

# A MONTE CARLO CODE FOR THE CALCULATION OF NEUTRON TRANSPORT THROUGH FILTER SYSTEMS

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## ABSTRACT

A Monte Carlo Code was developed and processed on the VAX-Computer system of the Reactors Department at Inshas to calculate the neutron flux and current of the output beam after a scatterer-filter combination systems, which are suggested for the generation of neutrons in the Intermediate and Epithermal energy range. The results was encouraging for constructing a horizontal channel in Inshas Reactor for the generation of neutrons in this energy range to be used for research experiments and calibration purposes.

## INTRODUCTION

In a previous work [1] an appropriate technique was proposed for the generation of Epithermal and Intermediate energy neutrons by using of filter-scatterer combination systems. This work introduces a computational model by which the reactor neutrons at experimental channel exit are simulated in their passage within different types of material combination systems in order to obtain their population after transmission. The Monte Carlo technique was employed to follow the neutron in its path by simulating the so called "random walk" of the neutron, and then by sampling its histories within the transported medium. The input data to the code are the neutron energy spectrum of Inshas reactor and the pointwise cross-sections of the filter and scatter materials. The output of the program is the transmission factor, which is used to make an estimation of the Intermediate neutron flux and current at the exit of the filter-scatterer combination. A combination of  $Ti^{48}$  -  $Co^{59}$  -  $Al^{27}$  was studied for the generation of 45 Kev neutrons and the exit beam shows high resolution and high intensity. Another combination of  $Mg^{24}$  -  $S^{32}$  produces 50Kev neutron beam.

## BASIC OUTLINES OF THE CODE

The Monte Carlo method is a numerical technique used to construct a simulation of certain diffusional elementary particles such as neutrons and photons [2,3]. The neutron paths through matter are simulated using input data describing the geometry and the nuclear properties of the

system [3]. A series of neutron histories is generated using the input data and a list of random numbers to choose interaction points and decide the outcome of an interaction. A final account of the fate of all the selected particles may then be used to calculate of the required estimator.

The essential feature common to all Monte Carlo methods is that, at some point in the calculations, one has to substitute for a standard random variable N. The advent of the digital computers has made a mathematical algorithm highly desirable. The multiplicative congruential method [4] adapted to the VAX-computer system of Inshas nuclear research center is used to generate standard random numbers.

In the Monte Carlo method, to represent a random variable, we must indicate the assumed values and what are the probabilities of these values. A continuous random variable X is defined by specifying the interval (a,b) of its variation and the function p(x), called the "Probability density" of the random variable. The X values can be found from the formula [19]

$$\int_a^x P(x)dx = N \quad (1)$$

Unless the integral of the function p(x) in Equation (1) is analytically integrable, the integration is performed numerically. Thus the normalized cumulative function P(x) can be obtained. The random variable X is selected by

interpolation for a set of  $P(x)$  values distributed uniformly, between (0,1). The  $X$  values are then tabulated. The proper values of random variable  $X$  can be selected from the table by using a uniformly distributed integer random number.

Figure (1) shows the flow-chart of the developed Monte Carlo code used to solve the problem of neutron transport through different filter-Scatterer systems.

The basic features of the program are described below:

