DYNAMICS OF MULTIPLE SPAN PIPES CONVEYING FLUID

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ABSTRACT

The dynamics of pipes conveying incompressible fluid, where the pipe is supported on equally-spaced elastic supports which exert both transverse and rotational restraints to the pipe motion has been investigated. The wave approach, which relies mainly on the notion of the propagation constant, is followed to investigate the case of steady flow. The effect of the flow velocity, the fluid pressure, the fluid/pipe mass ratio, the rotational stiffness and the transverse stiffness on the natural frequencies are examined.

NOMENCLATURE

- EI Flexural rigidity of the pipe.
- K_p non-dimensional transverse stiffness of the support.
- K_r non-dimensional rotational stiffness of the support.
- l Length of one span of the pipe.
- m_f Fluid mass per unit length
- m_p Pipe mass per unit length.
- M_l Applied bending moment at the left end.
- M_r Applied bending moment at the right end.
- R_A Receptance matrix at the extreme left end.
- R_B Receptance matrix at the extreme right end.
- R_{ij} Receptance matrix at point 'i' due to a unit applied load at point 'j'
- R_I Characteristic wave receptance matrix for the incident wave.
- T Tension in the pipe walls.
- t Time.
- u Non-dimensional flow velocity.
- v Flow velocity.
- V_l Applied shear force at the left end.
- V_r Applied shear force at the right end.
- x,y Axial and transverse coordinates.

Greek Letters

- α_{ij} Displacement at point 'i' due to a unit force at point 'j'
- β Fluid-pipe mass ratio.
- β_{ij} Displacement at point 'i' due to a unit moment at point 'j'
- Γ Non-dimensional pressure.
- γ_{ij} Rotation at point 'i' due to a unit force at point 'j'
- η Non-dimensional transverse coordinate.
- θ_{ij} Rotation at point 'i' due to a unit moment at point 'j'
- λ Wave number.
- μ Propagation constant.
- μ_I Propagation constant of the incident wave.
- μ_R Propagation constant of the reflected wave.
- υ Non-dimensional fluid pressure.
- ξ Non-dimensional axial coordinate.
- τ Non-dimensional time.
- φ Rotation coordinate at the end of a pipe-span.
- ψ Generalized coordinates.
- Ω Non-dimensional circular frequency.
- ω Circular frequency.

INTRODUCTION

A great surge of attention has been paid in recent years to the lateral vibrations of pipes conveying fluid as a branch of the wide area of flow induced vibrations. despite the flow induced vibrations are usually considered a secondary design parameter, a theory for the dynamics of elastic pipes carrying flowing fluid is of considerable interest.

In practice, the dynamical behaviour of such system is of considerable importance for the oscillations which have been observed in above ground oil pipelines, pump-discharge lines, various elements of high performance launch vehicle, missiles, reactor components such as fuel pins and monitoring and control rods, heat exchanger tube arrays and piping system in power generating plants and chemical and petrochemical industries.

As reported by Paidoussis and Issid [1], Marcel Brillouin was the first to investigate the self excited oscillations of the free end of a rubber. pipe in 1985. They reported also that Bourrieres studied the dynamics of flexible pipes conveying fluid. He examined the oscillatory instability of cantilevered pipes conveying fluid both theoretically and experimentally. Ashley and Haviland, [2], reactivated the interest in the study of dynamics of elastic pipes conveying fluid in connection with the vibration problems of the trans-arabian pipeline.

Housner [3] derived the equation of motion for a pipe conveying fluid with simply supported ends, while Long [4] studied the case of cantilevered pipes.

A study by Stein and Torbiner [5] was mainly concerned with infinitely long pipes conveying fluid. Paidoussis and Denise [6,7] studied the dynamics of very thin elastic pipes conveying fluid by applying thin-shell theory.

A derivation of the equations of motion of an initially stressed Timoshenko tubular beam subjected to a tensile follower load using Hamilton's principle was found by Laithier and Paidoussis [8]. On the other hand Tiny and Hosseinipour [9] extended a structural impedance approach for the dynamics of pipe structures conveying fluid flow.

The dynamic and stability of short tubes was examined by Paidoussis et al. [10]. They used Timoshenko's beam theory for the tube and a three dimensional fluid mechanical model for the fluid flow.

The objective of this study is to investigate the problem of multi-supported uniform pipes conveying incompressible steady fluid flow. The supports are considered elastic with both rotational and transverse stiffnesses.

