AN EVALUATION OF THE PERFORMANCE OF CAS-CAS AND HARTLEY TRANSFORMS IN IMAGE PROCESSING

George Ph. Fiani and Mona I. Loutfi
Electrical Engineering Department, Faculty of Engineering,
Alexandria University, Alexandria, Egypt.

ABSTRACT

Different types of two-dimensional (2-D) Hartley transforms appearing in the literature are defined in three categories, namely, the 2-D discrete Hartley transform, cas-cas transform and separable Hartley-like transform. The transforms are evaluated for the feasibility of fast computation. Their image compression performances are studied and compared. The performance criterion is taken as the mean-square error (MSE) between the original image and the reconstructed one. Threshold sampling and Zonal sampling compression techniques are used in this study. The results are presented and discussed.

INTRODUCTION

During the past decade, there has been an extensive research work devoted to image coding systems using orthogonal transformations [1], [2]. Many transforms have been considered for image compression, including the Karhunen-Loeve, cosine, Fourier, Slant, Walsh and Haar transforms [2]. The discrete Hartley transform (DHT), recently introduced by Bracewell [3], has gained considerable attention and proved to be a valuable tool in image processing. It has the attractive properties of using real arithmetic, of being its own inverse and of economy in memory utilization.

In one-dimensional applications, the fast Hartley transform (FHT) [4], [5], provides the same information as the fast Fourier transform (FFT) but with greater speed and efficiency when the input data are real. The compression properties and the image coding performance of the DHT were studied by the authors [6], [7] and found to be much superior to those of the FFT, although slightly inferior to those of the discrete cosine transform (DCT).

In the present work, we study and compare different types of two-dimensional discrete Hartley transforms and evaluate their performance for image processing. Both zonal and threshold coding filters are used in this study.

THE TWO-DIMENSIONAL HARTLEY TRANSFORM

In the literature, the two-dimensional (2-D) Hartley transform has been defined in different ways. We may classify these types under the following three approaches:

* The 2-D discrete Hartley transform (DHT)
* The cas-cas transform
* The Hartley-like transform

The 2-D discrete Hartley Transform

The 2-D DHT of an N × N image, [3], is defined by,

$$H(u,v) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} x(i,j)\, \mathrm{cas}[\frac{2\pi}{N}(iu+jv)] \qquad (1)$$

where $0 \le u \le N-1$; $0 \le v \le N-1$, and

$$\mathrm{cas}\,\theta = \cos\theta + \sin\theta \qquad (2)$$

The inverse 2-D DHT is defined by:

$$x(i,j) = \frac{1}{N^2} \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} H(u,v)\, \mathrm{cas}[\frac{2\pi}{N}(iu+jv)] \qquad (3)$$

where $0 \le i \le N-1$ and $0 \le j \le N-1$.

The kernel cas $[\frac{2\pi}{N}(iu+jv)]$ is similar to the kernel $\exp[\frac{2\pi}{N}(iu+jv)]$ of the FFT in that it has a simple physical interpretation as an incident wave at some general angle to the two-coordinate axes [8].

However, since $\mathrm{cas}(\alpha + \beta) \neq \mathrm{cas}\alpha\ \mathrm{cas}\beta$, the kernel of the 2-D DHT (unlike that of the 2-D FFT) is not separable into a product of factors. As a result, 1-D fast algorithms can not be generalized in a straightforward manner to the two-dimensional case. The 2-D DHT is thus not computationally efficient.

The Cas-Cas Transform

The cas-cas transform of an N × N image is defined by, [9], [10],

$$T(u,v) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} x(i,j)\, \text{cas}\left(\frac{2\pi iu}{N}\right) \text{cas}\left(\frac{2\pi jv}{N}\right) \quad (4)$$

where $0 \leq u \leq N\text{-}1$ and $0 \leq v \leq N\text{-}1$.

This transform has the advantage that the 1-D FHT algorithms may be used for its fast calculation. However, the kernel cas α cas β in this case has no physical meaning as an oblique incident wave.

The Separable Hartley-Like Transform

This transform, [8], [11], possesses the advantages of the previous two transforms. It has the intrinsic properties of the 2-D DHT and, at the same time, it is computationally competitive with the fast cas-cas transform and FFT.

