

# DIGITAL SIMULATION OF SCR VOLTAGE-CONTROLLED INDUCTION MOTOR

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## ABSTRACT

This paper presents a simplified model for digital simulation of a SCR voltage-controlled induction motor for both four-wire and three-wire star connections. This model utilizes the two phase time-invariant representation of induction machine with respect to orthogonal stationary axes. Comparison is made between transient and steady state behaviors both when the neutral wire is connected and when it is disconnected.

## NOTATIONS

$R_s, R_r$	Stator and rotor phase resistance respectively.
$L_s, L_r, M$	Apparent three phase stator, rotor, and mutual inductance respectively.
$s, r$	Suffixes indicating stator and rotor quantities respectively.
$q, p, o$	Suffixes indicating quadrature axes, direct axes, and zero sequence quantities respectively.
$\omega_r$	Rotor mechanical angular velocity.
$T_e, T_l, T_d$	Electrical load and drag torque respectively.
$T_x$	Thyristor x.
$T_xF$	Logical trigger signal for $T_x$ .
$T_xC$	Logical conduct signal for $T_x$ .
$V_xF$	Logical forward bias signal for $T_x$ .
$I$	Logical direction indicator for current $i$ .
$H$	Logical absolute magnitude indicator for $i$ .
$\bar{A}$	A bar under a variable indicates the use of the matrix notation.
$\bar{H}$	A bar above logical variable indicates the logical inverse "NOT(H)".

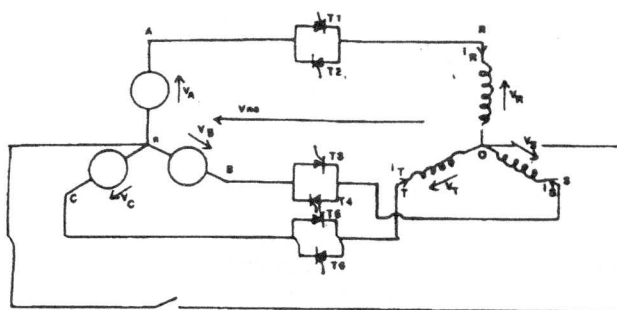


Figure 1. SCR voltage controlled induction motor.

The associated gate-control circuit must be capable of triggering at the same instant two thyristors, one in each of two phases. If short duration pulses are employed, each thyristor requires two pulses in each cycle, displaced by  $60^\circ$  to various thyristors in the same sequence as that of the supply voltage. On the other hand, if the star point of the induction motor stator windings is connected to the supply star point (four-wire system), each thyristor is fired once in each cycle and it is possible for the current to flow through an individual phase with the return path provided by the neutral. In this case the gate-control circuit provides six isolated pulses, one for each thyristor with a phase angle of  $60^\circ$  between two consecutive pulses [2].

Several authors have produced analyses and models to predict the behavior of SCR voltage controlled induction motors in the absence of the neutral connection. Lipo [4] used the symmetry of the controller to produce an analytical solution in closed form to give the steady state response of the SCR voltage controlled induction motor. Other workers [5, 6] use model analysis to determine the

## INTRODUCTION

SCR voltage controllers are increasingly used to control the speed of three phase induction motor, especially with variable torque loads such as fans [1-3]. They are also used to achieve soft starting for cage induction motors [3]. A typical thyristor controller for three phase star connected induction motor is shown in Figure (1).

In the absence of a neutral connection (three-wire system), at least two thyristors must conduct simultaneously to allow current to flow through the load.

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circuit configuration when zero, one, two, or three thyristors are conducting.

Different differential equations and different model structure are required for each mode. This creates further complexity in the resultant computer programs.

This paper presents a simplified model for simulation of a SCR voltage controlled induction motor when the neutral wire is connected and when it is disconnected. The model utilizes the two phase time-invariant representation of an induction machine with respect to orthogonal stationary axes [7]. This model has been programmed and results are compared with those from previous work [5, 6]. A comparison between system behaviors with and without neutral wire is made. Harmonic analysis for steady state torque, phase current, and phase voltage are carried out for both cases. The effect of changing the thyristors firing angle on the induction motor speed in both cases is investigated.

SYSTEM ANALYSIS

*Analysis of four-wire star connected induction motion*

The study of the transient performance of three phase induction motors and their behavior on non-sinusoidal supplies is facilitated by transformation of the machine variables to a stationary reference frame (d-q-o). This transformation eliminates the rotor angle dependent terms from the induction machine differential equations. The resultant equations describing the behavior of the induction machine may be expressed as [7]:

$$\underline{V} = \underline{R} \underline{i} + \underline{L} \underline{pi} \tag{1}$$

where

$$\underline{V} = \begin{bmatrix} V_{qs} & V_{ds} & V_{os} & V_{qr} & V_{dr} \end{bmatrix}^T$$

and for cage rotor

$$\underline{V} = \begin{bmatrix} V_{qs} & V_{ds} & V_{os} & 0 & 0 \end{bmatrix}^T$$

$$\underline{i} = \begin{bmatrix} i_{qs} & i_{ds} & i_{os} & i_{qr} & i_{dr} \end{bmatrix}^T$$

$$\underline{R} = \begin{bmatrix} R_s & 0 & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 & 0 \\ 0 & 0 & R_s & 0 & 0 \\ 0 & -\dot{\theta}_r M & 0 & R_r & -\dot{\theta}_r L_r \\ \dot{\theta}_r M & 0 & 0 & \dot{\theta}_r L_r & R_r \end{bmatrix},$$