MATHEMATICAL FORMULATION

The general equations of motion, of one span of a pipe, shown in Figure (1), are derived using the simple beam theory neglecting both shear deformation and rotary inertia, and given in non-dimensional from as follows.

$$\frac{\partial^4\eta}{\partial\xi^4}+\frac{\partial}{\partial\xi}[(\nu-\Gamma)\frac{\partial\eta}{\partial\xi}]+u^2\frac{\partial^2\eta}{\partial\xi^2}+2u\beta\frac{\partial^2\eta}{\partial\xi\,\partial\tau}+\beta\frac{\partial u}{\partial\tau}\frac{\partial\eta}{\partial\xi}+\frac{\partial^2\eta}{\partial\tau^2}=0 \quad (1)$$

and

$$\frac{\partial}{\partial\xi}(\nu-\Gamma)=-\beta\frac{\partial u}{\partial\tau} \quad (2)$$

where:

$$\xi=\frac{x}{l},\ \eta=\frac{y}{l},\ \nu=\frac{PAl^2}{EI},\ \Gamma=\frac{Tl^2}{EI},\ u=\sqrt{\frac{m_f}{EI}}\,vl,$$

$$\beta=\sqrt{\frac{m_f}{m_f+m_p}},\ \tau=\sqrt{\frac{EI}{m_f+m_p}}\cdot\frac{t}{l^2}$$

The assumption of steady flow imposes the condition that the pressure flow velocity constant which yields:

$$(\upsilon-\Gamma) = \text{constant} \quad (3)$$

Substituting equation (3) into equation (2) gives:

$$\frac{\partial^4\eta}{\partial\xi^4}+(u^2+\nu-\Gamma)\frac{\partial^2\eta}{\partial\xi^2}+2u\beta\frac{\partial^2\eta}{\partial\xi\,\partial\tau}+\frac{\partial^2\eta}{\partial\tau^2}=0 \quad (4)$$

which governs lateral vibration of a pipe with steady flow.

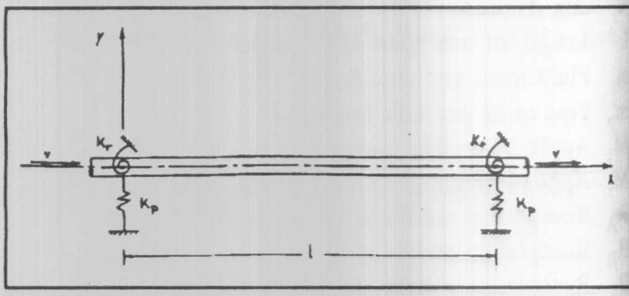

Figure 1. Pipe supports and coordinate system.

ANALYSIS OF THE PROBLEM

The problem may be analyzed by considering the wave approach which relies on an important physical quantity called "the propagation constant". This constant is estimated using the receptance functions which are obtained by solving the partial differential equation of motion along with the appropriate boundary condition.

ESTIMATION OF THE RECEPTANCES

The end receptances are found by solving equation (4) by the substitution of the following harmonic solution:

$$\eta(\xi,\tau) = \eta(\xi).e^{i\Omega\tau} \quad (5)$$

then equation (4) has been reduced to,

$$\frac{\partial^4\eta}{\partial\xi^4}+(u^2+\nu-\Gamma)\frac{\partial^2\eta}{\partial\xi^2}+i2u\beta\Omega\frac{\partial\eta}{\partial\xi}-\Omega^2\eta=0 \quad (6)$$

The solution of equation (6) is given by,

$$\eta(\xi) = \sum_{m=1}^{4} A_m e^{i\lambda m \xi} \tag{7}$$

Substituting equation (7) into equation (6) gives,

$$\lambda^4 - (u^2 + \upsilon - \Gamma)\lambda^2 - 2u\beta\Omega\lambda - \Omega^2 = 0 \tag{8}$$

Equation (8) is the characteristic equation which if solved gives four values for the wave number "λ" corresponding to each value of the frequency of motion. The values of the arbitrary constants, A_m, are determined using the boundary conditions at each end of the pipe segment, Figure (2). The boundary conditions are obtained by considering two infinitesimally short slices at the right and left ends of the pipe segment, Figure (3), and balancing both forces and moments on these slices. These boundary conditions are found to be:

$$V_l = \left(\frac{k_p}{2}\right)_l \eta(0) = \eta'''(0) = 0$$

$$\eta'''(1) = \left(\frac{K_p}{2}\right)_r \eta(1) + V_r = 0$$

$$-M_l + \left(\frac{K_r}{2}\right)_l . \eta'(0) - \eta''(0) = 0 \quad \text{and}$$

$$\eta''(1) + \left(\frac{K_r}{2}\right)_r . \eta'(1) - M_r = 0 \tag{9}$$

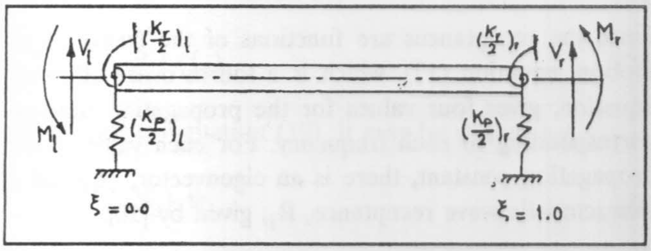

Figure 2. Porces and moments applied on the pipe segment.

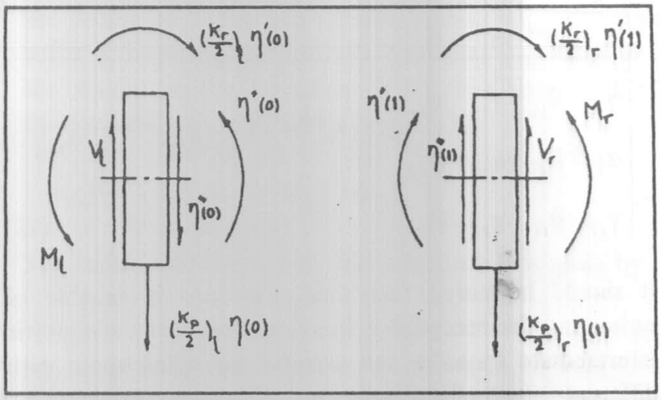

Figure 3. Forces and moments on both left and right slices of the pipe segment.