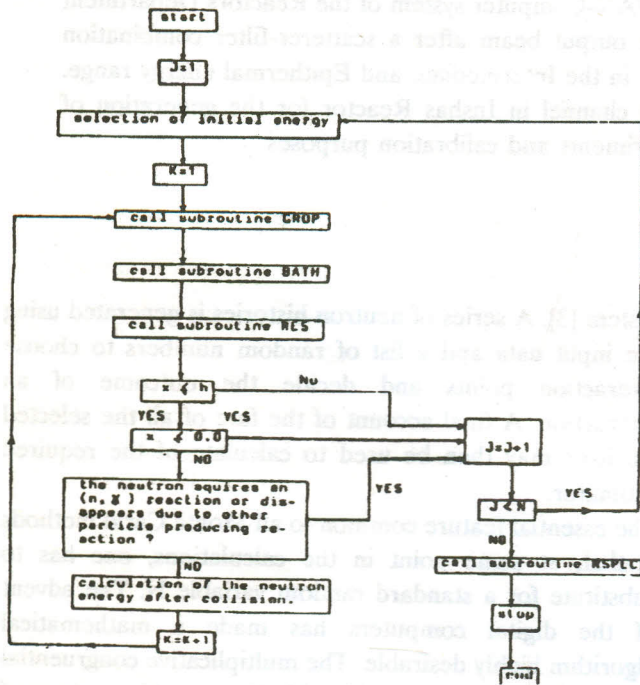


Figure 1. Flow chart of the developed monte cario program.

*Selection of The Neutron Initial Energy*

The initial energy of the neutron is selected from the Inshas reactor spectrum [5], shown in Figure (2), by drawing the cumulative distribution function  $P(E)$  of the initial neutron spectrum, shown in Figure (3). A 100 equally probable energies of the initial incident neutrons are given in a table form. This table is fed to the input data file of the program.

*Neutron Cross Section Data*

Once, the neutron energy is slected, the microscopic cross sections of the considered neutron reactions are obtained from the ENDF/B-V cross section data library [6]. The

macroscopic cross section values are calculated by multiplying the microscopic cross sections by the atom density of the filter material.

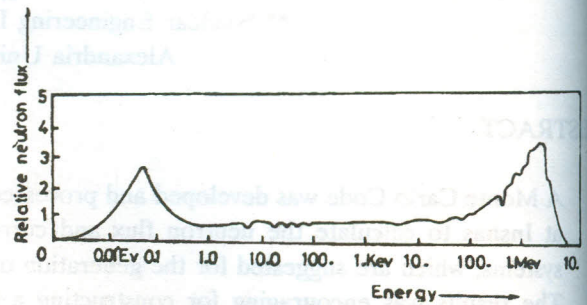


Figure 2. The neutron energy spectrum of inshas reactor.

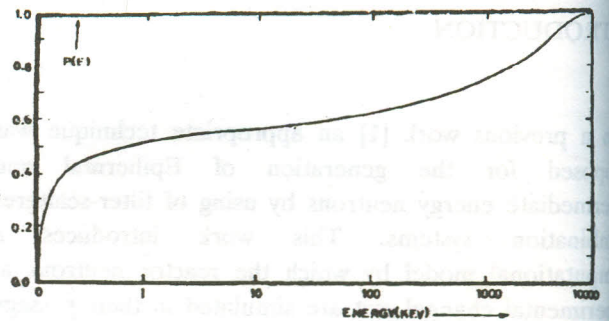


Figure 3. The normalized cumulative distribution function  $P(E)$  of the reactor energy spectrum.

*Particle Location Routine*

The geometry of the filter medium is taken to be a semi-infinite slab defined by;  $0 \leq x \leq h$ , where  $h$  is the slab thickness.

To determine the neutron location inside the medium, we assume the following:

- i)The incident neutron beam is monodirectional and perpendicular to the filter sample.
- ii)Any direction of "recoil" of a neutron from an atom in the sample is equally probable (i.e. isotropic scattering), and this is a suitable approximation in case of intermediate and heavy materials.

If the neutron has undergone the  $K$ -th collision inside the slab at the point with the abscissa  $x$  and started moving in the direction  $\mu_k$ . Then, the abscissa of the next collision is calculated from Figure (4) by;

$$x_{k+1} = x_k + \lambda_k \mu_k \tag{2}$$

The free path length  $\lambda$  is a random variable. It can take

any positive value with the probability density  $p(x)$ , given by,

$$p(x) = \Sigma_t e^{-\Sigma_t x} \quad (3)$$

Where,

$\Sigma_t$  is the total macroscopic cross section.