$$\underline{L} = \begin{bmatrix} L_s & 0 & 0 & M & 0 \\ 0 & L_s & 0 & 0 & M \\ 0 & 0 & L_{os} & 0 & 0 \\ M & 0 & 0 & L_r & 0 \\ 0 & M & 0 & 0 & L_r \end{bmatrix}$$

$$L_{os} = L_s - M$$

And the relationship between stator (d-q-o) term real stator variables is given by:

$$\begin{bmatrix} V_R \\ V_S \\ V_T \end{bmatrix} = \underline{C} \begin{bmatrix} V_{qs} \\ V_{ds} \\ V_{so} \end{bmatrix},$$

$$\begin{bmatrix} i_R \\ i_S \\ i_T \end{bmatrix} = \underline{C} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{so} \end{bmatrix}$$

where

$$\underline{C} = \begin{bmatrix} 1 & 0 & 1 \\ -1/2 & -\sqrt{3}/2 & 1 \\ -1/2 & \sqrt{3}/2 & 1 \end{bmatrix}$$

Equation (1) can be rewritten as:

$$\underline{pi} = \underline{L}^{-1} \underline{V} - \underline{L}^{-1} \underline{R} \underline{i}$$

where

$$\underline{L}^{-1} = \begin{bmatrix} -L_r/D & 0 & 0 & M/D & 0 \\ 0 & -L_r/D & 0 & 0 & M/D \\ 0 & 0 & 1/L_{os} & 0 & 0 \\ M/D & 0 & 0 & -L_s/D & 0 \\ 0 & M/D & 0 & 0 & -L_s/D \end{bmatrix}$$

$$D = M^2 - L_s L_r$$

Equation of motion is given by:

$$J p \dot{\theta}_r + T_d + T_L = T_e$$

$$\text{i.e., } p \dot{\theta}_r = (T_e - T_L - T_d)/J \quad (5)$$

where  $T_e = M(i_{qs} i_{dr} - i_{ds} i_{qr})$  p.u.

Equations (4) and (5) completely define the dynamic model for any transient analysis. These equations can be rearranged in state space format:

$$\dot{\underline{X}} = f(\underline{X}, \underline{V}) \quad (6)$$

$$\text{where } \underline{X} = [i_{qs} \ i_{ds} \ i_{dr} \ \dot{\theta}_r \ i_{os}]^T$$

Equation (6) are non-linear equations, so that numerical integration techniques are required to predict the system performance. For each integration step, motor phase voltages ( $V_R$ ,  $V_S$ , and  $V_T$ ) can be determined according to conducting state of the thyristors as follows:

i. When either thyristor in a phase is connected to the supply and its voltage will be equal to the corresponding supply voltage.

ii. When neither thyristor in a phase is conducting, that phase is open circuited and the open circuit e.m.f.  $V_{\text{phase oc}}$  appears across the winding.

To calculate the open circuit e.m.f. which appears across any phase, one of the following three cases may be considered:

Case 1- All phases are open circuited simultaneously:

$$i_R = i_S = i_T = 0, \text{ and consequently } i_{qs} = i_{ds} = i_{os} = 0,$$

$$\text{i.e. } x_1 = x_2 = x_6 = 0, \text{ and } px_1 = px_2 = px_6 = 0.$$

Substitution into equations (6) gives,

$$V_{qs} = Mx_4x_5 - (R_r M/L_r)x_3$$

$$V_{ds} = -Mx_3x_5 - (R_r M/L_r)x_4$$

$$V_{os} = 0$$

Substitution into equation (2) gives,

$$V_{Roc} = V_{qs} \quad (7.a)$$

$$V_{Soc} = (-V_{qs} - \sqrt{3} V_{ds})/2 \quad (7.b)$$

$$V_{Toc} = (-V_{qs} + \sqrt{3} V_{ds})/2 \quad (7.c)$$

Case 2- Two phases are open circuited simultaneously:

(a) With phases S and T opened

$$i_S = i_T = 0, \text{ and } V_R = V_A$$

Substitution into equation (3) gives,  $i_{qs} = 2i_{os}$ , and  $i_{ds} = 0$ , i.e.,  $x_1 = 2x_6$ , and  $x_2 = 0$ , and consequently  $px_1 = 2px_6$ , and  $px_2 = 0$ .

Substitution into equations (6) gives,

$$V_{ds} = -Mx_3x_5 - (R_r M/L_r)x_4 - (M^2/L_r)x_1x_5,$$

$$V_{qs} = (L_{os}R_sL_r x_1 - L_{os}R_r Mx_3 + L_{os}L_r Mx_4x_5 - 2R_sDx_6 - 2DV_A)/(L_{os}L_r - 2D)$$