Substituting equation (7) into equation (9) and rearranging, the resulting equations in a matrix form are given as

$$[L]\{A\} = \{c\} \tag{10}$$

where the elements of the "L" matrix are functions of λ_m (m = 1, 2, 3 and 4) and the transverse and rotation stiffness at both ends of the pipe segment. The column vector, {A} represents the unknowns to be evaluated according to the constant right hand vector in equation (10) and the column vector {c} takes one of the following forms:

$$\{c\} = \{1,0,0,0\}^T, \{0,1,0,0\}^T, \{0,0,1,0\}^T \text{ or } \{0,0,0,1\}^T \tag{11}$$

where the superscript T, denotes the transpose of the vector. The above four values of the column vector, {c}, represent respectively a unit applied shear force at the left end, a unit applied shear force at the right end, a unit applied moment at the left end and a unit applied moment at the right end.

Solving the system of simultaneous equations (10), with the constant vectors in equation (11), gives four column vectors for the arbitrary constant A_m . Each column may be used to have the solution of the differential equation (6) under the corresponding applied unit force or moment. These columns are given as:

$$A_{mj} = [a_{mj}] \text{ where } m = 1,2,3,4, \text{ and } j = 1,2,3,4.$$

Using these vectors, the receptance functions are estimated as follows:

$$\begin{bmatrix} \alpha_{ll} & \beta_{ll} & \alpha_{lr} & \beta_{lr} \\ \gamma_{ll} & \theta_{ll} & \gamma_{lr} & \theta_{lr} \\ \alpha_{rl} & \beta_{rl} & \alpha_{rr} & \beta_{rr} \\ \gamma_{rl} & \theta_{rl} & \gamma_{rr} & \theta_{rr} \end{bmatrix} = [1, e^{j\lambda_m}, j\lambda_m, j\lambda_m e^{j\lambda_m}]^T A_{m,j} \quad (12)$$

It should be noted that this procedure is capable of estimating the receptance functions for either symmetric intermediate spans or the extreme bounding spans with different values of stiffness.

DETERMINATION OF THE PROPAGATION CONSTANT

To evaluate the propagation constant associated with each wave travelling along an infinitely long pipe resting on equally spaced elastic supports, consider the system shown in Figure (4-a), which constitutes a block representation of the infinite pipe. In this figure each element is coupled through two lines which represent the two coupling coordinates between each adjacent elements, Figure (4-b). Acting at these coordinates are shear forces and bending moments from the adjacent elements.

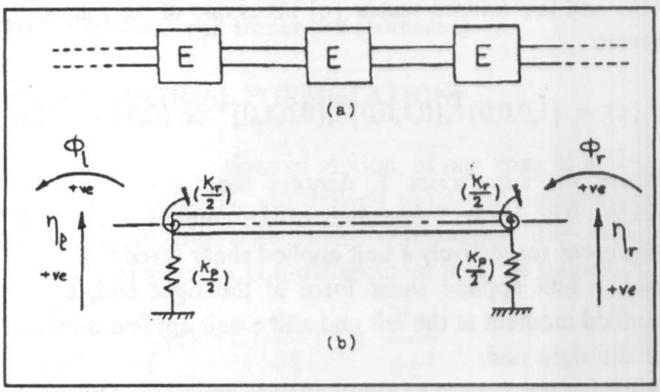

Figure 4. (a) Block representation of an infinite pipe.
(b) Single element of the system with the sign convention of the coupling coordinates.

The coordinates and forces are related through the receptance matrix, obtained in the previous section as follows:

$$\begin{bmatrix} q_l \\ q_r \end{bmatrix} = \begin{bmatrix} R_{ll} & R_{lr} \\ R_{rl} & R_{rr} \end{bmatrix} \begin{bmatrix} F_l \\ F_r \end{bmatrix} \quad (13)$$

where,

$$[q_l] = \begin{bmatrix} \eta_l \\ \varphi_l \end{bmatrix}, \quad [q_r] = \begin{bmatrix} \eta_r \\ \varphi_r \end{bmatrix},$$

$$[R_{ll}] = \begin{bmatrix} \alpha_{ll} & \beta_{ll} \\ \gamma_{ll} & \theta_{ll} \end{bmatrix}, \quad [R_{lr}] = \begin{bmatrix} \alpha_{lr} & \beta_{lr} \\ \gamma_{lr} & \theta_{lr} \end{bmatrix}$$

$$[R_{rl}] = \begin{bmatrix} \alpha_{rl} & \beta_{rl} \\ \gamma_{rl} & \theta_{rl} \end{bmatrix}, \quad [R_{rr}] = \begin{bmatrix} \alpha_{rr} & \beta_{rr} \\ \gamma_{rr} & \theta_{rr} \end{bmatrix}$$

$$[F_l] = \begin{bmatrix} V_l \\ M_l \end{bmatrix} \text{ and } [F_r] = \begin{bmatrix} V_r \\ M_r \end{bmatrix}.$$