To obtain the formula of random selection of the free path length, we substitute Eq. (3) into Eq. (1). Then, we get;

$$\lambda = - (1/\Sigma_t) \ln N \quad (4)$$

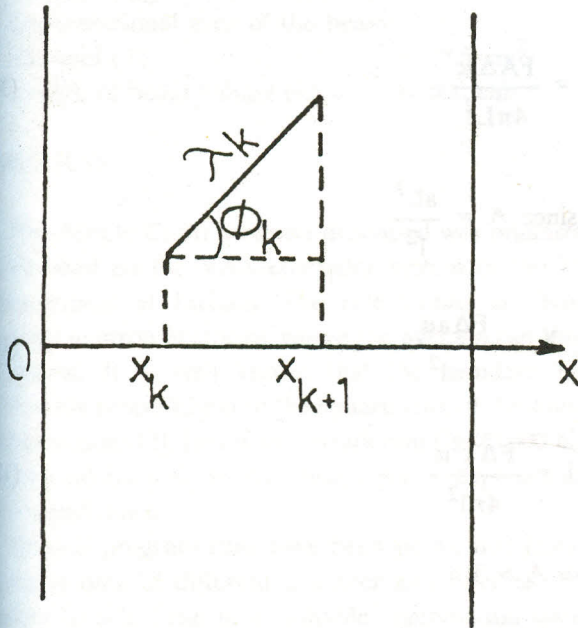


Figure 4. The one-dimensional slab geometry of neutron path through the medium.

The formula of random selection of the direction cosine ( $\mu$ ) is obtained in a similar way, taking into account the condition of equally probable directions. Then, we get [2]

$$\mu = 1 - 2 N \quad (5)$$

#### 4. Selection of The Interaction Type

We assume that the neutron acquires three types of interactions, namely; Elastic scattering, Radiative capture

and Inelastic scattering with threshold energy ( $E_{thr}$ ). Thus, for neutron energy less than ( $E_{thr}$ ), only the first two types of interactions are energetically possible.

When the neutron energy is greater than ( $E_{thr}$ ), we consider all the three types of interactions. Thus, we have to differentiate between the following two cases:

Case I: The neutron energy is less than ( $E_{thr}$ )

In this case, a random number is generated and compared with the ratio  $\Sigma_c/\Sigma_t$ , and the unit interval (0,1) is divided into two probability ranges; (0, to  $\Sigma_c/\Sigma_t$ ) and ( $\Sigma_c/\Sigma_t$  to 1.0). When the random number is less than or equal to the ratio  $\Sigma_c/\Sigma_t$ , the interaction is considered to be radiative capture and the history of the neutron is terminated and unity is added to the counter of the absorbed neutrons. If  $N > \Sigma_c/\Sigma_t$ , the interaction is considered to be elastic scattering, and a unity is added to the collision index and we start to follow the next interaction.

Case II: The neutron energy is greater than or equal to  $E_{thr}$

In this case, the three types of interactions are energetically possible. Accordingly, the unit interval is divided to three probability ranges;

- i. 0 to  $\Sigma_c/\Sigma_t$
- ii.  $\Sigma_c/\Sigma_t$  to  $\Sigma_c + \Sigma_e/\Sigma_t$
- iii.  $\Sigma_c + \Sigma_e/\Sigma_t$  to 1.0.

A random number is generated and compared with these probability ranges to determine the interaction type.

In this section,  $\Sigma_c$  and  $\Sigma_e$  are the macroscopic radiative capture and elastic scattering cross sections, respectively.

#### 5. Calculation of the Neutron Energy After Collision

The formula for selecting the energy  $E_f$  of the elastically scattered neutrons after collision is given by [7];

$$E_f = E_i (1 - N (1 - \alpha)) \quad (6)$$

Where,

$E_i$  is the incident neutron energy,

$$\alpha = (A - 1)^2 / (A + 1)^2,$$

A is the mass number of the sample material.