From equation (2) it can be seen that,

$$V_{qs} = (2V_A - V_{Soc} - V_{Toc})/3,$$

$$V_{ds} = (V_{Toc} - V_{Soc})/\sqrt{3}, \text{ and}$$

$$V_{os} = (V_A + V_{Soc} + V_{Toc})/3.$$

From these equations

$$V_{Soc} = V_A - (3V_{qs} + \sqrt{3} V_{ds})/2 \quad (8.a)$$

$$V_{Toc} = V_A - (3V_{qs} - \sqrt{3} V_{ds})/2 \quad (8.b)$$

(b) With phases R and T open circuited  $V_{Roc}$  and  $V_{Toc}$  are found to be:

$$V_{Roc} = V_B + (3V_{qs} + \sqrt{3} V_{ds})/2 \quad (8.c)$$

$$V_{Toc} = V_B + \sqrt{3} V_{ds} \quad (8.d)$$

where

$$V_{qs} = \{L_{os}L_rR_sx_1 + (L_{os}-3D/2L_r)(M^2x_2x_5 - R_rMx_3 + L_rMx_4x_5) + R_sDx_6 - (\sqrt{3}D/2L_r)(R_rMx_4 + L_rMx_3x_5 + M^2x_1x_5) + DV_B\} / (L_{os}L_r - 2D)$$

$$V_{ds} = \sqrt{3} V_{qs} + (\sqrt{3}/L_r)(M^2x_2x_5 - R_rMx_3 + L_rMx_4x_5) - (1/L_r)(R_rMx_4 + L_rMx_3x_5 + M^2x_1x_5)$$

(c) With phases R and S open circuited, the open circuit e.m.fs are,

$$V_{Roc} = V_C + (3V_{qs} - \sqrt{3} V_{ds})/2 \quad (8.e)$$

$$V_{Soc} = V_C - \sqrt{3} V_{ds} \quad (8.f)$$

where

$$V_{qs} = \{L_{os}L_rR_sx_1 + (L_{os}-3D/2L_r)(M^2x_2x_5 - R_rMx_3 + L_rMx_4x_5) - R_sDx_6 + (\sqrt{3}D/2L_r)(R_rMx_4 + L_rMx_3x_5 + M^2x_1x_5) + DV_C\} / (L_{os}L_r - 2D)$$

$$V_{ds} = -\sqrt{3} V_{qs} + (\sqrt{3}/L_r)(M^2x_2x_5 - R_rMx_3 + L_rMx_4x_5) - (1/L_r)(R_rMx_4 + L_rMx_3x_5 + M^2x_1x_5)$$

Case 3- One phase only is open circuited:

(a) With phase R open circuited

$$i_R = 0, V_S = V_B, \text{ and } V_T = V_C$$

substitution into equation (2) gives,

$$V_{qs} = (2V_{Roc} + V_A)/3$$

$$V_{os} = (V_{Roc} - V_A)/3$$

Substitution with  $i_R = 0$  in equation (3) gives,

$$x_1 = -x_6, \text{ and consequently } px_1 = -px_6.$$

Substitution into equation (6) gives,

$$V_{Roc} = \{(3L_{os})(R_sL_r x_1 + M^2x_2x_5 - R_rMx_3 + L_rMx_4x_5) - 3R_sDx_6 - (L_{os}L_r + D)V_A\} / (2L_{os}L_r - D) \quad (9.a)$$

(b) With phase S open circuited  $V_{Soc}$  is found to be

$$V_{Soc} = \{(3L_{os}/2)(R_rMx_3 - R_sL_r x_1 - M^2x_2x_5 - L_rMx_4x_5) + (3\sqrt{3} L_{os}/2)(R_rMx_4 + L_rMx_3x_5) + R_sL_r x_2 + M^2x_1x_5 + L_rMx_3x_5\} - 3R_sDx_6 - (L_{os}L_r - D)V_B / (2L_{os}L_r - D) \quad (9.b)$$

(c) With phase T open circuited  $V_{Toc}$  is found to be

$$V_{Toc} = \{(3L_{os}/2)(R_rMx_3 - R_sL_r x_1 - M^2x_2x_5 - L_rMx_4x_5) - (3\sqrt{3} L_{os}/2)(R_rMx_4 - R_sL_r x_2 + M^2x_1x_5 + L_rMx_3x_5) - 3R_sDx_6 - (L_{os}L_r - D)V_C\} / (2L_{os}L_r - D) \quad (9.c)$$

*Analysis of three-wire star connected induction motor*

The fact that there is no neutral connection means that

1. No zero sequence current can flow ( $i_{os} = 0$ ), i.e.,  $x_6 = 0$  (10)

2. The sum of motor phase voltages equals zero, i.e.,  $V_R + V_S + V_T = 0$  and consequently  $V_{os} = 0$  (11)

Therefore the system state space variables  $\underline{X}$  can be reduced to five states only by eliminating  $x_6$ .

$$\underline{X} = [i_{qs} \ i_{ds} \ i_{qr} \ i_{dr} \ \dot{\theta}_r]^T$$

and the system differential equations will be the same as equations (6) but with  $V_{os} = 0$ .

Substitution from equation (11) into equation (2) gives

$$V_{qs} = (2V_R - V_S - V_T)/3 \text{ and } V_{ds} = (V_T - V_S)/\sqrt{3}$$

From Figure (1) it can be seen that,

$$V_R = V_{Rn} - V_{on} \quad (12.a)$$

$$V_S = V_{Sn} - V_{on} \quad (12.b)$$

$$V_T = V_{Tn} - V_{on} \quad (12.c)$$

but  $V_R + V_S + V_T = 0$ , therefore,

$$V_{qs} = (2V_{Rn} + V_{Sn} + V_{Tn})/3 \quad (13.a)$$

$$V_{ds} = (V_{Tn} - V_{Sn})/\sqrt{3} \quad (13.b)$$

$$V_{on} = (V_{Rn} + V_{Sn} + V_{Tn})/3 \quad (13.c)$$

For each integration step the voltages  $V_{Rn}$ ,  $V_{Sn}$ , and  $V_{Tn}$  can be determined according to the conducting state of the thyristors as follows:

i. When either thyristor in each phase is conducting (all three phases are connected to the supply) then,

$$V_{Rn} = V_A, V_{Sn} = V_B, \text{ and } V_{Tn} = V_C \quad (14)$$

ii. When all thyristors are non-conducting (all three phases are open circuited) or when two phases are open circuited no current will flow through the motor and the open circuit e.m.f.s. appear across the windings. The value of these e.m.f.s. are the same as those given in equations (7-a), (7-b), and (7-c).

