GRAPHICAL METHOD FOR SYNTHESIZING LINKAGE MECHANISMS TO GENERATE FUNCTIONS OF TWO VARIABLES

Ahmed Hassan Metwally El-Sherif*
Department of Engineering Mathematics and Physics,
Faculty of Engineering, Beirut Arab University, Lebanon.

ABSTRACT

The paper presents a graphical method for synthesizing plane linkage mechanisms with two rotating inputs and one rotating output. The mechanisms are used to generate functions of two independent variables. The method is based on constructing circles passing through three points, and permits the mechanisms to satisfy six arbitrarily selected precision positions. Note; A precision position is a configuration of the system for which the values of the variables of the generated function coincide with those of the function which is to be synthesized.

NOMENCLATURE

Figure (1) shows the linkage mechanism and the letters for the designation of links and points. The following are definitions of symbols.

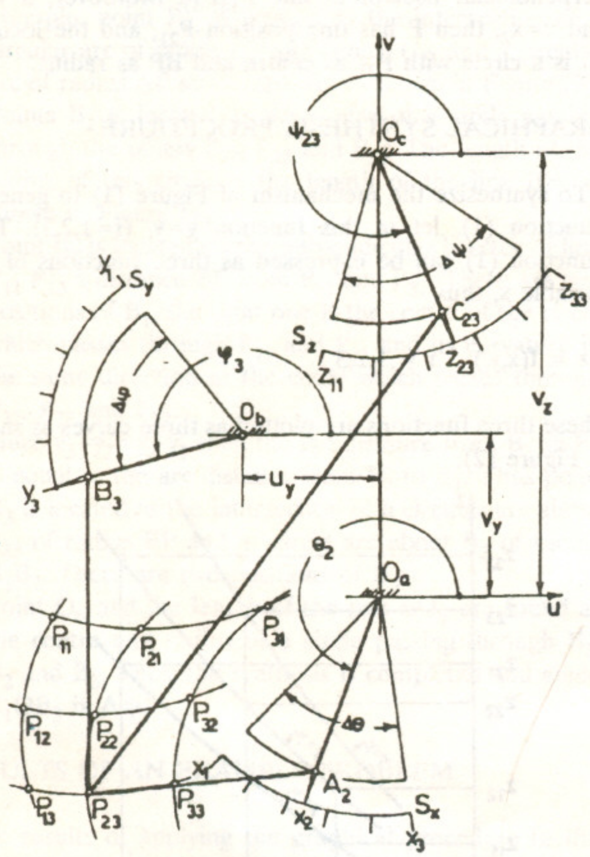

Figure 1.

x,y	the two input variables
z	the output variable
$z = f(x,y)$	the desired function
x_1, x_2, x_3	values of the input variables x
y_1, y_2, y_3	values of the input variables y
$z_{11}, z_{12}, \ldots, z_{33}$	values of the output variable such that $z_{12} = f(x_1, y_2)$
$\Delta x, \Delta y, \Delta z$	ranges of x,y and z.
$\theta_1, \theta_2, \theta_3$	input angles corresponding to x_1, x_2, x_3
ϕ_1, ϕ_2, ϕ_3	input angles corresponding to y_1, y_2, y_3.
$\psi_{11}, \psi_{12}, \ldots, \psi_{33}$	output angles corresponding to $z_{11}, z_{12}, \ldots, z_{33}$
$\Delta\theta, \Delta\phi, \Delta\psi$	angular ranges of θ, ϕ and ψ
R_x, k_y, k_z	scale factors of x,y,z
S_x, S_y, S_z	scale of x, y, z.

INTRODUCTION

Linkage mechanisms which mechanize functions of two variables have numerous applications. Typical examples include robotic components, automatic components and analog computer components. The synthesis problem of these mechanisms has been the subject of many investigations [1-5]. Papers [1] and [3] have presented analytical methods to synthesis 7-link mechanism with rotational inputs and output. In [2] Mruthyunijaya developed a graphical method to synthesis 7-link mechanism with siding inputs and output. Ramaiyan and others [4] proposed a graphical procedure for synthesizing

* On leave from the Faculty of Engineering, Alexandria University, Egypt.

problem is treated by synthesizing, graphically, three function generator mechanisms of one variable and an adder mechanism. When these three mechanisms are join together by the adder, the resultant mechanism performs as a function generator of two variables. Each mechanism of the three mechanisms consists of 6 links besides the adder which consists of 7 links. So the resultant mechanism of [5] contains many links and occupies large space.