For inelastically scattered neutrons, the energy of the emergent neutrons is distributed according to the function [8];

$$p(E) dE = (E/T^2) \exp(-E/T) dE \quad (7)$$

Where,

$p(E) dE$  is the probability that an inelastically scattered neutron will emerge with an energy between E, E + dE,

T is the neutron temperature =  $3.2\sqrt{E/A}$ .

The integral of Eq. (7) yields the cumulative probability function, and it is given by;

$$P(E) = (1 - e^{-y} (1+y)) / (1 - e^{-y'} (1+y')) \quad (8)$$

Where,

$$y = E/T \quad (9)$$

and  $y' = 10$  is taken to be the maximum value of this parameter.

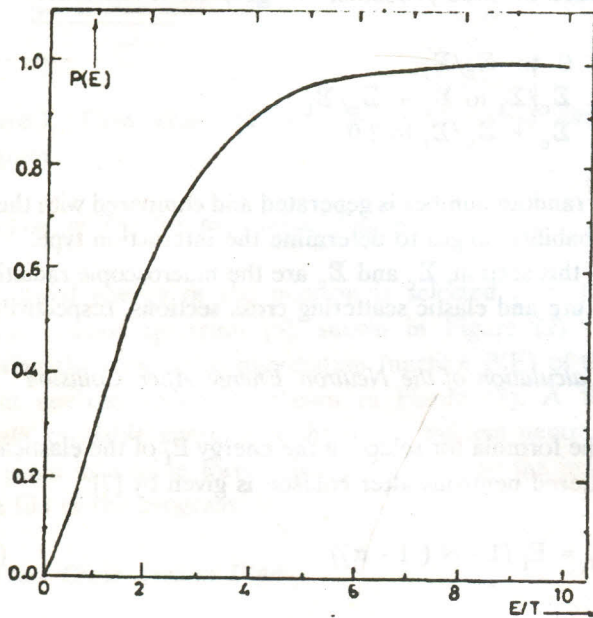


Figure 5. The normalized cumulative distribution function P(E) versus the relative energy of the inelastically scattered neutrons E/T.

The normalized cumulative distribution function of Eq. (9) is given in Figure (5), and a 100 equally probable values of the parameter y are selected from P(E) tabulated and fed to the input data file.

The selected value from the table is multiplied by the neutron temperature to get the proper energy of the inelastically scattered neutrons reactors which are versatile intermediate energy neutron facility utilising the filter systems described in section (2.5).

### 6. Filtered Beam Output Fluxes and Currents

The neutron flux and current of the output filtered beam can be determined by the aid of Figure (6). Where,

a) For  $A < T$  It can be seen from the figure that;

$$X = \frac{FA\Delta\alpha}{4\pi L^2} \quad (10)$$

and since  $A = \frac{aL^2}{l^2}$

$$\text{then, } X = \frac{F\Delta a\alpha}{4\pi l^2} \quad (11)$$

$$\text{and } I = \frac{F\Delta a^2\alpha}{4\pi l^2} \quad (12)$$

b) For  $A > T$

$$X = \frac{TF\Delta\alpha}{4\pi L^2} \quad (13)$$

$$\text{and } I = \frac{TFa\Delta\alpha}{4\pi L^2} \quad (14)$$

Where the symbols have the following meanings:

- T: Cross-sectional area of the beam channel (cm<sup>2</sup>)
- L: Length of the beam channel (cm).
- l: Length of filter (cm).
- a: Cross-sectional area of the filter (cm<sup>2</sup>)
- A: Area on the reactor core end of the beam channel

from which unscattered neutrons may reach a point on the exit end of the filter ( $\text{cm}^2$ ).

- X: Neutron flux at the exit end of the filter ( $\text{n/cm}^2.\text{s}$ )
- $\Delta$ : Lethargy width of the window ( $= \ln(E_2/E_1)$ ).
- I: Output neutron current ( $\text{n/s}$ ).
- $\alpha$ : Peak transmission
- F: Flux per unit lethargy at the base of the reactor beam channel ( $\text{n/cm}^2.\text{s}$ ).