iii. When neither thyristor in a phase is conducting while the other two phases are connected to the supply, then:

(a) If phase R is opened while phases S and T are connected to the supply, then  $i_R = 0$ , and therefore  $i_{qs} = 0$ , i.e.  $x_1 = 0$ , and consequently  $px_1 = 0$ . Substitution into equation (6) gives,

$$V_{Roc} = (M^2x_2x_5 - R_r Mx_3 + L_r Mx_4x_5) / L_r \quad (15-a)$$

from Figure (1) it can be seen that,

$$V_{Sn} = V_B \quad (15-b)$$

$$V_{Tn} = V_C \quad (15-c)$$

$$V_{Rn} = V_A - V_{AR}$$

where  $V_{AR}$  is the voltage across the blocking thyristors. It can be easily shown that,

$$V_{AR} = (3/2)(V_A - V_{Roc}) \quad (15-d)$$

giving,

$$V_{Rn} = (3V_{Roc} - V_A) / 2 \quad (15-e)$$

(b) If phase S is opened while phase R and T are connected to the supply, then

$$V_{Soc} = \{ \sqrt{3} (R_r Mx_4 + M^2x_1x_5 + L_r Mx_3x_5) + R_r Mx_3 - M^2x_2x_5 - L_r Mx_4x_5 \} / (2L_r) \quad (16-a)$$

$$V_{Sn} = (3V_{Soc} - V_B) / 2 \quad (16-b)$$

$$V_{Rn} = V_A \quad (16-c)$$

$$V_{Tn} = V_C \quad (16-d)$$

and the voltage across the blocking thyristors  $V_{BS}$  is

$$V_{BS} = (3/2)(V_B - V_{Soc}) \quad (16-e)$$

(c) If phase T is opened while phases R and S are connected to the supply, then

$$V_{Toc} = \{ -\sqrt{3} (R_r Mx_4 + M^2x_1x_5 + L_r Mx_3x_5) + R_r Mx_3 - M^2x_2x_5 - L_r Mx_4x_5 \} / (2L_r) \quad (17-a)$$

$$V_{Tn} = (3V_{Toc} - V_C) / 2 \quad (17-b)$$

$$V_{Rn} = V_A \quad (17-c)$$

$$V_{Sn} = V_B \quad (17-d)$$

and the voltage across the blocking thyristors  $V_{CT}$  is

$$V_{CT} = (3/2)(V_C - V_{Toc}) \quad (17-e)$$

#### Analysis of SCR voltage controller

Any thyristor is conducting if its gate is excited and it is forward biased or if it is already conducting and its current is in the forward direction and greater than its holding current.

In boolean terms:

$$T1C = T1F.V1F + T1C.IR.HR \quad (18-a)$$

$$T2C = T2F.\bar{V}1F + T2C.\bar{I}R.\bar{H}R \quad (18-b)$$

$$T3C = T3F.V3F + T3C.IS.HS \quad (18-c)$$

$$T4C = T4F.\bar{V}3F + T4C.\bar{I}S.\bar{H}S \quad (18-d)$$

$$T5C = T5F.V5F + T5C.IT.HT \quad (18-e)$$

$$T6C = T6F.\bar{V}5F + T6C.\bar{I}T.\bar{H}T \quad (18-f)$$

From Figure (1) for the case of four wire induction motor and from equations (15-d), (16-e), and (17-e) for the case of three wire induction motor, it can be seen that:

1. V1F is true if  $V_A$  is greater than  $V_{Roc}$ .
2. V3F is true if  $V_B$  is greater than  $V_{Soc}$ .
3. V5F is true if  $V_C$  is greater than  $V_{Toc}$ .

## COMPUTER SIMULATION PROGRAM

Figure (2) shows the simplified flow-chart for the computer simulation program of a SCR voltage controlled induction motor. The program was written in Pascal and used a fourth order Runge-kutta integration algorithm with nominal fixed step. Additional iterations were used to determine accurately the instant at which thyristors are fired and extinguished. The thyristors were triggered with a firing angle measured from the instant of voltage zero of the respective phases. Both program input data and output results were in per unit values. The program input data were machine parameters, supply voltage, supply frequency, thyristors firing angle and the integration step. The program output comprises the motor speed, torque, currents, phase voltages and line voltages.

## RESULTS AND DISCUSSION

In order to verify the developed computer program, the motor parameters and operating conditions used were the same as those quoted in [5]: viz.

- 10 hp, 4 pole, 60 Hz, 230 V, 26.2 A, star connected squirrel cage induction motor.  
 Base voltage  $V_{\text{base}} = 187.8$   
 Base current  $I_{\text{base}} = 37.05$  A  
 Base impedance  $Z_{\text{base}} = 5.075$  ohm  
 Base torque  $T_{\text{base}} = 55.37$  Nm  
 Base time  $t_{\text{base}} = 2.653$  ms  
 Thyristors firing angle =  $70^\circ$   
 Load torque  $T_L = 0.05 + 0.9\theta_r$  p.u. (19)

Figure (3-a) shows the simulated torque and speed pattern when the neutral wire is disconnected (three-wire). The steady state speed of the motor in this case was found to be 0.932 p.u. Figures (3-b) and (3-c) show the simulated phase current and phase voltage patterns at the same operating point. These results are practically identical to those given in [5, 6].

