# DEPTHS OF FLOW OVER RECTANGULAR BROAD CRESTED WEIR 

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## ABSTRACT

Since depths of free parallel flow over broad crested weirs are strongly related to the velocity coefficient $C_{v}$, it is considered an important parameter for studying characteristics of flow over such weirs. Water surface profile over broad crested weir depends on the ratio between the weir head H and its' length L . The flow was found to be parallel when $\mathrm{L} / \mathrm{H}$ varies between 2.5 to 10 [1], which in the present study clearly ranges from 4 to 9 . The depths of flow over broad crested weir are analyzed theoretically by applying the energy equation. Theoretical equations are developed for both depths of flow at the middle part of weir crest and the brink section. Also theoretical relationship is obtained for the discharge coefficient, $\mathrm{C}_{\mathrm{d}}$. The predicted relationships are checked by the experimental measurements in a laboratory flume and good results are found to exist.

## NOMENCLATURE

Froude number of the approaching flow for the brink section;
Gravity acceleration;
Head over weir;
Total head over weir, $\mathrm{H}_{\mathrm{o}}=\mathrm{H}+\frac{\alpha_{1} \mathrm{v}_{\mathrm{z}}^{2}}{2 \mathrm{~g}}$;
Depth of parallel flow over weir;
Depth of flow at the brink section;
Critical depth of flow;
Dimensionless depth, $K=\left(h_{c} / H_{o}\right)^{3 / 2}$;
Coefficient of pressure at the brink section;
Length of the weir;
Height of the weir;
Discharge per unit width of the weir;
Velocity of the approaching flow of the weir;
Velocity of flow over the weir;
Velocity of flow at the brink section;
$\alpha_{1}, \alpha_{2}$
and $\alpha_{3}$ Energy correction coefficients.

## 1. INTRODUCTION

Broad crested weirs are commonly used as control structures in hydraulic engineering practice. Study of flow characteristics over such weirs is required in their design and operation. The flow over broad crested weir is basically similar to the flow in short open channel ending with a free fall.

The free flow over broad weirs is mainly characterized by two depths; The first represents the depth of parallel flow $h$ over the middle part of weir crest, while the second is the flow depth at the downstream end of the weir crest, which is the brink depth $h_{b}$. The depth of flow $h$ is used to measure discharges passing over the weir. The brink depth is employed in the design of stilling basin downstream of the weir. Furthermore, the brink depth is considered a true flow meter in free overfalls.
The depth of parallel flow in some literatures $[2,3,4]$ is considered constant and is equal to the critical depth $h_{c}$ (depth corresponding to minimum specific energy), i.e., $h=h_{c}$. Other literatures [5] define $h$ as the depth of maximum discharge, i.e., $h=(2 / 3) H_{0}$, where $H_{o}$ is total head over the weir. The above two assumptions are valid for ideal case when losses are negligible, i.e. $\mathrm{C}_{\mathrm{v}}=1.0$. Due to the flow internal losses caused by the vertical contraction at the weir entrance, the coefficient $\mathrm{C}_{\mathrm{v}}$ will be less than unity. Accordingly, $\mathrm{h}<\mathrm{h}_{\mathrm{c}}$, and $\mathrm{h}<(2 / 3) \mathrm{H}_{\mathrm{o}}$.

The brink depth has been widely investigated experimentally and theoretically. Rouse[6] obtained the classical brink depth ratio, $\mathrm{h}_{\mathrm{b}} / \mathrm{h}_{\mathrm{c}}=0.715$ for parallel flow in rectangular channel of zero slope ending with a free fall. In a series of experiments in rectangular channel, an average values of $h_{b} / h_{c}$ appeared to be $0.705,0.781$ and 0.714 according to References [7], [8] and [9] respectively. Considering zero pressure distribution at the brink section and applying the momentum equation, Diskin [10] obtained a theoretical value of $h_{b} / h_{c}=0.67$ and 0.73 experimentally. Diskin's work was improved by