In this paper the adder mechanism which is developed in [5] for addition only is resynthesized graphically to generate functions of two variables in certain ranges of the independent variables. The only restrictions are that the functions in the ranges under consideration are bounded, single valued and continuous.

THE LINKAGE MECHANISM AND ITS PERFORMANCE

The linkage adder mechanism of [5] is shown in Figure (1). It has three fixed pivots O_a, O_b and O_c, three rotating links AO_a, BO_b and CO_a, and three floating links AP, BP and CP. Links AO_a and BO_b are the input links corresponding to the variables x and y. Link CO_c is the output link corresponding to the z variable. This mechanism is required to be synthesized to generate the function

$$z = f(x,y) \tag{1}$$

within the ranges $\Delta x = x_3 - x_1$, $\Delta y = y_3 - y_1$ and $\Delta z = z_{33} - z_{11}$. Where x_1, y_1 and z_{11} are the starting values of x,y,z and x_3, y_3 and z_{33} are their final values.

The independent variables x,y are represented mechanically by the angular rotations θ, ϕ of links AO_a and BO_b, while the dependent variable z is displayed by the angular rotation ψ of like CO_c. The angles θ, ϕ and ψ are measured anticlockwise from a fixed cartesian coordinate system (u,v) as shown in Figure (1). The relations between x,y, z and θ, ϕ, ψ are assumed to be linear and in the forms

$$x = k_x (\theta - \theta_1) + x_1 \tag{2}$$

$$x = k_y (\phi - \phi_1) + y_1 \tag{3}$$

$$z = k_z (\psi - \psi_1) + z_1 \tag{4}$$

where

$$k_x = \Delta x/\Delta\theta,\ k_y = \Delta y/\Delta\phi,\ k_z = \Delta z/\Delta\psi,\ \Delta\theta = \theta_3 - \theta_1,$$

$$\Delta\phi = \phi_3 - \phi_1, \text{ and } \Delta\psi = \psi_{33} - \psi_{11}.$$

The mechanism performs as follows: when the links AO_a and BO_b rotate through two angles, say θ_2 and ϕ_3, link CO_c will rotate through the angle ψ_{23}. The four points A, B, C and P will take the positions A_2, B_3, C_{23} and P_{23}. The three positions A_2, B_3 and C_{23} indicate on S_x, S_y and S_z to the values x_2, y_3 and z_{23} of x, y and z. The values x_2, y_3, z_{23} and their corresponding angles θ_2, ϕ_3, ψ_{23} must satisfy Equations (2), (3) and (4), and in the same time the value z_{23} must be equal to $f(x_2, z_3)$.

It is to be noted that, for a given value of y there are many positions of P corresponding to the values of x. For example, if $y=y_2$ and $x=x_i$ $(i=1,2,3)$, then P has three positions P_{12}, P_{22}, P_{32}. In this case B_2 is the centre of a circle passes through the points P_{12}, P_{22} and P_{32} with BP as radius. Also if $y=y_3$ and $x=x_i$ $(i=1,3)$, then P has two positions P_{13} and P_{33}, and the locus of B_3 is the perpendicular lisector of line $P_{13}P_{33}$. Moreover, if $y=y_1$ and $x=x_2$, then P has one position P_{21}, and the locus of B_1 is a circle with P_{21} as centre and BP as radius.

GRAPHICAL SYNTHESIS PROCEDURE

To synthesize the mechanism of Figure (1) to generate Function (1), let in this function $y=y_i$ $(i=1,2,3)$. Then Function (1) can be expressed as three functions of one variable x, thus

$$z = f(x,y_i) \quad i = 1,2,3$$

These three functions are plotted as three curves as shown in Figure (2).