When a characteristic wave travels through the system of identical elements with a propagation constant μ, the coordinate and the force vectors at the right hand side of the element are related to the corresponding vectors at the left side by [10].

$$\{q_r\} = e^{\mu}.\{q_l\} \quad (14)$$

$$\{F_r\} = -e^{\mu}.\{F_l\} \quad (15)$$

Substituting equations (14) and (15) into equation (13) and rearranging, we get:

$$[R_{ll} + R_{rr} - e^{\mu}.R_{lr} - e^{-\mu}.R_{rl}]\ \{F_l\} = 0 \quad (16)$$

For non-trivial solution of the above equation, the following condition must be satisfied,

$$|\ R_{ll} + R_{rr} - e^{\mu}.R_{lr} - e^{-\mu}.R_{rl}\ | = 0 \quad (17)$$

Since the receptances are functions of the frequency of motion, equation (17), which is a fourth order algebraic equation, gives four values for the propagation constant corresponding to each frequency. For each value of the propagation constant, there is an eigenvector, $\{f_I\}$ and a characteristic wave receptance, R_I, given by [10]:

$$R_I = [R_{ll} - e^{\mu I}.\ R_{lr}] \quad (18)$$

THE NATURAL FREQUENCIES OF A MULTI-SPAN PIPE:

Being disturbed at any point within any span of the infinite pipe model, four waves are excited to travel to the left and the right of the location of the disturbance. However, the presence of the two boundaries, A and B as shown in Figure (5) causes these waves to be reflected.

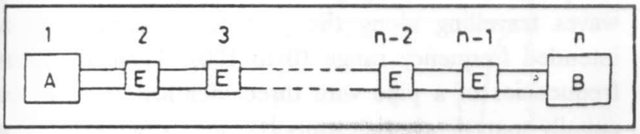

Figure 5. Block representation of a multi-span pipe.

The waves travelling from one end are reflected back upon reaching the other end so on. The reflection of any wave depends upon the properties of the boundary which are sufficiently represented by a receptance matrix, R_A or R_B. These matrices may be determined following the same procedure indicated previously. In order to estimate the natural frequencies of the n-span pipe, the compatibility of forces and displacements at each extreme boundary is assured. Suppose just one characteristic wave travels from left to right through the pipe at a given frequency. This is the incident wave, subscripted I, having the propagation constant μ_I, the force vector at the right end (F_{II}) and the displacement vector (q_{II}). This wave is the one which would be present if the pipe system was extended infinitely to the right. The normalized force vector, (f_{II}), corresponding to this characteristic wave, is related to a generalized wave coordinate ψ_I as follows:

$$\{F_{II}\} = \{f_{II}\}\ \psi_I \tag{19}$$

Upon multiplied by the characteristic wave receptance, this normalized force vector yield a normalized displacement vector given by,

$$\{S_{II}\} = [R_I]\ \{f_{II}\} \tag{20}$$

Analogous to equation (19), it may be stated that,

$$\{q_{II}\} = \{S_{II}\}\ \psi_I \tag{21}$$

The presence of the boundary causes two characteristic waves to be reflected [10]. The generalized wave coordinates, corresponding to these two reflected waves, are presented by the two element vector, (ψ_R), where the suffix R implying "Reflected". The total displacement in the system at the boundary, B, derives from both the incident and reflected waves is given by

$$\{q_B\} = \{S_{II}\}\ \psi_I + [S_{IR}]\ \{\psi_R\} \tag{22}$$

The total force exerted on the boundary B is given by

$$\{F_B\} = \{f_{II}\}\ \psi_I + [f_{IR}]\ \{\psi_R\} \tag{23}$$

The columns of the matrices, $[S_{IR}]$ and $[f_{IR}]$, are respectively the two normalized displacement and force vectors of the characteristic reflected waves.

The displacement and force vectors at the boundary B are related through the boundary receptance matrix by

$$\{q_B\} = [R_B].\ \{F_B\} \tag{24}$$

Substituting equations (22) and (23) into equation (24) yields

$$[[S_{IR}]-[R_B]\{f_{IR}\}]\ \{\psi_R\} = [[R_B]\{f_{II}\} -\{S_{II}\}]\ \psi_I \tag{25}$$

Substituting equations (18) and (20) into equation (25) gives,

$$[[R_{II} - R_B]\ [f_{IR}] - [R_{Ir}]\ [f_{IR}][e^{\mu R}]]\ \{\psi_R\} =$$

$$-[[R_{II} - R_B] - e^{\mu I}\ [R_{IR}]]\ \{f_{II}\}.\ \psi_I \tag{26}$$

Equation (26) gives the complex amplitudes of the reflected waves, (ψ_R), in terms of the magnitude of the single incident wave, ψ_I. In the general case there are two incident and two reflected waves. At the natural frequencies the modes of vibrations consist of these two positive going waves superimposing upon the two negative going waves to form a standing wave.