Rajaratnam [11,12,13] by deriving a general equation for nonzero pressure at the brink section. Rajaratnam obtained a theoretical value of $h_{b} / h_{c}$ equal to 0.705 , while it's average experimental value was 0.715 when Froude number F is 1.0. Assuming a free vortex motion at the brink section [14], theoretical and experimental values of $h_{b} / h_{c}$ were found to be 0.673 and 0.715 , respectively. For a broad crested weir of trapezoidal section [15], the ratio $h_{b} / h_{c}$ ranged from 0.7 to 0.76. A dimensionless plotting for water profiles near the brink section of waterfall was obtained [16] by studying the brink depth under zeroinertia condition. According to the momentum consideration [17] a theoretical value for $h_{b} / h_{c}$ as a function of Froude number $F$ was found to be 0.692 for $\mathrm{F}=1.0$. For a subcritical approaching flow of rectangular free overfall, a theoretical value of $h_{b} / h_{c}$ equal to 0.67 was obtained [18] according to Bousinisk equation. For a free waterfall under nonairated conditions [19], the pressure acting below the napp is experimentally correlated to the drop height in nondimensionless terms. The estimated pressure was used for theoretical evaluation of the brink depth by using the momentum equation [19]. Analyzing trapezoidal free overfall with an assumed pressure distribution based on previous measurements [20], an expression for the discharge as a function of brink depth was developed.
It is obvious from previous studies that the brink depth ratio ranges from 0.667 to 0.78 and the experimental values were always higher than the theoretical ones. This difference is due to the application of the momentum equation for which the evaluation of pressure at the brink section differs from one investigation to another. Furthermore, in all previous works, the pressure distribution upstream the brink section was assumed to be hydrostatic. Experimentally, the pressure upstream the brink section was found to be higher ( $5-15 \%$ ) than it's hydrostatic value in supercritical flow [21].
In the previous studies the energy equation was not employed to give a theoretical relationship for the brink depth. However in such cases the specific energy equation is more applicable, since the flow losses are easy to be estimated.

According to the energy considerations the present study establishes a new theoretical relationship for the flow depths in terms of velocity coefficient $\mathrm{C}_{\mathrm{v}}$ and Froude number F of the approaching flow.

## 2. THEORETICAL STUDY

### 2.1 Depth of Parallel Flow over Weir

Figure (1) shows a schematic representation of flow or rectangular broad crested weir. As shown in Figure (1 the depth of parallel flow h can be obtained by applyi the energy equation at section $1-1$ and $2-2$. In th equation the velocity coefficient, $\mathrm{C}_{\mathrm{v}}$ must be introduced account for the loss of energy between these two section


Figure 1. Flow over broad crested weir.

Equating the specific energy equations at the tw sections $1-1$ and 2-2 considering the crest surface as datum we get;

$$
H_{o}=h+\frac{\alpha_{2} v^{2}}{2 g}=h+\frac{\alpha_{2} q^{2}}{2 g C_{v}^{2} h^{2}}
$$

where

$$
H_{o}=H+\frac{\alpha_{1} v_{z}^{2}}{2 g}=H+\frac{\alpha_{1} q^{2}}{2 g(P+H)^{2}}
$$

Assuming that $\alpha_{1}=\alpha_{2}=1.0$ and substituting for $\frac{q^{2}}{g}=h_{c}^{3}$ in Equation (1), yields,

$$
h^{3}-H_{o} h^{2}+\frac{h_{c}^{3}}{2 C_{v}^{2}}=0
$$

Equation (2) is a cubic Equation for h. Using Cardan! solution [22], Equation (2) has the following three roots;

$$
\begin{align*}
& \mathrm{h}_{1}=\frac{\mathrm{H}_{0}}{3}\left[1+2 \cos \frac{\theta_{1}}{3}\right]  \tag{3-a}\\
& \mathrm{h}_{2}=\frac{\mathrm{H}_{0}}{3}\left[1-2 \cos \left(\frac{\theta_{1}}{3}+60^{\circ}\right)\right] \\
& \mathrm{h}_{3}=\frac{\mathrm{H}_{0}}{3}\left[1-2 \cos \left(\frac{\theta_{1}}{3}-60^{\circ}\right)\right]
\end{align*}
$$

where

## 1- The approximate $C_{v}-K$ relationship

The relationship between $\mathrm{C}_{\mathrm{v}}$ and K should be obtained for two ranges. The first when K varies from 0.385 to 0.544 , and the second for $\mathrm{K}<0.385$, as shown in Figures (3-a) and (3-b) respectively.