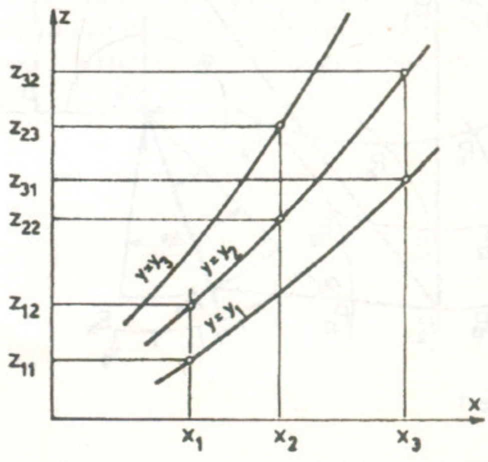

Figure 2.

ese curves six design points are chosen such that; nts (x_1, y_1, z_{11}) and (x_3, y_1, z_{31}) on the curve ree points(x_1, y_2, z_{12}), (x_2, y_2, z_{22}) and (x_3, y_2, z_{32}) urve $y=y_2$, and one point (x_2, y_3, z_{23}) on the curve o insure the linearity of S_y scale it is convenient $=y_2-y_1$. The graphical procedure for the synthesize nechanism is as follows:

locations of O_a and O_c, the lengths of links AO_a, CO_c and CP, the angles θ_1 and ψ_{11} and the scale ors k_x and k_z are chosen. Points A_1 and C_{11} are ted according to the values θ_1 and ψ_{11}.

angles θ_2 and θ_3 are calculated by substituting x_2 x_3 in Equation (2). Then, according to θ_2 and θ_3, nts A_2 and A_3 are located on an arc of a circle of ter O_a and radius AO_a.

substituting z_{11}, z_{31}, z_{12}, z_{22}, z_{32} and z_{23} into uation (4), the angles $\psi_{11}, \psi_{13}, \psi_{12}, \psi_{22}, \psi_{32}$ and ψ_{23} obtained. According to these angles points c_{11}, c_{31}, c_{22}, and c_{32} are located on an arc of a circle of tre O_c and radius CO_c.

nts P_{11}, P_{31}, P_{12}, P_{22}, P_{32} and P_{23} are located. For mple, point P_{21} is located as the intersection of a cular arc of radius CP and centre C_{21} and a circular of radius AP and centre A_2 as shown in Figure (3).

ints B_2 is located as the centre of a circle passing rough the points P_{12}, P_{22} and P_{32}. The length of the dius of this circle is the length of the link BP, as own in Figure (4).

oint B_1 is located on the perpendicular lisector of line $_1$ P_{13} a distance BP from P_{11} and P_{31}. These are two sitions of B_1, the right one is the centre of the circle nich passes through P_{11} and P_{31} and its curvature in e same direction as the circle which passes through $_{12}$, P_{22} and P_{32}.

nce $y_3 - y_2 = y_2 - y_1$ then the arc distance from B_3 to B_2 equal to the arc distance from B_2 to B_1. Thus point $_3$ is located as the intersection of a circular arc about $_{23}$ of radius BP and a circle arc about B_2 of radius $_1B_2$. There are two positions of B_3.

oint O_b and the length of the link BO_b are found as e centre and radius of a circle passing through B_1, $_2$ and B_3. Thus, the synthesis is completed and angle $_1OB_3$ is $\Delta\psi$.

ULTS OF AN EXAMPLE PROBLEM

e results of applying the graphical procedure to the tion $z=x^2\ y^{1.3}$ for $1 \le x \le 1.4$ and $1 \le y \le 2$ are $u_y=-2.15$, -2.67, $v_z = 15.65$, $AO_a = 5$, $BO_b = 15.6$, $CO_c = 4.8$, $= 8.7$, $BP = 9.77$, $CP = 9.2$, $\theta_1 = 48°$, $\phi_1 = 90°$, $=218.5°$, $\Delta\theta=44°$, $\Delta\psi=14.6°$, $\Delta\psi=68°$, $k_x=.0091$, $k_y = -.0685$ and $k_z = -.0563$. Figure (5) shows the desired and the generated functions. The design points are not points of zero error due to drawing inaccuracies.

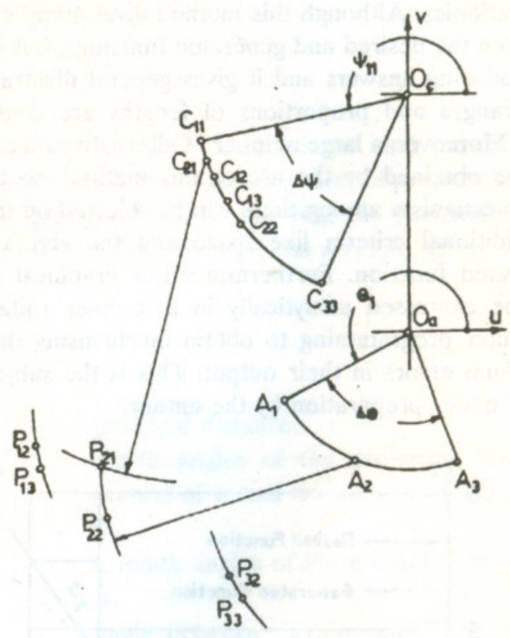

Figure 3.

