^r_{fr} = S^r_{br} = \frac{\beta}{\beta} = 1$$

These are indentical to those in the e-frame, since there is no steady-state relative motion between r and e frames (steady-state slip S = 0).

The circuit of Figure (1) agreed with the circuit established by [1,6] for the excited synchronous machine in the r-frame. Figure (1) can be easily checked by removing the field components from the network given in [1].

The f and b driving voltages are:

$$V'^r_{fs} = \frac{A}{\sqrt{2}}[\frac{v}{\beta}\sin(\beta t+\delta) + \psi_s\cos(\beta t-\alpha^r_s)]/S^r_{fs}$$

$$V'^r_{bs} = \frac{A}{\sqrt{2}}[\frac{v}{\beta}\sin(\beta t-\delta) + \psi_s\cos(\beta t+\alpha^r_s)]/S^r_{bs}$$

$$\psi_s = \sqrt{\psi^{r2}_{ds} + \psi^{r2}_{qs}},\quad \alpha^r_s = \tan^{-1}(\frac{\psi^r_{ds}}{\psi^r_{qs}})$$

$$\psi^r_{ds} = L_d i^r_{ds} \qquad \psi^r_{qs} = L_q i^r_{qs}$$

It is interesting to see that the equivalent circuit of the reluctance motor in r-frame Figure (1) relates to that for induction motor [10] in r-frame. The saliency of the former causes the f, b waves to be cross-coupled, whereas in the cylindrical induction motor they co-exist without interaction. Because of this cross-coupling, it seems clear that we should only expect to find circuit of reasonable simplicity for reluctance motor in either r or e frames: only in these frames do f and b waves have the same frequency, so that they can be readily depicted as interacting.

The same problem for the driving voltage in induction motor equivalent circuit in r-frame [10] appears in Figure (1) for a reluctance motor in complete variables. It involves steady-state voltage v in a manner that is unsymmetrical, although it involves a symmetrical term in steady-state flux linkage. This problem is physically hard to understand and shall be removed in section 4 when a circuit developed in e-frame.

3 TRANSFORMATION MATRICIES AND RELATIVE VELOCITIES AMONG PRINCIPAL REFERENCE FRAMES

When the rotor rotates with velocity w in the direction from d to q at a steady-state operation, the positive direction of the rotation is conventionally defined from q to d, that is the load angle δ is assumed to be positive for motoring as measured from q to d. Therefore, the relative motion of the rotor with respect to stator reference frame is $p\theta_r^s = -\omega$, where $\theta_r^s = -(\omega t - \delta)$ from q to d. Consequently, the stator speed as it refered to the rotor is $p\theta_s^r = \omega$, and $\theta_s^r = \omega t - \delta$. Since the relative velocity between the rotor and the synchronous flux wave reference frame is zero in the reluctance motor (because the slip is zero), then the steady-state angle of displacement is $\theta_r^e = 0$, consequently the stator coil is seen to be rotating in e-frame with velocity $p\theta_s^e = \omega$, where $\theta_s^e = \theta_s^r = \omega t - \delta$.

From the above discussion, we conclude that when the steady-state variables (voltages and currents) are determined in r-frame, they are identical to those determined in e-frame, but variables in either r or e frame are different from their corresponding variables in s-frame. Thus, the transformation matrices $\underline{D}$ of the steady-state variables can then be defined [8] among principal reference frames as follows:

$$\underline{D}^{es} = \underline{D}^{rs} = \begin{bmatrix} \cos(\omega t-\delta) & \sin(\omega t-\delta) & & \\ -\sin(\omega t-\delta) & \cos(\omega t-\delta) & & \\ & & \cos(\omega t-\delta) & \sin(\omega t-\delta) \\ & & -\sin(\omega t-\delta) & \cos(\omega t-\delta) \end{bmatrix} \begin{matrix} ds \\ qs \\ kd \\ kq \end{matrix}$$

$= \underline{D}^{seT} = \underline{D}^{srT}$, $\underline{D}^{er} = \underline{D}^{re} = \underline{I}$ = identity matrix

By perturbing the shaft under forced oscillation of constant amplitude A at frequency β in the direction from d to q, the rotor perturbation velocity as seen by stator reference frame is conventionally $p\Delta\theta_r^s = -A\cos\beta t$ from q to d, where $\Delta\theta_r^s = -A\sin\beta t/\beta$. Consequently, the stator coil is seen to have relative velocity in r-frame equals $p\Delta\theta_s^r = A\cos\beta t$; $\Delta\theta_s^r = A\sin\beta t/\beta$ such behaviour of the wave having no oscillation between s-frame and e-frame, that is $\Delta\theta_s^e = 0$, but the rotor wave is seen to be oscillating by velocity $p\Delta\theta_r^e = -A\cos\beta t$ in e-frame in the positive direction from q to d, where $\Delta\theta_r^e = -A\sin\beta t/\beta$.