Figures (4-a), (4-b), and (4-c) show the simulated torque, speed, phase current, and phase voltage patterns when the neutral wire is connected assuming the same load torque. Thyristors firing angle is chosen to be  $88.2^\circ$  so that the steady state speed of the motor will be 0.932 p.u. as in the previous case.

Comparison between transient responses with and without neutral wire Figures (4-a) and (3-a) show that when the neutral wire is connected (four-wire) the system is more damped and the magnitude of torque pulsations is reduced. By applying furrier analysis to the simulated wave forms Figures (3) and (4), the amplitude of

harmonic components for steady state torque, phase current, and phase voltage were calculated when neutral wire is connected and when it is disconnected. The results are summarized in Table (1). Fourier analysis shows that the current in the four-wire case has a substantial third harmonic component due to the neutral connection which permits the flow of third harmonic current. Comparison results in Table (1) shows that:

1. In the four-wire case, where third harmonic is present, the fifth and the seventh harmonics currents are greatly reduced.
2. The dominant (sixth) harmonic component of torque in the four-wire case is about 50% less than that in the three wire case. This result was expected since the sixth harmonic torque is developed from the interaction between the magnetizing current and both fifth and seventh harmonic currents.
3. The current harmonic distortion  $[(\sum I_n^2)^{1/2} / I_{1,n=2,3,4,5,6,7,8,9,10} \dots]$  is found to be:  
 H.D. = 0.176 for the three-wire case,  
 H.D. = 0.468 for the four-wire case,  
 which means that the current in the three-wire case is less distorted than that in the four-wire case.

The torque-slip curves for the SCR voltage controlled induction motor at different thyristors firing angles are given in Figures (5-a) and (5-b) when the neutral wire is disconnected and when it is connected respectively. From these curves and for the load torque given by equation (19), the relation between the thyristors firing angle  $\alpha$  and the motor speed can be obtained Figure (6).

## CONCLUSION

An efficient method of predicting the performance of SCR voltage controlled star connected induction motor with and without neutral wire has been demonstrated. The method produces stator phase information directly and does not require matrix inversion at each computational interval. No mode changes are required. This makes the resultant digital computer program relatively simple and reduces the C.P.U. time. The simulated results show that the four wire system is more damped and has less harmonic torque components than the three-wire system. However, the four-wire system has a substantial third harmonic current which is a disadvantage as it increases the motor losses and increases the harmonic distortion of the current.

By choosing the appropriate thyristors firing angles, the proposed model (the four-wire case) can be used for simulation of other types of phase controlled induction motors, as the phase-amplitude modulation controlled induction motors [3].



