CONDENSATION OF CONTROL POINTS IN SURVEYING

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ABSTRACT

This paper gives directly a straight forward solution for the problem of condensation and extension of surveying control points which are performed by distance measurements. Traditionally due to practical region and time limitations beside the invention of EDM and progressing in their techniques most of new stations in processes of condensation and extension are performed according to these new techniques. This paper offers also some useful formulae for computation in the field of the fixation of stations process which are considered to be very important operations in surveying. As many cases of condensation for new points are presented and throughly discussed.

1. INTRODUCTION

The fixation of stations process is an important operation in the field of surveying.

Moreover to determine the coordinates of points linear or angular observations are needed and these observations or measurements must be adjusted to obtain the probable values of the coordinates of such stations. Recent progress in accuracy standard of the distance measuring equipment will be more emphasized on this type of measurement than direction measurements. In this paper the linear measurements will be used in determining and fixing new position for a number of new points as condensation or extension for a certain system of stations so it is important and necessary to study the resulting accuracy of these stations in its new forms and methods.

. CASES OF CONDENSATION OF THE NET

2.1 *Calculation of the coordinates of a set of between two fixed control points*

As depicted in Figure (1), A and B are two points where their coordinates X_A, Y_A and X_B, Y_B are known. Assume a set of new points 1,2,3,..., and n are to be located on the straight line between A and B. Distances between each pair of points are measured (s_1, s_2, ..., and s_{n+1}) as shown in the same Figure.

$$\text{If } S = s_1 + s_2 + ... + s_{n+1} = [s_i] = \sum_{i=1}^{n+1} s,$$

the summation of these distances will very from the distances calculated from the known coordinates of A and B i.e.,

$$S = s_1 + s_2 + ... + s_{n+1} = \sum_{i=1}^{n+1} s \neq D \quad (1)$$

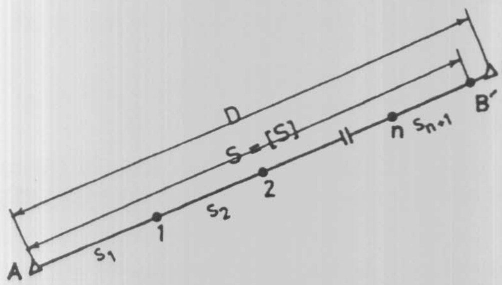

Figure 1.

where:

$$D = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2} \quad (2)$$

Assuming V_i is the correcting vector to each distance s_i, in this case Equation 1 can be written as:

$$s_1 + v_1 + s_2 + v_2 + ... + s_{n+1} + v_{n+1} = D \quad (3)$$

The conditional equation will be in the form:

$$v_1 + v_2 + ... + v_{n+1} + w = 0 \quad (4)$$

besides, the absolute value of w will be:

$$w = S - D$$

where S and D are as stated in Equations 1 and 2.

The vector of correction v_i can be calculated according to (Lazaref and Camyshken), (1980) as follows:

$$v_i = \frac{f_s}{[\frac{1}{p}]} \cdot \frac{1}{p_i} \qquad (5)$$

where,

f_s = the variation between the measured distance and the calculated one (S and D respectively)

f_s = - w = D -S (6)

p_i = the weight which is equal to:

$$p_i = \frac{1}{s_i} \qquad (7)$$

substituting from (7) to (5) we get:

$$v_i = \frac{f_s}{[s]} . s_i = \frac{f_s}{S} . s_i \qquad (8)$$

From (6) and (8) the last one will be rewritten as:

$$v_i = \frac{D - S}{S} . s_i = \frac{D}{S} . s_i - s_i \qquad (9)$$

Then, the formula for vector of correction for any distance will be:

$$s_i' = s_i + v_i \qquad (10)$$

or

$$s_i' = \frac{D}{S} . s_i \qquad (11)$$

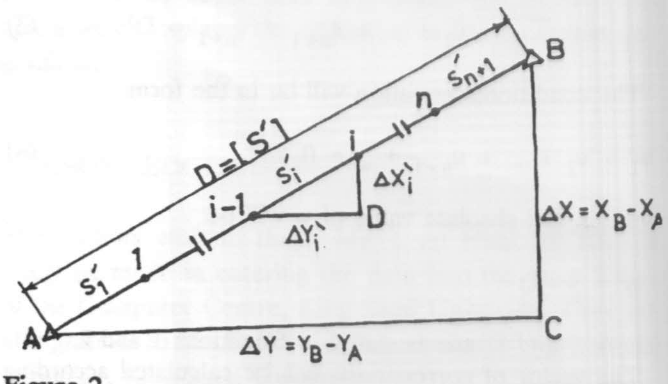

Figure 2.