Therefore, it is shown from the above that the transformation matrices $\underline{\Delta D}$ of the perturbation variables of the reluctance motor are easy to obtain, as follows:

$$\underline{\Delta D}^{sr} = -\begin{bmatrix} \sin(\omega t-\delta) & \cos(\omega t-\delta) & & \\ -\cos(\omega t-\delta) & \sin(\omega t-\delta) & & \\ & & \sin(\omega t-\delta) & \cos(\omega t-\delta) \\ & & -\cos(\omega t-\delta) & \sin(\omega t-\delta) \end{bmatrix} A\frac{\sin\beta t}{\beta}$$

$= \underline{\Delta D}^{rsT}$, $\underline{\Delta D}^{es} = \underline{\Delta D}^{se} = 0$

$$\underline{\Delta D}^{er} = \begin{bmatrix} & -1 & & \\ 1 & & & \\ & & & -1 \\ & & 1 & \end{bmatrix} A\frac{\sin\beta t}{\beta} = \underline{\Delta D}^{reT}$$

Furthermore, the following are satisfied: $\underline{v}^e = \underline{D}^{er}\underline{v}^r = \underline{v}^r$, $\underline{i}^r = \underline{D}^{re}\underline{i}^e = \underline{i}^e$, $\underline{v}^s = \underline{D}^{sr}\underline{v}^r$, $\underline{i}^e = \underline{D}^{es}\underline{i}^s$, ... etc. Also

$\underline{\Delta v}^e = \underline{\Delta v}^r + \underline{\Delta D}^{er}\underline{v}^r$, $\underline{\Delta i}^s = \underline{D}^{se}\underline{\Delta i}^e$, $\underline{\Delta i}^r = \underline{D}^{rs}\underline{\Delta i}^s + \underline{\Delta D}^{rs}\underline{i}^s$, ... etc.

The analysis is extended for the transformation process of the voltage/current relation between two different reference frames. This can simply be observed such as, for example, the relation in the r-frame is transformed to e-frame as follows:

$$\underline{v}^r_{dq} = \underline{D}^{re}\underline{v}^e_{dq} = [\underline{R} + \underline{L}p + \underline{G}_s\omega]\underline{D}^{re}\underline{i}^e_{dq}$$

but $\underline{D}^{re} = \underline{I}$, then $\underline{v}^e_{dq} = [\underline{R} + \underline{L}p + \underline{G}_s\omega]\underline{i}^e_{dq}$

Similarly, for the s-frame, we have:

$$\underline{\Delta v}^s_{dq} = \underline{D}^{rsT}[\underline{R} + \underline{L}p + \underline{G}_s\omega]\underline{D}^{rs}\underline{\Delta i}^s_{dq}$$
$$= [\underline{R}^s + \underline{L}^s p + \underline{G}^s_s\omega]\underline{\Delta i}^s_{dq}$$

It seems from the above that the coefficients in both r and e frames are constant (independent of time), but they are time-variant in the s-frame. This is because of the saliency factor and due to the nonzero velocity between r and s frames. Some elements in each matrix of $\underline{R}^s$, $\underline{L}^s$ and $\underline{G}^s_s$ being periodic in time, where

$$\underline{R}^s = \underline{D}^{sr}\underline{R}\underline{D}^{rs}$$

In this case, therefore, equivalent circuit in s-frame cannot simply be developed, since the frame is non-autonomous, hard to investigate, and its analysis does not pursued in this paper.

The f and b components of the complete perturbation currents in r-frame can be expressed in terms of the true currents ($\underline{\Delta i}^s_{fbs}$, $\underline{\Delta i}^r_{fbr}$) by the following analysis:

$$\underline{\Delta i}^r_{dqs} = \underline{D}^{rs}\underline{\Delta i}^s_{dqs} + \underline{\Delta D}^{rs}\underline{D}^{se}\underline{i}^e_{dqs}$$
$$\underline{K\Delta i}^r_{fbs} = \underline{D}^{rs}\underline{K\Delta i}^s_{fbs} + \underline{\Delta D}^{re}\underline{i}^e_{dqs}, \text{ where}$$
$$\underline{\Delta D}^{re} = \underline{\Delta D}^{rs}\underline{D}^{se}, \quad \underline{i}^e_{dqs} = \underline{i}^r_{dqs} \tag{4}$$
$$\therefore \underline{\Delta i}^r_{fbs} = \underline{K}^{-1}\underline{D}^{rs}\underline{K\Delta i}^s_{fbs} + \underline{K}^{-1}\underline{\Delta D}^{re}\underline{i}^r_{dqs}$$

Similarly, the f and b components of perturbation currents in e-frame are expressed as follows:

$$\underline{\Delta i}^e_{dqs} = \underline{D}^{es}\underline{\Delta i}^s_{dqs}, \text{ where } \underline{\Delta D}^{es} = 0, \underline{D}^{es} = \underline{D}^{rs}$$
$$\therefore \underline{\Delta i}^e_{fbs} = \underline{K}^{-1}\underline{D}^{rs}\underline{K\Delta i}^s_{fbs} \tag{5a}$$

$$\underline{\Delta i}^e_{dqr} = \underline{D}^{er}\underline{\Delta i}^r_{dqr} + \underline{\Delta D}^{er}\underline{i}^r_{dqr}, \text{ where}$$
$$\underline{D}^{er} = \underline{I}, \; \underline{i}^r_{dqr} = (i_{kd}, i_{kq}) = 0 \tag{5b}$$
$$\therefore \underline{\Delta i}^e_{fbr} = \underline{\Delta i}^r_{fbr}$$

from (4) and (5), it can be proved that perturbation currents in e-frame are true currents and those in r-frame are as follows:

$$\underline{\Delta i}^r_{fb} = \Delta i^e_{fbu} + \underline{K}^{-1}\underline{\Delta D}^{re}\underline{i}^r_{dq}$$