Allowing for the presence of all incident waves on the right boundary, expanding equation (26) yields,

$$[[R_{II} - R_B][f_{IR}] - [R_{Ir}]\ [f_{IR}][e^{\mu R}]]\ (\psi_R) +$$

$$[[R_{II} - R_B][f_{II}]- [R_{Ir}]\ [f_{II}][e^{\mu I}]]\ (\psi_I) = 0 \tag{27}$$

At the other end of the pipe, N-spans to the left, the reflected waves corresponding to (ψ_R) impinge on the boundary A which is characterized by the receptance matrix $[R_A]$.

These waves are reflected at A and become the positive going waves (ψ_I) which ultimately arrive back at B.

The displacement vector $\{q_{Aj}\}$ at the boundary A due to a single wave ψ_{Rj} is given by:

$$\{q_{Aj}\} = \{S_{IRj}\}e^{-N\mu Rj} \cdot \psi_{Rj} + \{S_{IIj}\}e^{-N\mu Ij}\, \psi ij \qquad (28)$$

When all pairs of waves are present,

$$\{q_A\} = [S_{IR}][e^{-N\mu R}]\,\{\psi_R\} + [S_{II}]\,[e^{-N\mu I}]\,\{\psi_I\} \qquad (29)$$

The total force exerted upon the boundary A by these waves is given by,

$$\{F_A\} = -[f_{IR}][e^{-N\mu R}]\,\{\psi_R\} - [f_{II}]\,[e^{-N\mu I}]\,\{\psi_I\} \qquad (30)$$

The displacement and force vectors, at the boundary A, are related by

$$\{q_A\} = [R_A]\,\{F_A\} \qquad (31)$$

Substituting equations (29) and (30) into equation (31) gives,

$$[[S_{IR}] + [R_A]\,[f_{IR}]]\,[e^{-N\mu R}]\,\{\psi_R\} +$$

$$[[S_{II}] + [R_A]\,[f_{II}]]\,[e^{-N\mu I}]\,\{\psi_I\} = 0 \qquad (32)$$

Finally, substituting the value of $[S_{II}]$ and $[S_{IR}]$ into equation (32) gives,

$$[[R_{II} + R_A]\,[f_{IR}] - [R_{Ir}][f_{IR}]\,[e^{\mu R}]\,[e^{-N\mu R}]\,\{\psi_R\} +$$

$$[[R_{II} + R_A]\,[f_{II}] - [R_{Ir}]\,[f_{II}][e^{\mu I}][e^{-N\mu I}]\,\{\psi_I\} = 0 \qquad (33)$$

It is obvious that equation (33) has the same general form of equation (27). These two groups constitute four simultaneous algebraic equations with $\{\psi_R\}$ and $\{\psi_I\}$ as the unknowns.

For a non-trivial solution of these equations the following condition must be satisfied.

$$|\,F(\Omega)\,| = 0 \qquad (34)$$

Where $F(\Omega)$ is the coefficient matrix of the above mentioned simultaneous equations. The roots of equation (34) are the natural frequencies of the N-span pipe.

RESULTS AND DISCUSSION

A computer program and some common subroutines written in FORTRAN IV are used for estimating the end receptances of a single pipe span (inner identical spans or extreme one) of a system consisting of a pipe with infinite number of spans on equally spaced identical elastic supports (K_p and K_r). Upon estimating the end receptances, the propagation constants of the flexural waves travelling along the pipe are obtained over the intended frequency range (0 to 100). Then, the natural frequencies of a pipe with three identical spans on four equally-spaced elastic supports are calculated as an example of N-span pipes.

Figures (6), (7), (8), (9) and (10) show the direct and transfer receptances, when the pipe is considered rigidly supported with zero rotational stiffness (pinned supports). The velocity of flow u is taken equal to 2.0 and the mass ratio $\beta = 0.5$. It is obvious that all receptances approach infinity at certain frequencies, which constitute the natural frequencies of a pinned-pinned single span pipe. Figures (12) through (15) show the effect of the different parameters on the natural frequencies of the pipe, shown in Figure (11), in the Ω versus u-plane for the first three modes of vibration.

Figure (12) shows the effect of the fluid pressure on the natural frequencies With two values of fluid pressure taken to be 0.0 and 2.0, it can be seen in this figure that at the same flow velocity, the increases of ν decreases the natural frequencies of the pipe associated with all the three modes of vibration.

The effect of the mass ratio is shown in Figure (13) through the two extreme values of the mass ratio which are 0.0 and 1.0. It is clear from this figure that the mass ratio has very small effect where the increase of the mass ratio slightly decreases the natural frequencies of the pipe.

Figure (14) explains the effect of the rotational stiffness with $K_r = 0.0$ and 2.0. It can be seen that the increase of the rotational stiffness shifts all curves to higher frequencies. This effect is more pronounced with lower modes. At the same flow velocity, the increase in the rotational stiffness leads to the increase of the natural frequencies of the pipe.

Three values for the transverse stiffness are considered, $K_p = 10^5, 10^3, 0.5 \times 10^3$, to show its effect on the natural frequencies. It can be seen from figure (15) that the decrease in K_p shifts all curves to lower frequencies. The amount of shift is more significant with higher modes.