a) $k=0.385$ 10 0.544

b) $K<0.385$

Figure 3. Interpolation of $\cos \theta_{1}=f\left(C_{v}\right)$.
Accordingly, the average value of $\mathrm{C}_{\mathrm{v}}$ for values of K ranging from 0.385 to 0.544 is;

$$
\begin{align*}
& \mathrm{C}_{\mathrm{v}}=0.919 \mathrm{~K}+0.5  \tag{9}\\
& \text { and for } \mathrm{K}<0.385 \\
& \mathrm{C}_{\mathrm{v}}=2.218 \mathrm{~K} \tag{10}
\end{align*}
$$

## 2- The exact $C_{\nu}-K$ relationship

Integrating Equation (4) with respect to $\mathrm{C}_{\mathrm{v}}$ from $\mathrm{C}_{\mathrm{v}}=\sqrt{3.375} \mathrm{~K}$ to $\mathrm{C}_{\mathrm{v}}=1.0$ for a constant values of K varying from 0.385 to 0.544 we get;

$$
\left.\begin{array}{rl}
\int_{C_{v}}^{C_{v}} & =\sqrt{3.375} \mathrm{~K}
\end{array} 1-6.75\left(\frac{\mathrm{k}}{\mathrm{C}_{v}}\right)^{2}\right] \mathrm{dC} \mathrm{C}_{\mathrm{v}} .
$$

Dividing Equation (11) by the difference $\Delta \mathrm{C}_{\mathrm{v} 1}=1-\sqrt{3.375} \mathrm{~K}$ and equating the product to the right hand side of Equation (4) we get;

$$
\begin{equation*}
C_{v}=1.355 \sqrt{K} \tag{12}
\end{equation*}
$$

Following the same procedure for $\mathrm{K}<0.385$, and $\mathrm{C}_{\mathrm{v}}$ ranges from $\sqrt{3.375} \mathrm{~K}$ to $\sqrt{6.75} \mathrm{~K}$, we get;
$\mathrm{C}_{\mathrm{v}}=2.185 \mathrm{~K}$
A maximum deviation of $1.5 \%$ occurs in $\mathrm{C}_{\mathrm{v}}$ values between Equations (9) and (12), and also between Equations (10) and (13).
From the relationships (12) and (13), a general expression for the critical depth may be obtained in the following forms;
(i) For $\mathrm{C}_{\mathrm{v}}=0.84-1.0, \mathrm{~h}_{\mathrm{c}}=0.667 \sqrt[3]{\mathrm{C}_{\mathrm{v}}^{4}} \mathrm{H}_{\mathrm{o}}$, and
(ii) For $\mathrm{C}_{\mathrm{v}}<0.84, \mathrm{~h}_{\mathrm{c}}=0.594 \sqrt[3]{\mathrm{C}_{\mathrm{v}}^{2}} \mathrm{H}_{\mathrm{o}}$

According to the above two expressions for $h_{c}$, and tie discharge equation of the weir $\mathrm{q}=2 / 3\left(\mathrm{C}_{\mathrm{d}} \sqrt{2 \mathrm{~g}} \mathrm{H}_{0}^{3 / 2}\right)$, the corresponding relationships for the discharge coefficient $\mathrm{C}_{\mathrm{d}}$ are;

$$
\begin{equation*}
C_{d}=0.5775 C_{v}^{2} \tag{15-d}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{C}_{\mathrm{d}}=0.4857 \mathrm{C}_{\mathrm{v}} \tag{15-b}
\end{equation*}
$$

Equation (15-a) is valid for values of $\mathrm{C}_{\mathrm{v}}$ ranges from 0.44 to 1.0. For values of $\mathrm{C}_{\mathrm{v}}<0.84$, Equation (15-b) should be used to obtain $\mathrm{C}_{\mathrm{d}}$.