4 EQUIVALENT CIRCUIT FOR TRUE VARIABLES IN e-frame

Substituting for the complete currents of (6) into equation (3), the left hand side of (3) remains the same in e-frame, but the right hand side will be as follows:

$$A\underline{K}^{-1}\left(\begin{bmatrix} v\cos\delta \\ v\sin\delta \\ 0 \\ 0 \end{bmatrix}\frac{\sin\beta t}{\beta} + \begin{bmatrix} L_q i^r_{qs} \\ -L_d i^r_{ds} \\ 0 \\ 0 \end{bmatrix}\cos\beta t\right)$$
$$-\underline{K}^{-1}[\underline{R} + \underline{L}p + \underline{G}_s\omega]\underline{\Delta D}^{re}\underline{i}^r_{dq}$$

which can be rearranged by considering the steady-state voltage equations: $v\cos\delta = R_s i^r_{qs} + L_d\omega i^r_{ds}$ and $-v\sin\delta = R_s i^r_{ds} - L_q\omega i^r_{qs}$ and assuming that the differential operator p operates normally on $\sin\beta t$. Therefore the right hand side is:

$$-\frac{A}{\sqrt{2}\,\beta}\begin{bmatrix}(L_d-L_q)(i^r_{qs}\cos\beta t - i^r_{ds}\sin\beta t)(\beta+\omega)\\ (L_d-L_q)(i^r_{qs}\cos\beta t + i^r_{ds}\sin\beta t)(\beta-\omega)\\ (M_d i^r_{qs}\cos\beta t + M_q i_{ds}\sin\beta t)\,\beta\\ (M_d i^r_{qs}\cos\beta t - M_q i^r_{ds}\sin\beta t)\,\beta\end{bmatrix}$$

Multiplying the first row by $\beta/(\beta+w)$ (similarly as in equation (3) and the network of Figure (1)) and the second by $\beta/(\beta-w)$, we derive the circuit of true variables in e-frame, Figure (2).

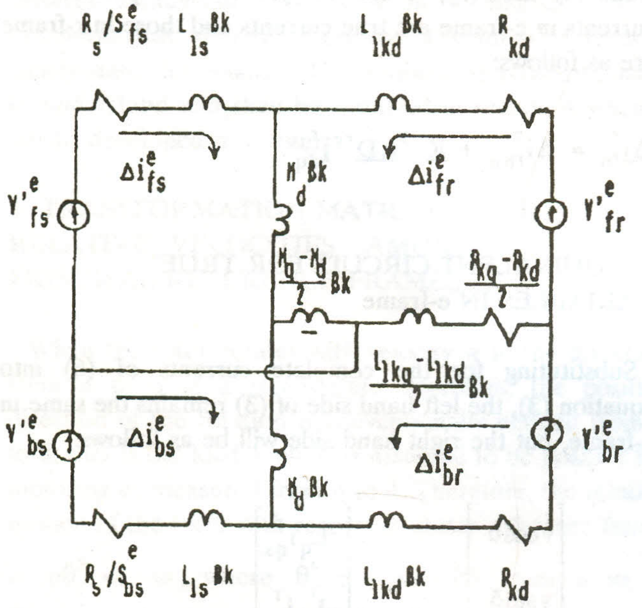

Figure 2. Perturbation network for reluctance motor in e-frame.

The f and b driving voltages are:

$$V'^e_{fs} = -\frac{A}{\sqrt{2}}(L_d-L_q)\,i_s\cos(\beta t+\varepsilon^e_s)$$

$$V'^e_{bs} = -\frac{A}{\sqrt{2}}(L_d-L_q)\,i_s\cos(\beta t-\varepsilon^e_s)$$

$$V'^e_{fr} = -\frac{A}{\sqrt{2}}\psi_{rs}\cos(\beta t-\alpha^e_{rs})$$

$$V'^e_{br} = -\frac{A}{\sqrt{2}}\psi_{rs}\cos(\beta t+\alpha^e_{rs})$$

where

$$i_s = \sqrt{i^{e2}_{ds} + i^{e2}_{qs}}$$

$$\psi_{rs} = \sqrt{(M_d i^e_{qs})^2 + (M_q i^e_{ds})^2}$$

$$\varepsilon^e_s = \tan^{-1}\left(\frac{i^e_{ds}}{i^e_{qs}}\right)$$

$$\alpha^e_{rs} = \tan^{-1}\left(\frac{M_d i^e_{ds}}{M_d i^e_{qs}}\right)$$

In both Figures (1,2), the negative inductors are depicted as capacitors (following [1]). However, this is not good symbolism, since the modulus values of these impedances are proportional to β, whereas a capacitor would of course show inverse proportionality. The circuit of true variables· Figure (2) is somewhat similar to Figure (1) except the driving voltage are on both sides of the circuit with four different sources, two for forward and two for backward. In these effective voltages, terms of mutual flux only are involved, there is no term involving v, as it occur in the sourses of complete variables. Furtheremore, the powers demanded at the shaft can be calculated from the synchronous powers which represented by the sum of Δi^2R in the true circuit, just as the case of induction motor [10], which will be discussed in section 5. We also note that the equivalent circuit of true variables Figure (2) is equally the equivalent circuits of induction motor observed in [10] under the conditions of $S = 0$, $\delta = 0$ and non-saliency.