From similarity in Figure (2) and analogously we obtain:

$$\left.\begin{aligned} \frac{\Delta x_i'}{s_i'} &= \frac{\Delta x}{D} \\ \frac{\Delta y_i'}{s_i'} &= \frac{\Delta y}{D} \end{aligned}\right\} \qquad (12)$$

which means that $\Delta x_i'$ and $\Delta y_i'$ can be calculated as follows:

$$\Delta x_i' = \frac{\Delta x}{D} . s_i' = \frac{\Delta x}{D} . \frac{D}{S} s_i$$

$$= \frac{\Delta x}{S} . s_i = a . s_i \qquad (13)$$

and

$$\Delta y_i' = \frac{\Delta y}{D} . s_i' = \frac{\Delta y}{D} . \frac{D}{S} . s_i$$

$$= \frac{\Delta y}{S} . s_i = b . s_i \qquad (14)$$

where,

$$\left.\begin{aligned} a &= \frac{\Delta x}{S} = \frac{x_B - x_A}{S}, \\ b &= \frac{\Delta y}{S} = \frac{y_B - y_A}{S} \end{aligned}\right\} \qquad (15)$$

Consequently from Equations (13) and (14), the variati[on] of coordinates of any point can be calculated b[y] multiplying factors a and by the measured distances.

Moreover, factors a and b can be checked as follows:

$$a^2 + b^2 = \frac{\Delta x^2 + \Delta y^2}{S^2} = \frac{D^2}{S^2} \approx 1 \qquad (1$$

The final adjusted coordinates for any new points can expressed as:

$$\left.\begin{aligned} x_i &= x_{i-1} + \Delta x_i' \\ y_i &= y_{i-1} + \Delta y_i' \end{aligned}\right\} \qquad ($$

2.2 *The New Points which are lying on the Extension of Line AB*

In this case Equation 17 can be easily used to obtain coordinates of any new points.

2.3 *The New Points are Perpendicular to Line AB*

As stated before coordinates of point A and point B are known, and from Figure (3) coordinates of point 2 will be:

$$\left.\begin{aligned} x_2 &= x_A + \Delta x_1 - \Delta x_2' \\ y_2 &= y_A + \Delta y_1 + \Delta y_2' \end{aligned}\right\} \qquad (18)$$

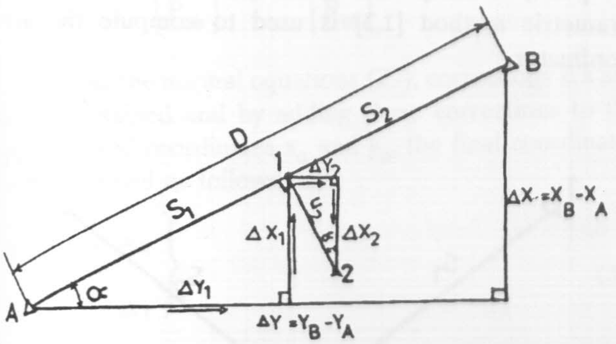

Figure 3.

According to Equations (13) and (14), the variation of coordinates for line A-1 are:

$$\Delta x_1 = a\ s_1 \text{ and } \Delta y_1 = b\ s_1 \qquad (19)$$

Also, the variation of coordinates for line 1-2 can be computed in a similar form as follows:

$$\frac{\Delta x_2}{U_2} = \frac{\Delta y}{D} = \frac{\Delta Y}{S} = b,$$

$$\frac{\Delta y_2}{U_2} = \frac{\Delta x}{D} = \frac{\Delta x}{S} = a,$$

this means:

$$\left.\begin{aligned} \Delta x_2 &= b\ U_2 \\ \Delta y_2 &= a\ U_2 \end{aligned}\right\} \qquad (20)$$

substituting from Equations (19) and (20) to equation (18) we obtain:

$$\left.\begin{aligned} x_2 &= x_A + a\ s_1 - b\ U_2 \\ y_2 &= y_A + b\ s_1 + a\ U_2 \end{aligned}\right\} \qquad (21)$$

Equation (21) can be obtained if point 2 is on the right hand side of the line AB and variation of coordinates calculated from point A to point B, in this case $U_i > 0$ and U_i will be positive in Equation (21).