### 2.3 Brink Depth

The brink depth at the free overfall of the rectangular broad crested weir is analyzed with regard to the following characteristics: (1) Relation between brink depth $h_{b}$ and total head over the weir $\mathrm{H}_{\mathrm{o}}$; (2) Relationship between brink depth $h_{b}$ and critical depth of parallel flow, $h_{c}$; and (3) Variation of brink depth ratios $h_{b} / H_{o}$ and $h_{b} / h_{c}$ with both Froude number of the approaching flow F and coefficient of velocity $\mathrm{C}_{\mathrm{v}}$.
The depth of flow at the brink section can be expressed according to the following two assumptions;
1- From the analysis of Equation (4) for both $\cos \theta_{1}=0$ and -1 , two expressions were obtained for the flow depth over the weir as shown by Equations (6) and (8). It is obvious from these two expressions that the depth of free flow over the weir h, estimated by Equation (3-b), has two values. The physical meaning for these two values is that, the lower one, which is expressed by Equation (6), represents the minimum depth of flow over the weir which is the depth of flow at the brink section. The higher value of $h$, characterized by Equation (8), simulates the maximum value of flow depth at the middle part of weir crest, over which the parallel flow occurs. consequently, from Equation (6) the brink depth is;

$$
\begin{equation*}
\mathrm{h}_{\mathrm{b}}=0.529 \sqrt[3]{\left(\frac{\mathrm{C}_{\mathrm{v}}}{\mathrm{~F}}\right)^{2}} \mathrm{H}_{\mathrm{o}} \tag{16}
\end{equation*}
$$

The brink depth as a function of the critical depth $h_{c}$ can be obtained by dividing Equation (16) by Equation (14-a), from which

$$
\begin{equation*}
h_{b}=0.7937 \sqrt[3]{\frac{1}{\left(C_{v} F\right)^{2}}} h_{c} \tag{17}
\end{equation*}
$$

Equating the specific energy for the two section 2-2 and 3-3 shown in Figure (1), and neglecting the losses between these two sections yields,

$$
\begin{equation*}
\mathrm{h}_{\mathrm{o}}=\mathrm{h}_{\mathrm{b}}+\frac{\alpha_{3} \mathrm{~V}_{\mathrm{b}}^{2}}{2 \mathrm{~g}} ; \quad \text { or } \mathrm{h}_{\mathrm{b}}^{3}-\mathrm{h}_{\mathrm{o}} \mathrm{~h}_{\mathrm{b}}^{2}+\frac{\mathrm{h}_{\mathrm{c}}^{3}}{2}=0 \tag{18}
\end{equation*}
$$

Equation (18) is a cubic one and can be solved for $h_{b}$ using similar procedure as in Equation (2), hence,

$$
\begin{equation*}
\cos \theta_{2}=1-6.75\left(\frac{h_{\mathrm{c}}}{\mathrm{~h}_{\mathrm{o}}}\right)^{3} \tag{19}
\end{equation*}
$$

substituting for $h_{o}=0.667 \sqrt[3]{C_{v}^{2}} H_{o}$ in Equation (19), gives

$$
\cos \theta_{2}=1-\frac{22.78}{\mathrm{C}_{\mathrm{v}}^{2}}\left(\frac{\mathrm{~h}_{\mathrm{c}}}{\mathrm{H}_{\mathrm{o}}}\right)^{3} ;
$$

from which for the case of $\cos \theta_{2}=-1$,

$$
\begin{equation*}
h_{b}=\frac{4}{9} \sqrt[3]{C_{v}^{2}} H_{o} \propto 0.444 \sqrt[3]{C_{v}^{2}} H_{o} \tag{20}
\end{equation*}
$$

Dividing Equation (20) by Equation (14-a), we obtain $h_{b}$ as a function of $h_{c}$ as follows;

$$
\begin{equation*}
h_{b}=\frac{2}{3} \sqrt[3]{\frac{1}{C_{v}^{2}}} h_{c} \approx 0.667 \sqrt[3]{\frac{1}{C_{v}^{2}}} h_{c} \tag{21}
\end{equation*}
$$