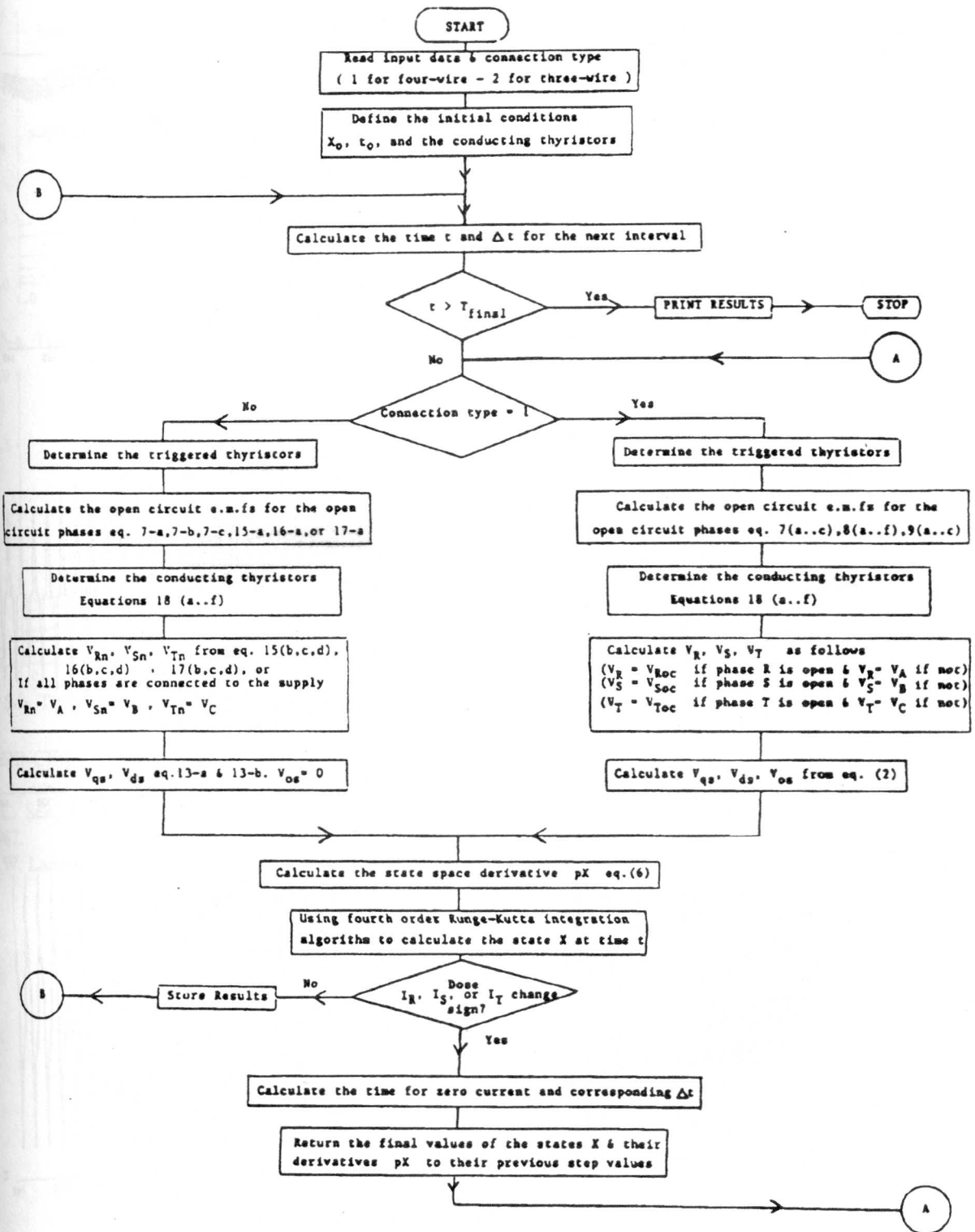
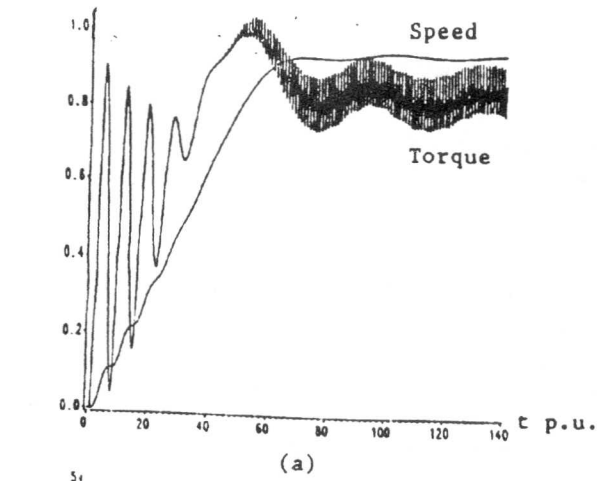
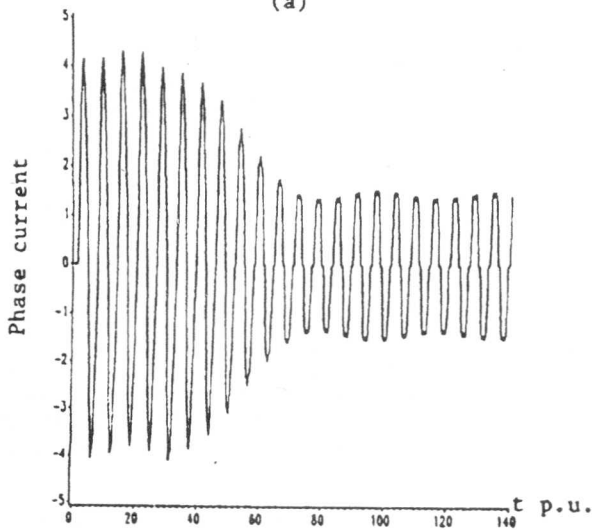


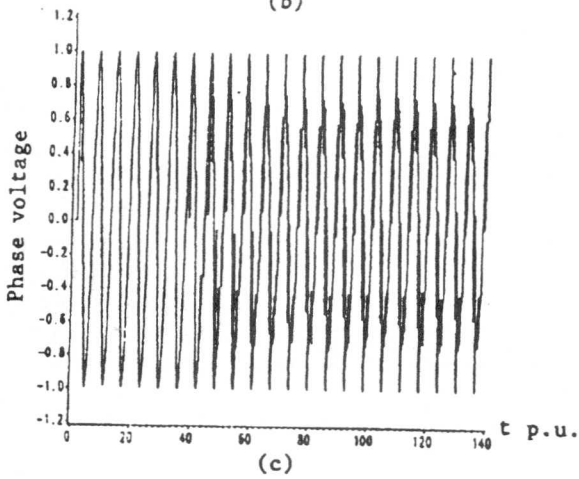
Figure 2. Program flow-chart.



(a)

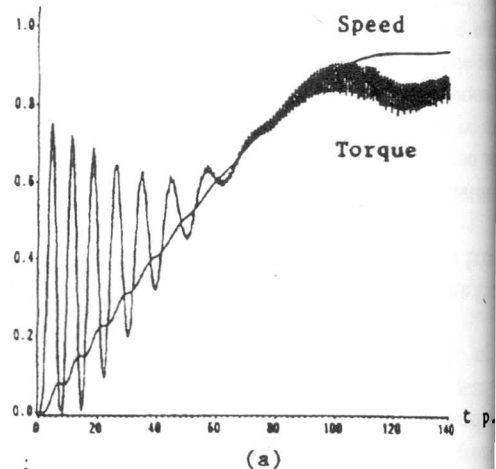


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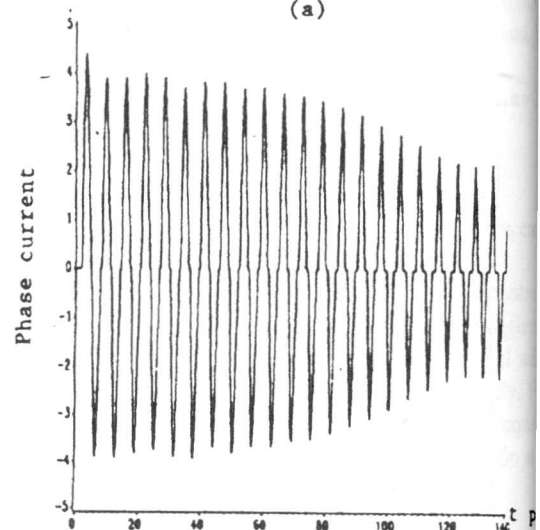