In case of the research reactor at Inshas, the different values of the above parameters are:

- Reactor power density = 12.44 Kw/litre.
- Average thermal flux =  $10^{13}$   $\text{n/cm}^2.\text{s}$ .
- Ratio of intermediate to thermal Flux = 0.01.
- Calculated intermediate flux (F) =  $10^{11}$   $\text{n/cm}^2.\text{s}$ .
- Cross-sectional area of the beam Channel (T) =  $78.5 \text{ cm}^2$
- Length of beam channel (L) = 400 cm

RESULTS

The Monte Carlo program described was organized and processed on the VAX-computer system of the Reactor department at Inchass. The calculations are based on 20,000 neutron histories, where the average run time is 20 minutes. It is well known that the standard error is inversely proportional to the square root of the number of observations [2], therefore considering the 20,000 histories in the calculation, the statistical error is expected to be of no significance.

Several program runs have been performed using cross section data of different scatterer and filter materials in order to select the most suitable combination system for the generation of Intermediate energy neutrons. The neutron transport through two systems is discussed and the results are analysed:

a)  $\text{Ti}^{48}$  -  $\text{Co}^{59}$  -  $\text{Al}^{27}$  Filter System

A successful combination that consists of 2 cm of  $\text{Ti}^{48}$ , 3 cm of  $\text{Co}^{59}$  and 50 cm of  $\text{Al}^{27}$  shows high quality of the output filtered beam.  $\text{Co}^{59}$  has a scattering peak energy that coincide with the peak transmission energy (window) of  $\text{Ti}^{48}$  and the energy of peak transmission is 1.8 Kev. Then the use of both  $\text{Ti}^{48}$  and  $\text{Co}^{59}$  as scattering foils constitutes a good "source of the 1.8 Kev energy neutron. Neutrons of this energy are suitable to be transmitted

through the  $\text{Al}^{27}$  filter that has a transmission window at this nearly energy. Figure (7) shows a plot of the relative neutron transmission versus log energy. The peak transmission for this system is 27% at 1.5 Kev with an energy width at half maximum equals to  $\pm 0.3$  Kev.

The neutron flux and current of the output filtered beam is calculated using Equation (13) & Equation (14). We have length of Aluminum filter (l) = 50 cm  
 diameter of filter (d) = 5 cm  
 cross sectional area (a) =  $19.625 \text{ cm}^2$   
 area (A) =  $0.44 \text{ m}^2 > T$   
 maximum transmission at 1.5 Kev ( $\alpha$ ) = 0.2734  
 Lethargy width at half maximum ( $\Delta$ ) = 0.4052

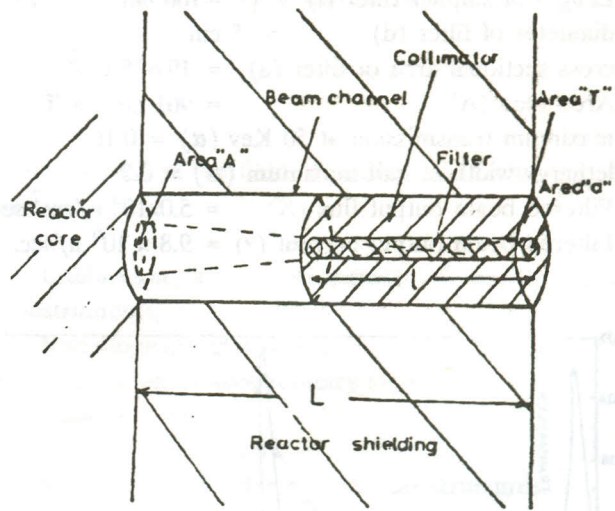


Figure 6. Geometrical relationship of parameters associated with the filter in reactor beam channel.