If point 2 lies on the left hand side of line AB, then, $U_i < 0$ and U_i is negative and Equation (21) will take thee following formula in a general form:

$$\left.\begin{aligned} x_2 &= x_A + a\ s_1 + b\ U_2 \\ y_2 &= y_A + b\ s_1 - a\ U_2 \end{aligned}\right\} \qquad (22)$$

Figure (4) illustrate how coordinates of points on each side of line AB are calculated

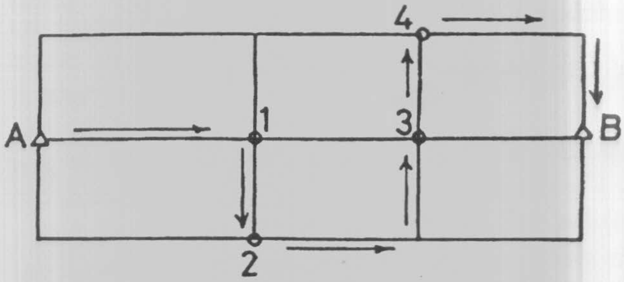

Figure 4.

2.3.1 *General Case*

The method illustrated in (2.3) can be easily used to calculated the coordinates of a group of new points and consequently these new points can be determined in triangulation system.

Assuming the main line in Figure (5) is AB then coordinates of any point can be calculated as follows:

For point (i-1)

$$\left.\begin{aligned} x_{i-1} &= x_A + a\ s_{i-1} - b\ U_{i-1} \\ y_{i-1} &= y_A + b\ s_{i-1} + a\ U_{i-1} \end{aligned}\right\} \qquad (23)$$

and for point i, the general form of coordinates and their amount of correction can be gives as:

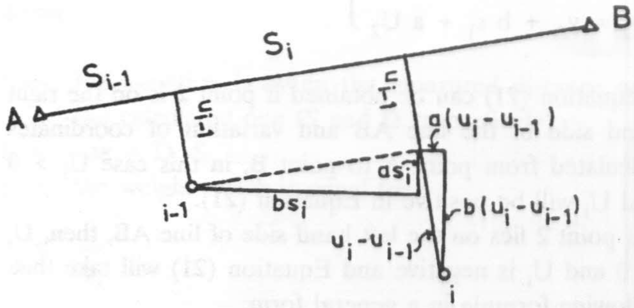

Figure 5.

$$x_i = x_A + a\,(s_{i-1} + s_i) - b\,U_i \qquad (24.a)$$

$$y_i = y_A + b\,(s_{i-1} + s_i) + a\,U_i \qquad (24.b)$$

$$x_i - x_{i-1} = a\,s_i - b\,(U_i - U_{i-1}) \qquad (24.c)$$

$$y_i - y_{i-1} = b\,s_i + a\,(U_i - U_{i-1}) \qquad (24.d)$$

finally, the coordinates of points i are:

$$\left.\begin{aligned} x_i &= x_{i-1} + a\,s_i - b\,(U_i - U_{i-1}),\\ y_i &= y_{i-1} + b\,s_i + a\,(U_i - U_{i-1}) \end{aligned}\right\} \qquad (25)$$

If points i lies on the left hand side of AB then $U_i < 0$.

Figure (6) shows the case when (i-1) is no the right hand side and point i is on the left side of AB, in this the following formula can be used:

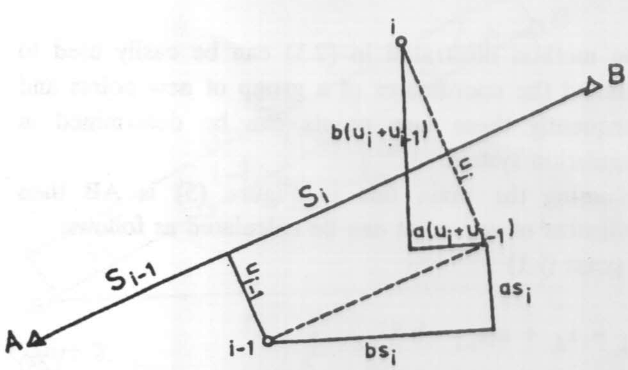

Figure 6.

$$\left.\begin{aligned} x_i &= x_{i-1} + a\,s_i + b\,(U_i + U_{i-1}),\\ y_i &= y_{i-1} + b\,s_i - a\,(U_i + U_{i-1}) \end{aligned}\right\} \qquad (26)$$

2.4 Calculating the Coordinates of a Point by Intersecting Two Points or More

In Figure (7), the coordinates of a new point T can be traditionally calculated by the intersection, if distance from this point to the known stations (1, 2,..., n)are measured.

In this case, it is quite enough to measure two distances from two known stations to obtain the coordinates of the new point T, but practically it is necessary to use the coordinates of the three points and its distances to the new point, so computations can be checked. In this case parametric method [1,3] is used to compute the new coordinates.