5 TORQUE COMPONENTS AND POWERS IN THE CIRCUIT

5.1 *Torque components from torque equation*

The electrical perturbation torque expression on the rotor (= - the torque on stator) of the reluctance motor is, in the general matrix form [7] in r-frame, as follows:

$$T^r_s = i^{rT}_{dq}(G_s + G^T_s)i^r_{dq} = -T^r_r$$

$$\text{where } G_s + G^T_s = \begin{bmatrix} & L_d-L_q & & -M_q \\ L_d-L_q & & M_d & \\ & M_d & & \\ -M_q & & & \end{bmatrix} \quad (7)$$

$$\underline{\Delta i}_{dq} = \underline{K\Delta i}^r_{fb}$$

Expression (7) is divided into two separate terms by the transformation matrix $\underline{K}$, one for forward and one for backward, as follows:

$$\Delta T^r_{fs} = -\Delta T^r_{fr}$$
$$= \frac{1}{\sqrt{2}}(L_d-L_q)[i^r_{qs}\Delta i^r_{fs} - i^r_{ds}k\Delta i^r_{fs}] + M_d i^r_{qs}\Delta i^r_{fr} + M_q i^r_{ds}k\Delta i^r_{fr}$$
$$\Delta T^r_{bs} = -\Delta T^r_{br}$$
$$= \frac{1}{\sqrt{2}}(L_d-L_q)[i^r_{qs}\Delta i^r_{bs} + i^r_{ds}k\Delta i^r_{bs}] + M_d i^r_{qs}\Delta i^r_{br} - M_q i^r_{ds}k\Delta i^r_{br} \quad (8)$$

(Note that Equations (8) are also available in the same form for e-frame.)

To obtain expressions for the damping and synchronising torque coefficients for f and b components, we algebraically solve equation (3) for RM in r-frame and obtaining the perturbation f and b currents, then substituting these into (8), in a similar way as induction motor [10], to get the torque coefficients, as follows:

$$\Delta T^r_{fr} = -\Delta T^r_{fs} = A(T^r_{Sf}\frac{\sin\beta t}{\beta} + T^r_{Df}\cos\beta t)$$

$$\Delta T^r_{br} = -\Delta T^r_{bs} = A(T^r_{Sb}\frac{\sin\beta t}{\beta} + T^r_{Db}\cos\beta t)$$

The coefficients T^r_{Sf}, T^r_{Df}, T^r_{Sb} and T^r_{Db} are obtained in terms of machine parameters, but their expressions are not written here in this paper because of lenghthy and complexity. Figures (3) show the computed results of the f, b components of these coefficients in the complete r-frame, which will be discussed in section 6.

To obtain expressions for torque coefficients of f and b components in e-frame, we substitute for complete currents in (8) from (6) in terms of the true currents in e-frame. The forms of the forward and backward components of torque in this frame are:

$$\Delta T^e_{fru} = T^r_{Sf}A\frac{\sin\beta t}{\beta} + T^r_{Df}A\cos\beta t - \frac{T_s}{\beta}A\cos\beta t$$
$$- \frac{A}{2\beta}(L_d-L_q)(i^{r2}_{ds} - i^{r2}_{qs})\sin\beta t$$
$$= T^e_{Sfu}A\frac{\sin\beta t}{\beta} + T^e_{Dfu}A\cos\beta t$$

$$\Delta T^e_{bru} = T^r_{Sb}A\frac{\sin\beta t}{\beta} + T^r_{Db}A\cos\beta t + \frac{T_s}{\beta}A\cos\beta t$$
$$- \frac{A}{2\beta}(L_d-L_q)(i^{r2}_{ds} - i^{r2}_{qs})\sin\beta t$$
$$= T^e_{Sbu}A\frac{\sin\beta t}{\beta} + T^e_{Dbu}A\cos\beta t$$

where:

$$T^e_{Sfu} = T^r_{Sf} - \frac{1}{2}(L_d-L_q)(i^{r2}_{ds} - i^{r2}_{qs})$$
$$T^e_{Sbu} = T^r_{Sb} - \frac{1}{2}(L_d-L_q)(i^{r2}_{ds} - i^{r2}_{qs})$$
$$T^e_{Dfu} = T^r_{Df} - \frac{T_s}{\beta} \quad (9)$$
$$T^e_{Dbu} = T^r_{Db} + \frac{T_s}{\beta}$$
$$T_s = (L_d-L_q)i^r_{ds}i^r_{qs}$$
= steady-state torque

The coefficients are:

$$T^e_{Su} = T^r_S - (L_d-L_q)(i^{r2}_{ds} - i^{r2}_{qs})$$
$$T^e_{Du} = T^r_D$$

It is interesting to see from (9) that the true f, b components of T_D, T_S have the same behaviour to those obtained for induction motor [10]. They are different from the corresponding coefficients of complete components by a similar expressions as induction machine. The physical meaning of that is: in the r-frame, the steady-state stator currents produce apparent currents which interact with the salient-pole shape of the rotor to produce apparent torque components as seen in that frame. But in the e-frame, there are no apparent stator currents to interact with rotor saliency, and so no apparent torque components are produced. As with the induction motor, T_{Sfu} and T_{Sbu} do not include a component of T_S due to steady-state currents only, which must be added separately, as equations (9) show.

In summary, T_S is invariant in any reference frame, as for the induction motor. But for the reluctance motor it happens that the e-frame yields perturbation torque variables that are true (because of the absence of steady-state rotor current). Consequently, in e-frame, T_S is found to be the sum of T^e_{Sfu} plus T^e_{Sbu} which are the true components of T_S. In the r-frame T^r_{Sf}, T^r_{Sb} (which appear in (9) and calculated from (8)) are physically meaningless, since they involve complete perturbation currents that contain apparent terms. There is thus a close comparison between the situations in induction motor and reluctance motor for T_S, and similarly for T_D.

5.2 *Relation between torque and powers in the circuit of true variables*

The important results between the power and torque which were observed for the induction motor [10] are reflected here in case of the reluctance machine. It can be demonstrated that the f, b components of T_D and T_S are directly related, respectively, to the time-averaged power and reactive power in the circuit of true variables in e-frame.