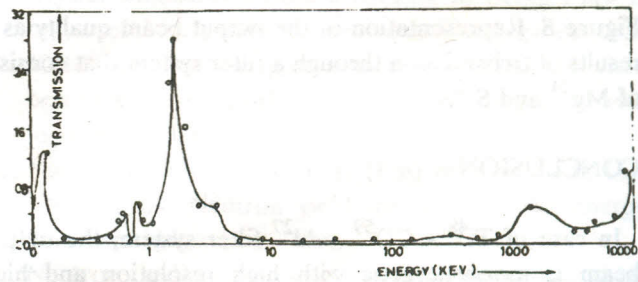


Figure 7. Representation of the output beam quality as a result of transmission through a filter system that consists of  $\text{Ti}^{48}$ ,  $\text{Co}^{59}$  and  $\text{Al}^{27}$ .

Substituting these values in Equation (13) and Equation (14), we get:

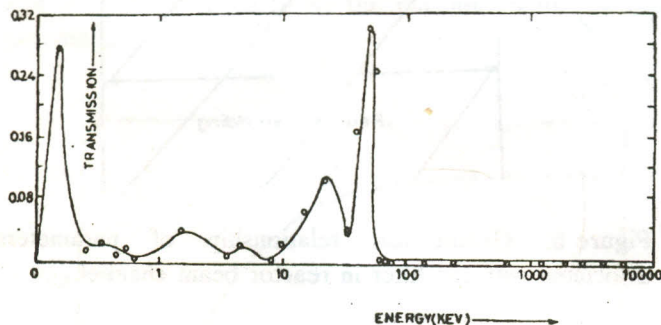
Filtered beam output flux  $X = 6.9 \times 10^6 \text{ n/cm}^2 \cdot \text{s}$   
 Filtered beam output current  $I = 1.36 \times 10^8 \text{ n/s}$

**b.  $\text{Mg}^{24} - \text{S}^{32}$  Filter system**

Another useful filter system is that consists of 2 cm of  $\text{Mg}^{24}$  and 100 cm of  $\text{S}^{32}$ .  $\text{Mg}^{24}$  has a peak scattering energy of 60 Kev and a peak transmission energy of 50 Kev.  $\text{S}^{32}$  has also a peak transmission window at 50 Kev. Figure (8) shows the relative transmission versus log energy. The peak transmission for this system is 16% with an energy width at half maximum equals to  $\pm$  Kev.

The neutron flux and current of the output filtered beam is calculated using Equation (13) & Eq. (14) we get:

- Length of sulphur filter (l) = 100 cm
- diameter of filter (d) = 5 cm
- cross sectional area of filter (a) =  $19.625 \text{ cm}^2$
- Area seen (A) =  $961 \text{ cm}^2 > T$
- maximum transmission at 50 Kev ( $\alpha$ ) = 0.16
- lethargy width at half maximum ( $\Delta$ ) = 0.2
- Filtered beam output flux (X) =  $5.0 \times 10^5 \text{ n/cm}^2 \cdot \text{sec.}$
- Filtered beam output current (z) =  $9.8 \times 10^6 \text{ n/sec.}$



**Figure 8.** Representation of the output beam quality as a results of trcbssmission through a filter system that consists of  $\text{Mg}^{24}$  and  $\text{S}^{32}$ .

**CONCLUSION**

In case of  $\text{Ti}^{48} - \text{CO}^{59} - \text{Al}^{27}$  filter system, the output beam is monoenergetic with high resolution and high intensity. The  $\gamma$ -ray contamination from (n,  $\gamma$ ) reaction in  $\text{Co}^{59}$  and  $\text{Ti}^{48}$  scattering foils is expected to be low due to the small thickness of these foils. In case of  $\text{Mg}^{24} - \text{S}^{32}$  filter system, there is some contamination from the 20 Kev energy neutrons in the output filtered beam, but there is a complete cut-off for the fast spectrum. Neutron

transport through other filter systems can be calculate and analysed to adapt neutron sources of Epithermal and Intermediate Energy range.

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