(c)

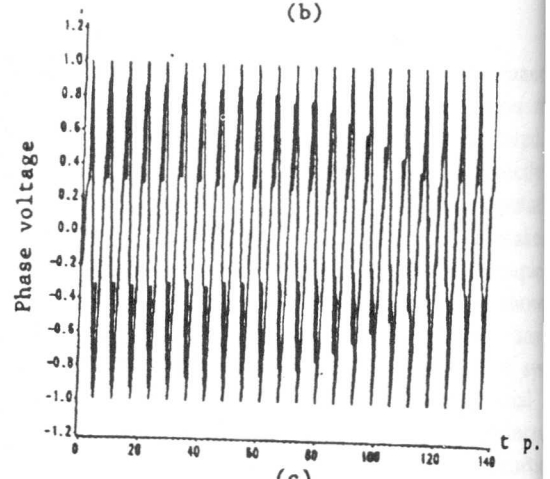
Figure 3. Simulated results in p.u. (three-wire).



(a)



(b)



(c)

Figure 4. Simulated results in p.u. (four-wire).



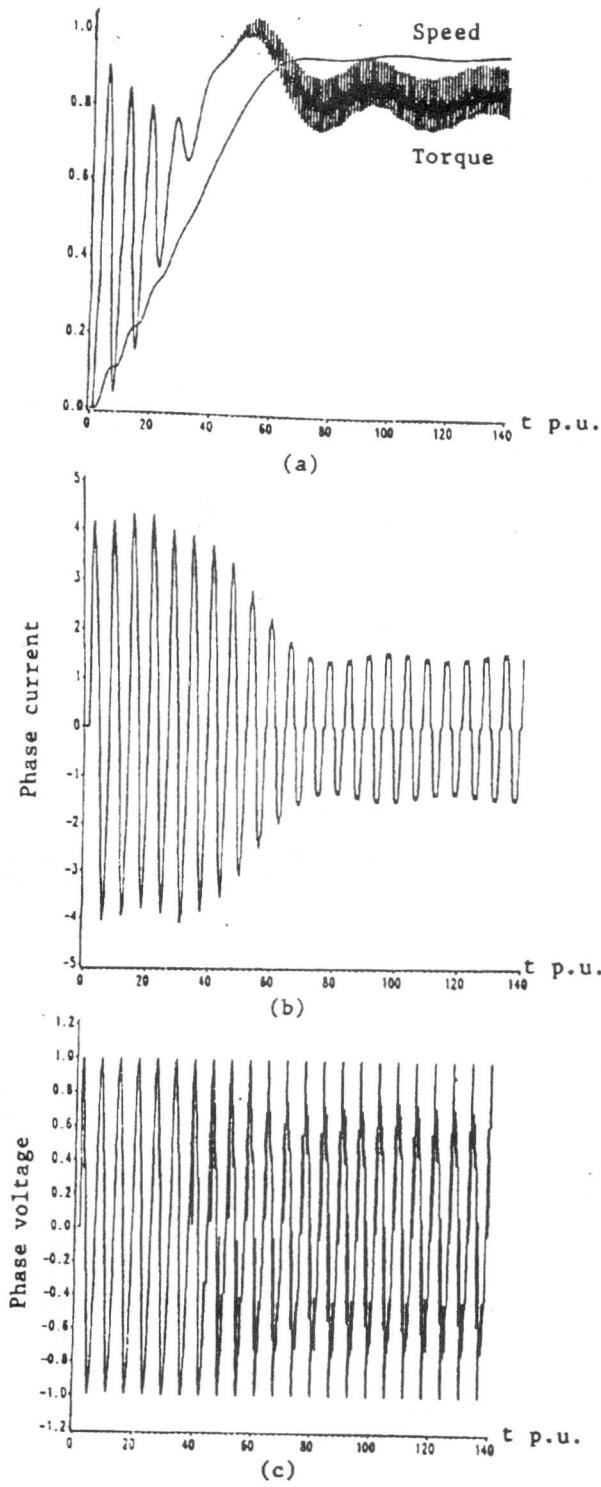


Figure 3. Simulated results in p.u. (three-wire).