Thus, by considering the forward part of the network Figure (2), the currents are given by:

$$\Delta i^e_{fsu} = \Delta \hat{i}^e_{fsu} \sin(\varepsilon^e_{fsu} - \beta t),$$
$$\Delta i^e_{fru} = \Delta \hat{i}^e_{fru} \sin(\varepsilon^e_{fru} - \beta t)$$

It is immediately seen that the time-averaged power is:

$$\frac{A}{2\sqrt{2}}[(L_d - L_q) i_s \Delta \hat{i}^e_{fsu} \sin(\varepsilon^e_{fsu} + \varepsilon^e_s) + \psi_{rs} \Delta \hat{i}^e_{fru} \sin(\varepsilon^e_{fru} - \alpha^e_{rs})]$$

and the reactive power is:

$$\frac{A}{2\sqrt{2}}[(L_d - L_q) i_s \Delta \hat{i}^e_{fsu} \cos(\varepsilon^e_{fsu} + \varepsilon^e_s) + \psi_{rs} \Delta \hat{i}^e_{fru} \cos(\varepsilon^e_{fru} - \alpha^e_{rs})]$$

These are related to T_{Df} and T_{Sf} respectively, which are obtained from (8) by the substitution of currents Δi^e_{fsu} and Δi^e_{fru}; in which the same expressions of (8) are available in e-frame; and therefore, the time-averaged power = $A^2 T^e_{Dfu}/2$ and the reactive power = $A^2 T^e_{Sfu}/2\beta$. Similarly for the backward part of the network.

These results can be seen to follow immediately, that each component of T_{Du} relates to the sum of the perturbation i^2R losses in the corresponding part of the circuit, while each component of T_{Su} relates to the sum of the perturbation i^2X in the corresponding part of the circuit However, in the circuit of true variables in e-frame the cros-coupled saliency branch between the forward part and the backward part gives a further component of the coefficients T_{Du}, T_{Su}. This component involves both forward and backward rotor perturbation currents, which produced due to saliency. The total coefficient of T_{Du} is therefore, the sum of forward T_{Dfu} plus the backward T_{Dbu} plus the saliency component T_{Dfb}. Similarly for $T_{Su} = T_{Sfu} + T_{Sbu} + T_{Sfb}$

The power losses in the circuit of Fig(2) is:

$$\Delta \hat{i}^{e2}_{fsu} \frac{R_s \beta}{\beta + \omega} + \Delta \hat{i}^{e2}_{fru} R_{kd} + \Delta \hat{i}^{e2}_{bsu} \frac{R_s \beta}{\beta - \omega} + \Delta \hat{i}^{e2}_{bru} R_{kd} + (\Delta \hat{i}^e_{fru} - \Delta \hat{i}^e_{bru})^2 \frac{R_{kd} - R_{kq}}{2}$$

$$= \Delta \hat{i}^{e2}_{fsu} \frac{R_s \beta}{\beta - \omega} + \Delta \hat{i}^{e2}_{fru} \frac{R_{kd} - R_{kq}}{2} + \Delta \hat{i}^{e2}_{bsu} \frac{R_s \beta}{\beta - \omega} + \Delta \hat{i}^{e2}_{bru} \frac{R_{kd} - R_{kq}}{2} + \Delta \hat{i}^e_{fru} \Delta \hat{i}^e_{bru} (R_{kd} - R_{kq})$$

The components of T_{Du} are:

$$T^e_{Dfu} = \frac{2}{A^2}\left(\Delta\hat{i}^{e2}_{fsu}\frac{R_s\beta}{\beta+\omega} + \Delta\hat{i}^{e2}_{fru}\frac{R_{kd}+R_{kq}}{2}\right)$$

$$T^e_{Dbu} = \frac{2}{A^2}\left(\Delta\hat{i}^{e2}_{bsu}\frac{R_s\beta}{\beta-\omega} + \Delta\hat{i}^{e2}_{bru}\frac{R_{kd}+R_{kq}}{2}\right) \quad (10)$$

$$T^e_{Dfb} = \frac{2}{A^2}\Delta\hat{i}^{e}_{fru}\Delta\hat{i}^{e}_{bru}(R_{kd}-R_{kq})$$

Similarly for T_{Su}.

The results of (10) show that f and b components of T_D and T_S whose obtained from the complete variables Eqn(8) or from the variables in e-frame Eqn(9) are mathematically correct but physically ambiguous, since the complete variables involve apparent components of torque which is not true, and the saliency effect plays a rule for producing an independent component of torque, specially for the reluctance motor. The f and b components of torque coefficients must now obtained from the real and reactive powers associated in the circuit of true variables in the e-frame. This new result seems to be applied as for the induction machine [10] that each resistive term in the f, b circuit may be precisely related to a separate secondary circuit of a parasitic reluctance machine. There are now five parasitic machines, four are similar to those observed for the induction machine [10] plus one further is particularly generated by the saliency coupling between the two waves. Again, the perturbed shaft oscillation is the mechanically primery member and all the five parasitic machines which represented by the equivalent circuit are the secondary members.

Figures (4) illustrate the three true components of T_D, whose obtained from the power losses in the circuit of Figure (2). The detailed expression of (10) in terms of machine parameters are not quoted here because of comlexity and lenghthy. Computed results are given by Figures (3,4) for f, b components, using a program made by the author.