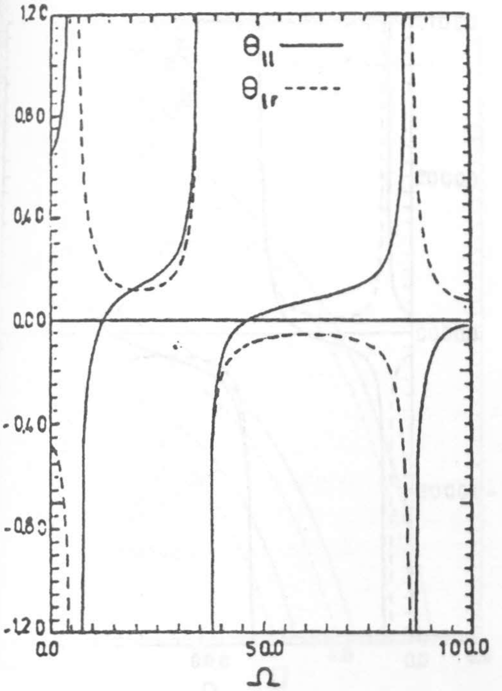

Figure 10. The direct and cross receptances, θ_y and θ_{lr}, for a single pipe-span.

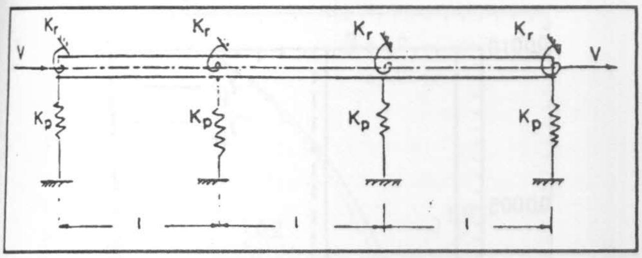

Figure 11. Three-span pipe on identical equally spaced elastic supports.

It can also be seen that the curves corresponding to each mode diverge with the increase of the velocity of flow until they meet each other at the horizontal axis at zero frequency. These points of intersection represent the conditions of the onset of static buckling associated with the first three modes. Therefore, it is clear that the transverse stiffness has no effect at all on the onset of the divergence instability.

CONCLUSIONS

1. The increase of the fluid pressure decreases the natural frequencies of the pipe for all modes of vibration by nearly the same value.
2. The increase of the mass ratio slightly decreases the natural frequencies especially with high velocities of flow.
3. The increase of the rotational stiffness increases the natural frequencies of the pipe and this is more significant for lower modes.
4. The decrease of the transverse stiffness decreases the natural frequencies of the pipe and this is more significant for higher modes.

REFERENCES

[1] Paidoussis, M.P. and Issid, N.T., "Dynamic Stability of Pipes Conveying Fluid", *J. Sound and Vibration*, (1974) v33 (3), pp. 267-294.

[2] Ashley, H. and Haviland, G., "Bending vibration of a pipeline containing flowing fluid", *J. applied mechanics*, (1950) v17, pp. 229-232.

[3] Housner, G.W., "Bending Vibration of a Pipeline Containing Flowing Fluid", *J. applied Mechanics*, (1952) v19 p. 205-208.

[4] Long, R.H., "Experimental and Theoretical Study of Transverse Vibration of a Tube Containing Flowing Fluid", *J. Applied Mechanics*, (1955) v22 pp. 65-68.

[5] Stein, R.A. and Torbiner, M.W., "Vibrations of Pipes Containing Flowing Fluids", *J. Applied Mechanics*, (1970) v92 pp. 906-916.

[6] Paidoussis, M.P. and Denise, J.P., "Flutter of Cylindrical shells Conveying Fluid", *J. Sound and Vibration*, 1971, v16, pp. 459-461.

[7] Paidoussis, M.P. and Denise, J.P., "Flutter of Thin Cylindrical Shells Conveying Fluid", *J. Sound and Vibration*, 1972, v20, pp. 9-26.

[8] Laithier, B.E. and Paidoussis, M.P., "The Equations of Motion of Initially Stressed Timoshenko Tubular Beams Conveying Fluid", *J. Sound and Vibration*, 1981, v79, n2, pp. 175-195.

[9] Ting, E.C. and Hosseinipour, A., "A Numerical Approach for Flow-Induced Vibration of Pipe-Structures", *J. Sound and Vibration*, 1983, v 88, n3, pp. 289-298.

[10] Paidoussis, M.P., Luu, T.P. and Laithier, B.E., "Dynamics of Finite-Length Tubular Beams Conveying Fluid", *J. Sound and Vibration*, 1986, v106, pp. 311-331.

[11] Mead, D.J., "Wave Propagation and Natural Modes in Periodic Systems. Part I: Mono-Coupled Systems", *J. Sound and Vibration*, 1975, v40, n1, pp. 1-18.