Its calculation may be summarized in two steps. In the first step, an intermediate array T(u,v) is obtained by taking the fast 1-D DHT of the rows (one by one) of the original image, then the fast 1-D DHT of the columns of the resulting array. In the second step, the array T(u,v) is converted to the desired Hartley-like transform H(u,v) as follows,

$$H(u,v) = \tfrac{1}{2}[T(u,v) + T(N-u,v) + T(u,N-v) - T(N-u,N-v)] \quad (5)$$

where $1 \leq u \leq N\text{-}1;\ 1 \leq v \leq N\text{-}1;$

$$H(u,0) = T(u,0) \quad (6)$$

and

$$H(0,v) = T(0,v) \quad (7)$$

NUMBER OF OPERATIONS REQUIRED

The total number of operations required for each of the above mentioned types of 2-D Hartley transforms may be calculated as follows,

2-D DHT: The number of multiplications and additions required for the computation of the 2-D DHT is given by:

$$M_{Ht} = N^3 \quad (8)$$

$$A_{Ht} = N^2(N-1) \quad (9)$$

Some approaches have been made to reduce this number [8].

Cas-cas transform: Applying the decimation-in-time FHT, we have for N columns, $N(N\log_2 N-3N+4)$ multiplications and $N[(3N\log_2 N-3N+4)/2]$ additions. Repeating this for N rows, we get a total number of multiplications M_{cas},

$$M_{cas} = 2N(N\log_2 N-3N+4) \quad (10)$$

and total number of additions A_{cas},

$$A_{cas} = 2N[(3N\log_2 N-3N+4)/2] \quad (11)$$

Separable Hartley-like transform: By using the cas-cas approach to calculate the intermediate array T(u,v), we end up with three extra additions for each element of H(u,v), according to (5), except for the (2N-1) unchanged elements defined by (6) and (7). The total number of multiplications required, $M_{L\text{-}Ht}$, is :

$$M_{L\text{-}Ht} = M_{cas} \quad (12)$$

The total number of additions is, $A_{L\text{-}Ht}$,

$$A_{L\text{-}Ht} = A_{cas} + 3[N^2-(2N-1)] \quad (13)$$

Table I gives the total number of operations as a function of the array dimension N, for the three approaches.

TRANSFORM IMAGE CODING USING THE DHT

DHT image coding achieves a reduction in the correlation among the image pixels intensities as well as energy compaction [6], [7]. This is exploited for image compression by discarding the low-energy transform coefficients or by coding them with a lower number of bits. Bandwidth reduction and low transmission rates may thus be obtained with negligible image degradation.

Table I. Comparison of the number of operations required for each transform

	2-D DHT			cas-cas transform			Separable Hartley-like transform		
N	M_{Ht}	A_{Ht}	Total (M+A)	M_{cas}	A_{cas}	Total (M+A)	$M_{L\text{-}Ht}$	$A_{L\text{-}Ht}$	Total (M+A)
4	64	48	112	0	64	64	0	91	91
8	512	448	960	64	416	480	64	563	627
16	4096	3840	7936	640	2368	3008	640	3043	3683
32	32768	31744	64512	4352	12416	16768	4352	15299	19651
64	262144	258048	520192	25088	61696	86784	25088	73603	98691
128	2097152	2080768	4177920	132096	295424	427520	132096	343811	475907

In the present section, we compare the image compression performance of the cas-cas transform and the separable Hartley-like transform. In order to evaluate the degradation in the reconstructed image $\hat{x}(i,j)$, with respect to the original one $x(i,j)$, a performance index P.I. has been adopted, in the form of a percent mean-square error criterion.

$$\text{P.I.} = \frac{\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}[x(i,j)-\hat{x}(i,j)]^2}{\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}[x(i,j)]^2} \times 100 \qquad (14)$$

In order to eliminate the low-energy transform coefficients, two approaches have been used: the zonal sampling technique and the threshold sampling technique.

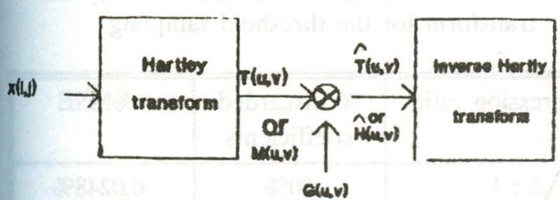

Figure 1. Block diagram of the zonal sampling process.

Zonal Sampling

In the zonal sampling technique, the image is reconstructed with a subset of the transform coefficients lying in certain pre-specified geometric zones, usually the low-frequency coefficients (low index region). This is equivalent to a low-pass filtering process in which the filter transfer function G(u,v) controls the bandwidth reduction. Figure (1) shows a simplified block diagram of the zonal sampling process. Different filter functions and pass-bands have been designed and tried:

a) Rectangular filter

$$G(u,v) = 1 \quad u \le L$$
$$= 0 \quad u > L$$

where the pass-band L is such that $L < N - 1$
In this work, two values of L are considered,

$$L = (N/2)-1 \text{ and } L = (3N/4)-1$$

b) Triangular filter

$$G(u,v) = 1 \quad u+v \le N-1$$
$$= 0 \quad u+v > N-1$$

c) Elliptical filter

$$G(u,v) = 1 \quad u^2 + {}^2 \le a^2$$
$$= 0 \quad \text{otherwise}$$

Two values of a are chosen, a=N-1 and a=N-3.