6 EXAMPLES OF APPLICATIONS

In this section, we discuss some examples of computed torque coefficients and components for specific machines, as functions of perturbation frequency. In Figs(3,4) the vertical axis represents components or coefficient magnitudes in per-unit, while ß is plotted horizontally. The comments offered below are for the machine as represented, at its specified working frequency w and specified load, where the characteristics are plotted to a ß-value significantly exceeding w. The characteristics applies to all the figures are: (—) for T_D(a) or T_S(b), (–-) for T_{Df}(a) or T_{Sf}(b), (-.-) for T_{Db}(a) or T_{Sb}(b), and (---) for T_{Dfb}(a).

In all Figures the machine parameters are in per-unit, specified as follows

stator resistance	R_s	= 0.045
rotor resistances	R_{kd}	= 0.03, R_{kq} = 0.015
leakage inductances	L_{ls}	= L_{lkd} = 0.1, L_{lkq} = 0.08
mutual inductances	M_d	= 2.0, M_q = 0.5
moment of inertia	J	= 77

The operating condition is at $v = \omega = 0.5$, and $\delta = 0$ for Figures (3a1,3b1,4a1,4b1) and is at $v = \omega = 0.8$ and $\delta = 30°$ for the figures (3a2,3b2,4a2,4b2).

Figure (3a1)

This machine works at zero load angle and half rated frequency and voltage. The complete components of T_D in r-frame are shown to be incorrect in their magnitude; however, their behaviour demonstrate that T_{Df} is positive over all ß-range and T_{Db} exhibts negative range just below ß=w which is in keeping with the behaviour of the bs parasitic machine

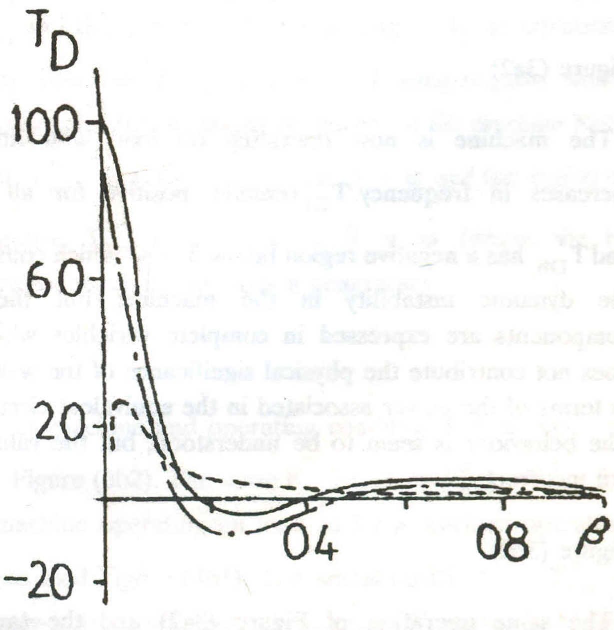

Figure 3a1. Complete components of T_D.

Figure (3b1)

The machine and operating condition are the same as for Fig(3a1). The complete components of T_S in r-frame are physically meaningless. The steady-state component of T_S is absorbed (in two equal halves) in T_{Sf} and T_{Sb} respectively, so that $T_S = T_{Sf} + T_{Sb}$ and the only coefficient having physical significance is T_S.

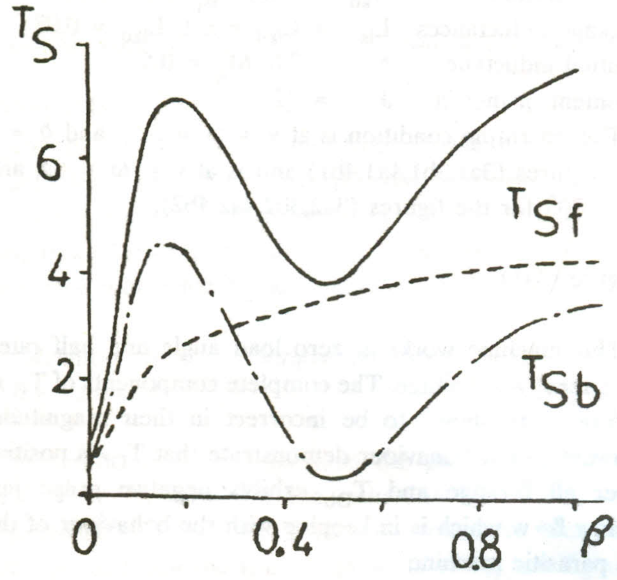

Figure 3b1. Complete components of T_s.

Figure (3a2)

The machine is now operating on load with little increases in frequency. T^r_{Df} remains positive for all ß and T^r_{Db} has a negative region below ß = ω which couses the dynamic instability in the machine, but these components are expressed in complete variables which does not contribute the physical significance of the waves in terms of the power associated in the equivalent circuit. The behaviour is seem to be understood, but the values are incorrect.

Figure (3b2)

The same operation of Figure (3a2) and the same characteristics of T^r_{Sf}, T^r_{Sb} components of Figure (3b1).

$T^r_S = T^r_{Sf} + T^r_{Sb}$ is the synchronising torque coefficient which causes the static instability. This may be occur when T^r_S is negative at ß = 0 [11].

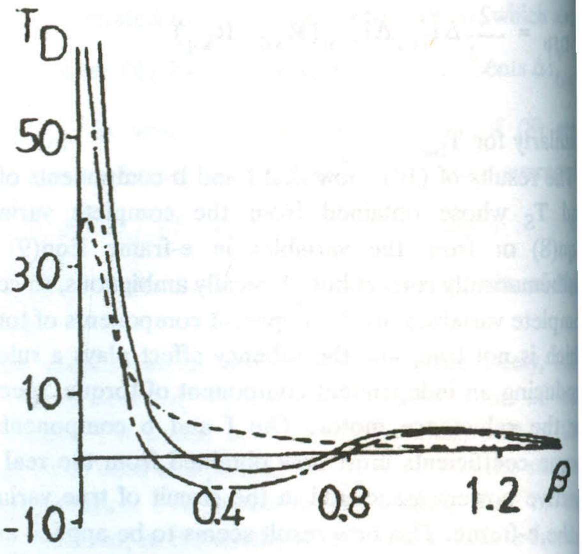

Figure 3a2. Complete components of T_D.