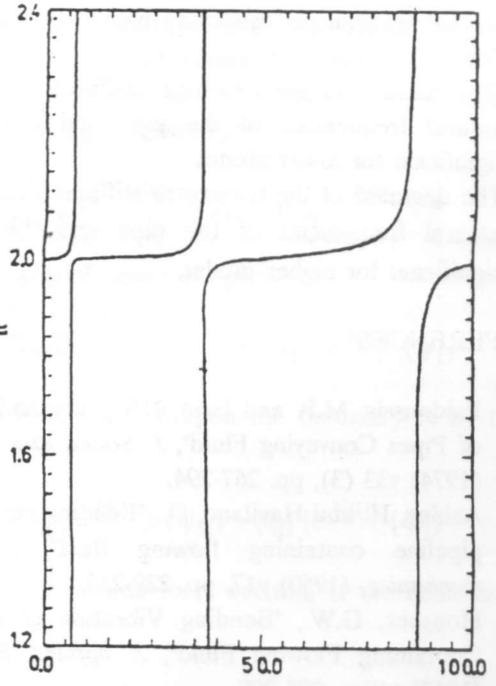

Figure 6. Direct receptance, α_{ll}, for a single pipe-span.

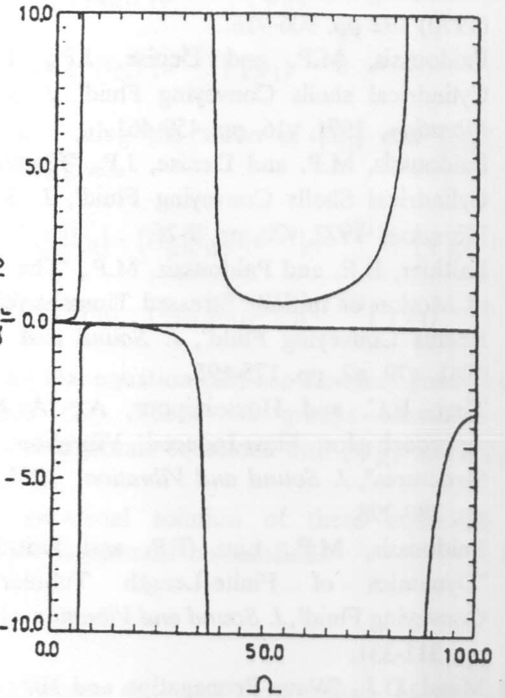

Figure 7. Cross receptance, α_{lr}, for a single pipe-span.

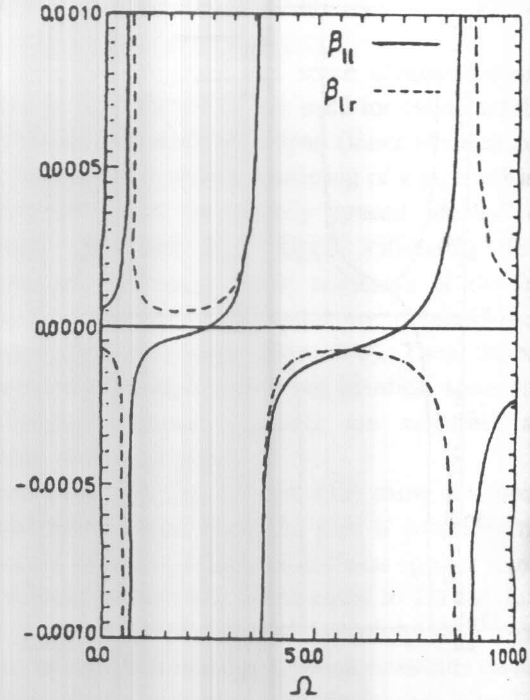

Figure 8. The direct and cross receptances, β_{ll} and β_{lr}, for a single-span.

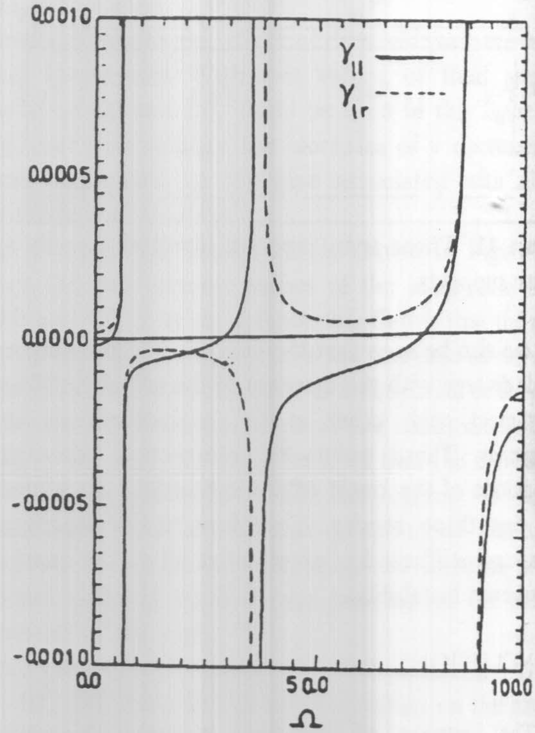

Figure 9. The direct and cross receptance, γ_{ll} and γ_{lr}, for a single-span.