Threshold Sampling

In the threshold sampling technique, the image is reconstructed with a subset of the image transform coefficients which are larger than a specified threshold. By changing this threshold value, V_{th}, different compression ratios are obtained. Figure (2) shows a simplified block diagram of the threshold sampling process.

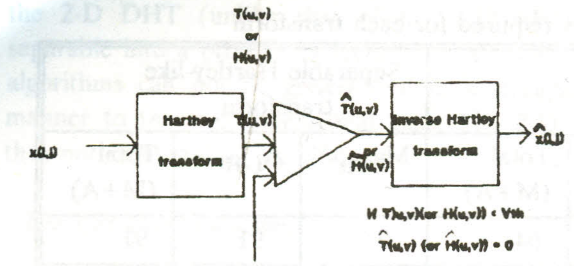

Figure 2. Block diagram of the threshold sampling process.

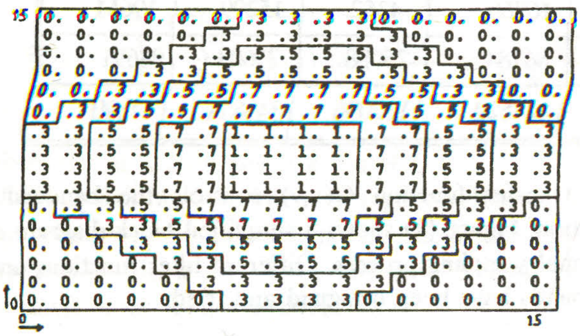

Figure 3. *The selected image x (i,j).*

The Image Compression Performance

Different N×N images have been used. The 16×16 image shown in Figure (3) has been used as a reference for comparing the image compression performance of the various 2-D Hartley transforms.

Results using the cas-cas transform defined in (4) are summarized in Table II [for the zonal sampling technique, with different filtering functions] and in Table III [for the threshold sampling technique with different compression ratios]. Corresponding results are given respectively in Tables IV and V for the separable Hartley-like transform (5).

Table II. Results using the cas-cas transform for the zonal sampling

Filter type	% discarded coefficients	%MSE
Rectangular (L=7)	50%	16.5%
Rectangular (L=11)	25%	15.5%
Triangular	46.9%	3%
Elliptic (a=15)	24.6%	2.3%
Elliptic (a=13)	43%	29.9%

Table III. Results using the cas-cas transform for the threshold sampling

Compression ratio	% discarded coefficients	%MSE
2 : 1	50%	0.008%
4 : 1	75%	0.28%
6 : 1	83.3%	0.67%
8 : 1	87.5%	1.06%
16 : 1	93.75%	1.94%
32 : 1	96.9%	3.16%

Table IV. Results using the separable Hartley-like transform for the zonal sampling

Filter type	% discarded coefficients	%MSE
Rectangular (L=7)	50%	16.5%
Rectangular (L=11)	25%	15.4%
Triangular	46.9%	2.5%
Elliptic (a=15)	24.6%	2%
Elliptic (a=13)	43%	29.5%

Table V. Results using the separable Hartley-like transform for the threshold sampling

Compression ratio	% discarded coefficients	%MSE
2 : 1	50%	0.0248%
4 : 1	75%	0.49%
6 : 1	83.3%	0.89%
8 : 1	87.5%	1.21%
16 : 1	93.75%	2.06%
32 : 1	96.9%	3.17%

DISCUSSION AND CONCLUSIONS

It has been noted that for each transform considered, the MSE resulting when using threshold sampling is much lower than that resulting when using the zonal sampling for the same degree of image compression.

The cas-cas transform has been found to give the better performance, when the threshold sampling technique is considered. It has yield lower values of MSE than the separable 2-D Hartley-like transform. When the zonal sampling technique is used, approximately the same values of MSE are obtained for both transforms.

On the other hand, the cas-cas fast algorithm possesses a smaller total number of operations, compared to the 2-D DHT and the separable 2-D Hartley like transform, and it has proved substantial reduction in computation time.

Then, the adoption of the cas-cas transform is recommended for image coding and compression purposes.

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