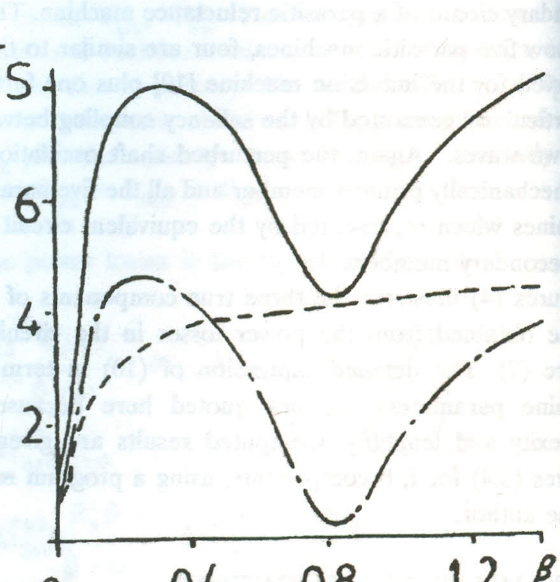

Figure 3b2. Complete components of T_s.

Figure (4a1)

The machine and operating condition are the same for Figure (3a1). T^e_{Dfu} and T^e_{Dbu} are the true f, b components of T_D in e-frame which calculated from the power losses in the equivalent circuit of true variables in e-frame Figure (2). T^e_{Dfu} is positive as well as the saliency component T_{Dfb} over all ß-range, while T^e_{Dbu} exhibts negative region below $ß = \omega$. These components are correct and physically significance.

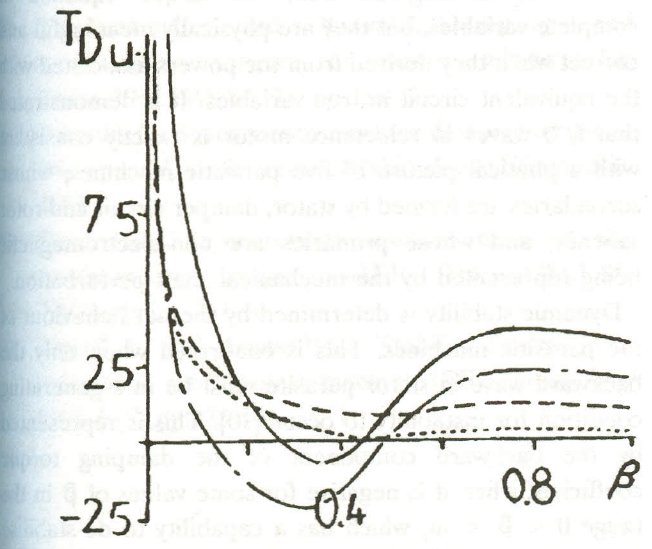

Figure 4a1. True components of T_{Du}.

Figure (4b1)

The same machine of Fig(3b1). The steady-state component of T_S disappeared from the true coefficient T_{Su}, which seems to be negative at $ß = 0$, while the machine is statically stable since $\delta = 0$ [11]. The reason is that T^r_S is the correct coefficient shown in Fig(3b1), where $T^r_S = T_{Su}$ + a steady-state component, but T^e_{Sfu} and T^e_{Sbu} are the true components which calculated from i^2X in the circuit, which when the steady-state component is added, it contributes T_S in the correct manner. Note that, $T_S = T_{Sfu} + T_{Sbu} + T_{steady\text{-}state} = T^r_{Sf} + T^r_{Sb}$ and $T^e_{Su} = T^e_{Sfu} + T^e_{Sbu}$.

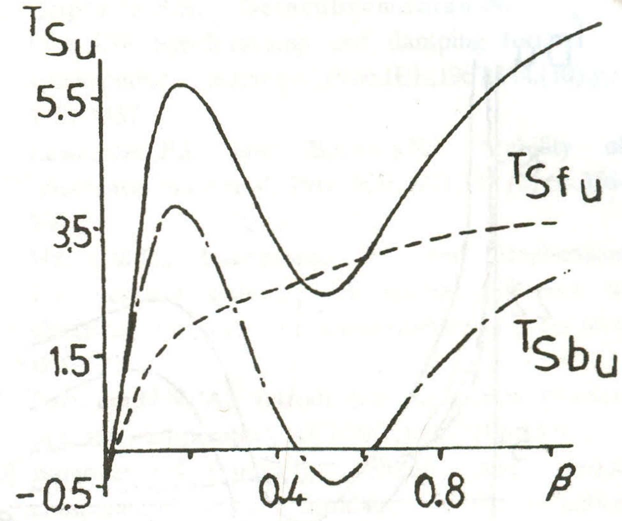

Figure 4b1. True components of T_{su}.

Fig(4a2)

The machine and operating condition are the
Figure (3a1) Complete components of (T_D - β) curves
Figure (4a1) True components of (T_{Du} - β) curves
Figure (3a2) Complete components of (T_D - β) curves
Figure (4a2) True components of (T_{Du} - β) curves

same as for Figure (3a2). T^e_{Dfu} is always positive, T^e_{Dfb} is positive also in all cases over all ß-range, since $R_{kd} > R_{kq}$ and the f, b currents are in magnitude, as equations (10) show, but T^e_{Dbu} is capable of being negative which accounts for the instability properties of the machine Note that T^e_{Dbu} is always positive for $\beta > \omega$ and the region of negative T^e_{Dbu} occurs below $\beta = \omega$ (where the bs parasitic machine becomes a generator).