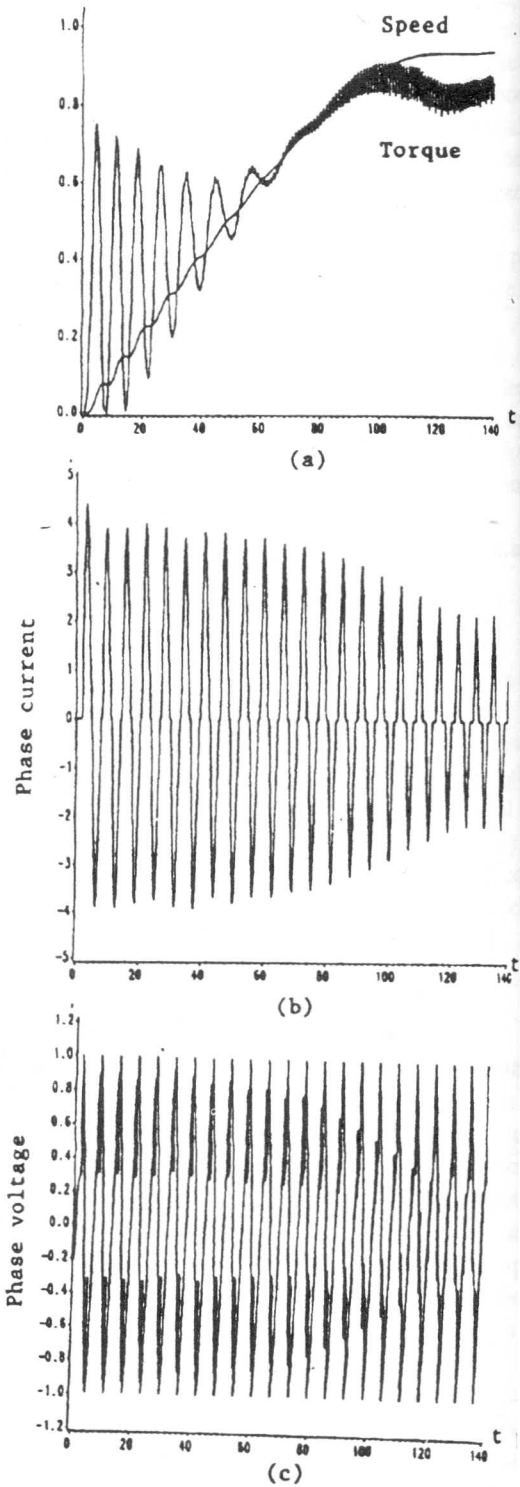


Figure 4. Simulated results in p.u. (four-wire)

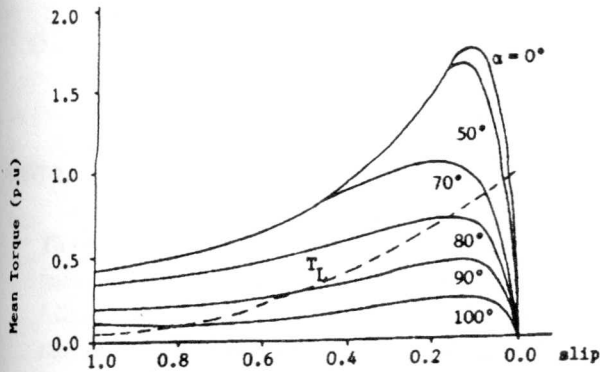


Figure 5-a. Torque/Slip for three-wire case.

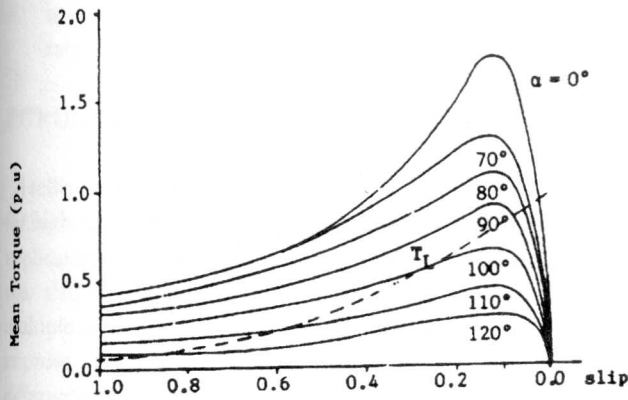


Figure 5-b. Torque/Slip for four-wire case.

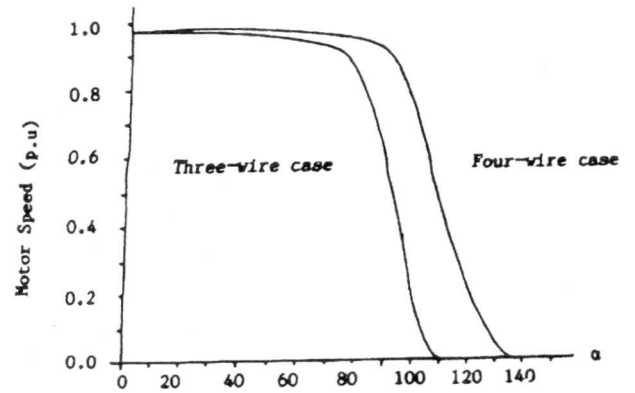


Figure 6. Relation between motor speed and thyristors firing angle.

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Table (1) Amplitude of Harmonic Components in p.u.

Harmonic No.	Torque		Current		Phase Voltage	
	3-wire	4-wire	3-wire	4-wire	3-wire	4-wire
0	0.8363	0.8370	0.0000	0.0000	0.0000	0.0
1	0.0000	0.0000	0.1559	0.1559	0.7841	0.7849
3	0.0000	0.0000	0.0000	0.7215	0.1112	0.2159
5	0.0000	0.0000	0.2428	0.0145	0.2984	0.1025
6	0.0658	0.0323	0.0000	0.0000	0.0000	0.0000
7	0.0000	0.0000	0.1291	0.0872	0.1742	0.0152
9	0.0000	0.0000	0.0000	0.0742	0.1084	0.0824
12	0.0113	0.0046	0.0000	0.0000	0.0000	0.0000