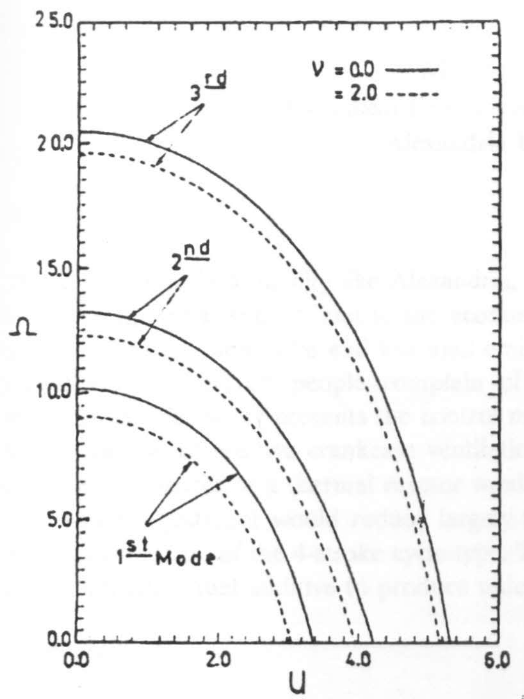

Figure 12. Effect of fluid pressure, ν, on the natural frequencies of the pipe.

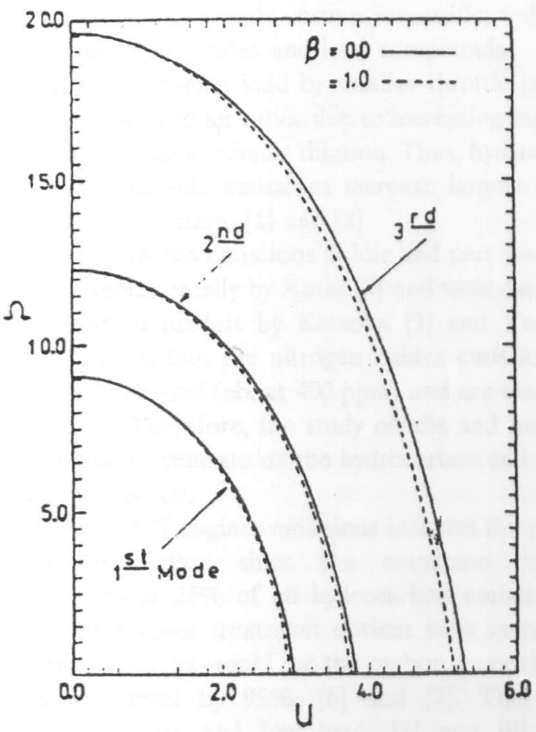

Figure 13. Effect of the mass ratio, β, on the natural frequencies of the pipe.

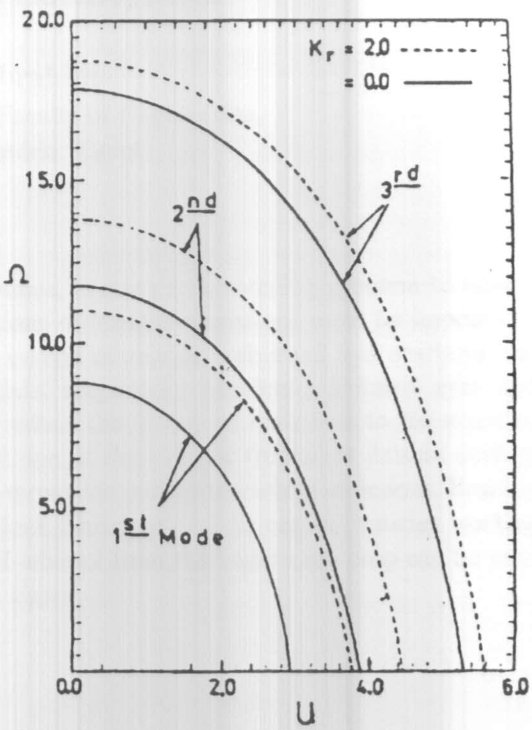

Figure 14. Effect of rotational stiffness, K_r, on the natural frequencies of the pipe.

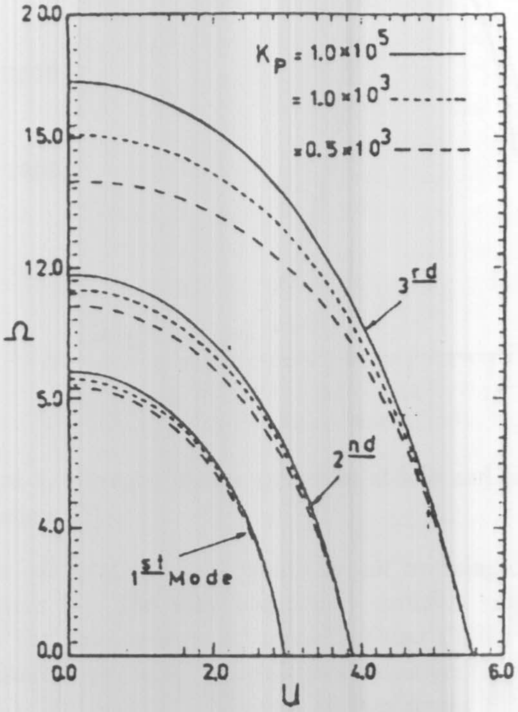

Figure 15. Effect of transverse stiffness, K_p, on the natural frequencies of the pipe.