Figure (4b2)

This machine and operating condition are the same as for Figure (3b2). The same behaviour of T^e_{Sfu}, T^e_{Sbu} for a machine operating on load as for a machine operating at no load Figure (4b1). The actual coefficient is T^e_{Su} + steady-state component, which equals T_S shown in Figure (3b2).

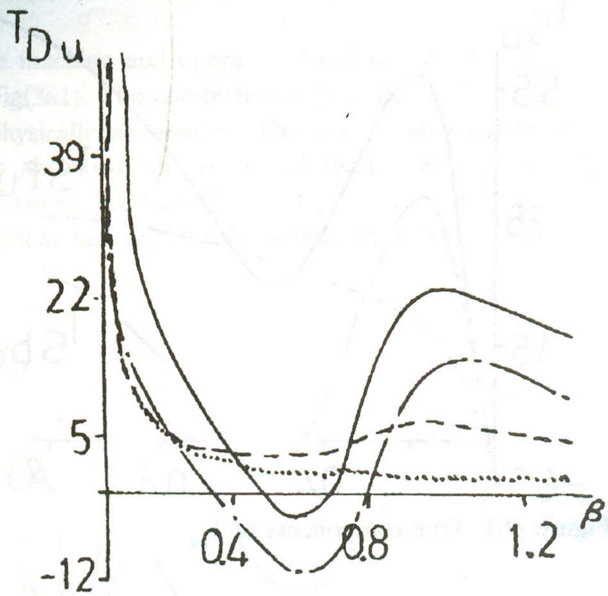

Figure 4a2. True components of T_{Du}.

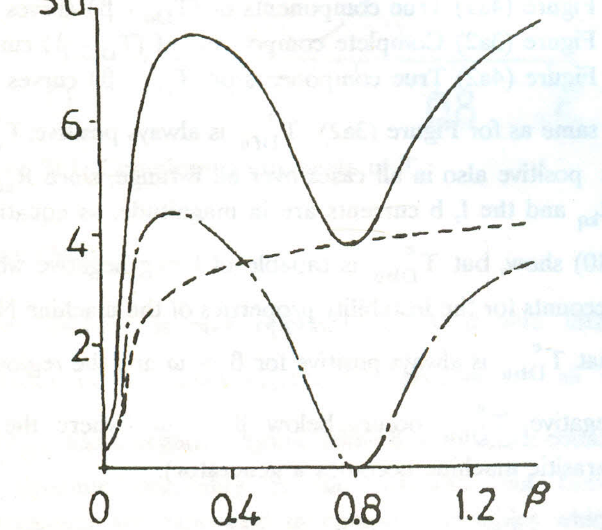

Figure 4b2. True components T_{Su}.

7 CONCLUSION

Equivalent circuits for f and b waves for the reluctance synchronous machine are developed. These are derived in both complete variables in r-frame and true variables in e-frame In r-frame the voltage source occurs in the stator side, while in e-frame the driving voltage occurs on both stator and rotor sides; typical behaviour for f and b waves like induction motor [10] is investigated. Circuit in tru variables discusses physical behaviour of f and b waves which is correct in all respects, while circuit

Fig(3b1) Complete components of (T_S - β) curves
Fig(4b1) True components of (T_{Su} - β) curves
Fig(3b2) Complete components of (T_S - β) curves
Fig(4b2) True components of (T_{Su} - β) curves

in complete variables fails in all respects. The f and b components of the torque coefficients are derived and computed. It is shown that they are physically ambiguous when they investigated from the torque equation in complete variables, but they are physically meaningful and correct when they derived from the powers associated with the equivalent circuit in true variables. It is demonstrated that f, b waves in reluctance motor is exactly consistent with a physical picture of five parasitic machines, whose secondaries are formed by stator, damper circuit and rotor saliency, and whose primaries are non-electromagnetic being represented by the mechanical shaft perturbation.

Dynamic stability is determined by the net behaviour of the parasitic machines. This is confirmed when only the backward wave in stator parasite must be in a generating condition for instability to occur [10]. This is represented by the backward component of the damping torque coefficient when it is negative for some values of β in the range 0 < β < ω, which has a capability to de-stabilise the machine. While the forward component of the same coefficient can never cause instability, because it is positive for all possible values of β in the range 0 < β < ∞, a the analytical expresions indicated and confirmed by th results. Static stability is determined by the total reactiv power in the circuit at steady-state condition; i.e. when th synchronising torque coefficient at β = 0 is negative, th instability occurs [11].

It might be important to investigate the characteristics o the f and b waves for the excited synchronous motor. Th work is left for future, because of that problem (introduced in the introduction) which associated with th field components in e-frame. The 2-axis model of tha machine, with one shorted coil in rotor axis, may becom 8^{th} order in e-frame, while it is 7^{th} order in r-frame. Th problem, therefore, should be carefully studied.

On the other hand, it may be useful to investigate th characteristics of the f and b components of T_D and T_S the machine in time domain. This may show how far th stability margin, represented by the backward wave, fro the load condition and speed indication for cotributi instability. This can be measured in a range of sensible values of machine parameters at a particular operating condition. However, it should be noted that the time characteristics depend on the total coefficients T_D and T_S plus inertial torque; but not the f and b components.

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