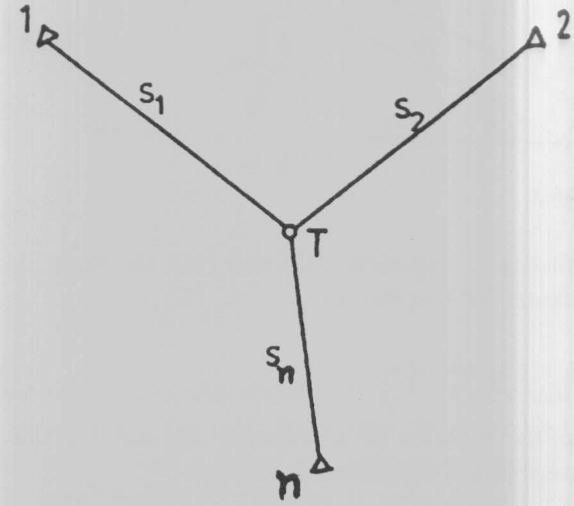

Figure 7.

In parametric method, observation equations, for the measured distances are as follows:

$$v_i = a_i\,\Delta x + b_i\,\Delta y + f_i \quad (i = 1,2,...,n) \qquad (27)$$

where factors a_i and b_i are computed from:

$$a_i = \cos n_i = \frac{x_o - x_i}{s_i^o}, \qquad (28.a)$$

$$b_i = \cos n_i = \frac{y_o - y_i}{s_i^o}, \qquad (28.b)$$

and f_i from:

$$f_i = s^o_i - s_i \qquad (28.c)$$

and

$$s_i^o = \sqrt{(x_o - x_i)^2 + (y_o - y_i)^2}$$

From the set of observation equations (27), the normal equations will be:

$$\begin{bmatrix} P_{aa} \\ P_{ab} \end{bmatrix} \Delta x + \begin{bmatrix} P_{ab} \\ P_{bb} \end{bmatrix} \Delta y + \begin{bmatrix} P_{af} \\ P_{bf} \end{bmatrix} = 0$$

By solving the normal equations (29), corrections Δx and Δy are obtained and by adding these corrections to the approximated coordinates x_o and y_o, the final coordinates are determined as follows:

$$\left.\begin{aligned} x &= x_o + \Delta x \\ y &= y_o + \Delta y \end{aligned}\right\} \qquad (30)$$

The initial (approximate) coordinates x_o and y_o can be obtained from maps or by estimation from the different distances. This last techniques is used for any number of observations and sets when applying the collinearity equations stated before.

3. ESTIMATE OF ACCURACY

The precision of the coordinates of the new point can be evaluated and expressed by mean square error "Priori" as follows:

$$\sigma = \frac{\sqrt{[Pvv]}}{n-1}$$

where σ is the mean square error for unit weight and σ_x and σ_y will be the mean square errors for the coordinates of the new points and computed from:

$$\sigma_x = \sigma\sqrt{Q_{xx}}$$
$$\sigma_y = \sigma\sqrt{Q_{yy}}$$

where,

$$Q_{xx} = \frac{[bb]}{D},$$

$$Q_{yy} = \frac{[aa]}{D}$$

$$D = \begin{bmatrix} [aa] & [ab] \\ [ab] & [bb] \end{bmatrix}$$

$$= [aa]\,[bb] - [ab]^2$$

4. CONCLUSIONS

The determination of new points in triangulations and traverses net is necessary and it is considered as a basis in the subject of survey. The coordinates of a new point can be computed from Equations (17,21,22,25, and 26) if distances are measured either between any two points which their coordinates are known before or on the extension of these points. Points which are perpendicular to the line joining the two points are valid. Equation (30) is used to determine the coordinates of a new point in case of intersection.

At the end, it can be concluded that the coordinates of the new point depend mainly on the coordinates of known points, the distances from the known and new points and the coordinates difference between any two points which their coordinates are known before. And in case of intersection the parametric method is the best way to obtain the coordinates of the new points. the accuracy of the position of the new point and its error ellipse can be computed by the use of matrix algebra.

REFERENCES

[1] Hazay, D.Sc. (Techn.)-Corresponding Member of the Hungarian Acdemy of Sciences-, "Adjusting caculation in surveying", Budapest, 1970.

[2] Lazaref, G.E. and Camyshken, E.M., *Higher Geodesy,* Moscow, 1980 (in Russian).

[3] Mikhailofh, K., *Higher Geodesy*, Moscow, 1984 (in Russian).