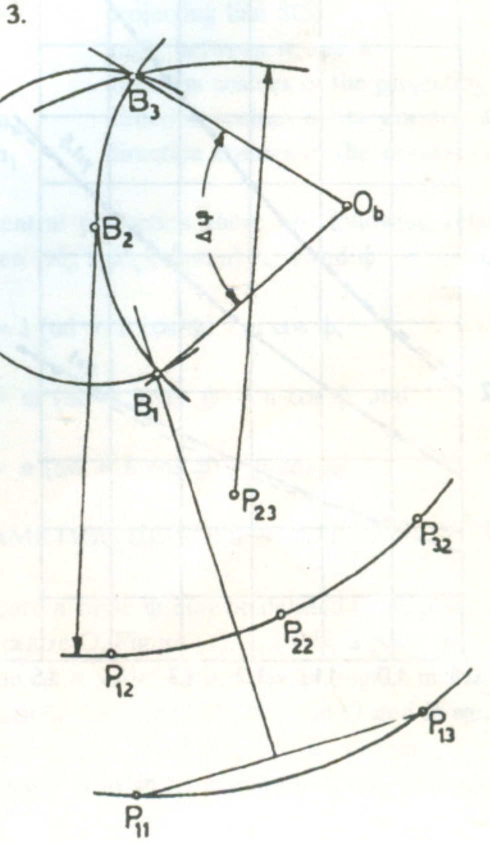

Figure 4.

CONCLUSION

Represented in this paper a graphical method of synthesize a linkage mechanism to generate functions of two variables. Although this method gives some errors in between the desired and generated functions, but it is fast in producing answers and it gives general illustration, so that ranges and proportions of lengths are constant in view. Moreover, a large number of alternative mechanisms can be obtained by the use of this method, so that the best mechanism among these can be selected on the basis of additional criteria like space and the errors in the generated function. Furthermore, this graphical method can be expressed analytically in a manner suitable for computer programming to obtain mechanisms that have minimum errors in their output. This is the subject of a paper under preparation by the author.

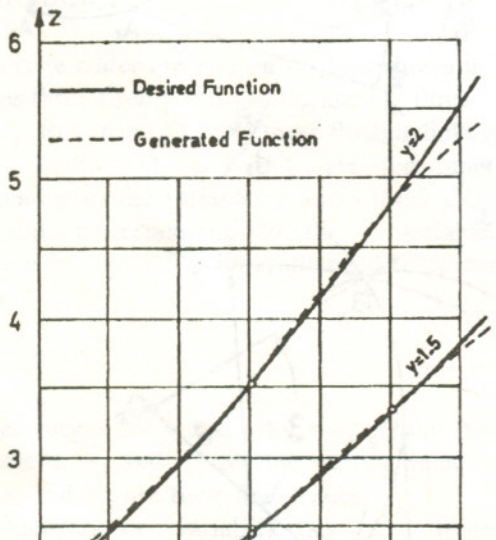

Figure 5.

REFERENCES

[1] Philip, R.E. and Freudensten, F., "Synthesis of Two-Degree of Freedom Linkages", *J. Mechanism*. Vol. 1, pp. 9-21, 1966.

[2] Mruthunjaya, T.S., "Synthesis of Plane Linkages to Generate Functions of Two Variables Using Point Position Reduction-II. sliding Inputs and Outputs", *Mechanism and Machine Theory*, Vol. 7, pp. 399-405, 1972.

[3] Kohli, D. and Soni, A.H., "Synthesis od Seven-Link Mechanisms", *Journal of Engineering for Industry. Trans. ASME. Series B,* Vol. 95, pp 533-540, May 1973.

[4] Ramaiyan, G., Lakshminarayana, K. and Narayanamurth, R.G., "Nine-Linke Plane Mechanisms for Two-Variable Function Generation-II Synthesis", *Mechanism and Machine Theory*. Vol. 11, pp. 193-199, 1976.

[5] El Sherif, Ahmed H.M. "Design of Plane Linkage to Generate Function of Two Variables", *Ph.D. Thesis*, Faculty of Eng., Alex. Univ. 1983.