### 2.4 Discharge-Brink Depth Relationship

As mentioned earlier, the brink depth is considered a true measuring device for discharge over free overfall. Considering Equation (21) and using $h_{c}=\sqrt[3]{\left(q^{2} / g\right)}$, the discharge per unit width, q can be obtained as follows; $\mathrm{q}=5.75 \mathrm{C}_{\mathrm{v}} \mathrm{h}_{\mathrm{b}}^{3 / 2}$
where $h_{b}$ is in ms. and $q$ is in $\mathrm{m}^{2} / \mathrm{sec}$.

## 3. EXPERIMENTAL PROGRAM

The present experiments were performed in a horizontal rectangular channel, 4.0 m long and 0.19 m wide. The channel was fabricated from 8 mm . perespex sheet supported by a steel frame. A coated wooden model for the broad crested weir with sharp edged corners 0.40 m long was inserted into the channel. Depths of flow were measured by point gauge. The discharges passing over the
weir was measured by a V-notch weir.
The experiments were conducted by increasing the weir height in increments from 4 to 12 cm . For each weir height a number of discharges ranging from 2.52 to 10.14 litre/sec. were allowed to pass over the weir covering a range of L/H from 4 to 9 and $\mathrm{H} / \mathrm{P}$ from 0.4 to 2.5 . For each discharge, the head over the weir H , the depth of parallel flow h and the brink depth $\mathrm{h}_{\mathrm{b}}$ were measured.

## 4. RESULTS AND DISCUSSION

The experimental results for 41 experiments were used to check the developed theoretical equations as given in Table 1.
The values of $\mathrm{K}=\left(\mathrm{h}_{\mathrm{c}} / \mathrm{H}_{\mathrm{o}}\right)^{3 / 2}$ and $\mathrm{C}_{\mathrm{r}}$, calculated using Equation (12) were used to determine the depth of the free parallel flow over the weir h according to Equations (3-b) and (4). The calculated values of $h$ are compared to the measuring ones as shown in Figure (4). The figure indicates a good agreement with maximum deviation of $\pm 6 \%$.


Figure 4. Comparison between measured and calculated values of free flow depth $h$ by relation (3-b).

The coefficient of discharge $C_{d}$ is calculated using Equation (15-a) and compared with the experimental value as given in Table 1. An excellent agreement occurs with maximum deviation $\pm 3.7 \%$.
The values of Froude number F for the flow upstream
the brink section are obtained, as $\mathrm{F}=(\mathrm{q} / \mathrm{h} \sqrt{\mathrm{gh}})$. The brink depth as a function of both the total head $\mathrm{H}_{\mathrm{o}}$ and the critical depth $h_{c}$ is calculated using Equations (16), (17), (20) and (21) using the above values of $\mathrm{C}_{\mathrm{v}}$ and F . Figures (5) and (6) show that a good agreement exists between the measured and the calculated values of $h_{b}$ with a maximum deviation of $\pm 5 \%$.


Figure 5. Comparison between measured and calculated values of brink depth according to; a) eq. (16), b) eq. (17).


Figure 6. Comparison between measured and calculated values of brink depth according to a) eq. (20), b) eq. (21).

A comparison between the values of $h_{b} / h_{c}$, calculated according to energy consideration using Equation (17) and the calculated values using the free vortex theory [10] is shown in Figure (7). It is clear that for both assumptions values of $h_{b} / h_{c}$ decreases as Froude number increase. For $C_{v}=0.72$ the two assumptions give the same value of $h_{b} / h_{c}$ for values of $\mathrm{F} \geq 3.5$.
In Equation (21) when $\mathrm{C}_{\mathrm{v}}=1.0$ the brink depth ratio $h_{b} / h_{c}$ is equal to 0.667 , which is equal to the value obtained employing the momentum equation and close to that obtained using free vortex theory as given in Table (2).


Figure 7. Variation of $h_{b} / h_{c}$ with $F$.
Table 2. Comparison between theoretical and experimenta results for brink depth ratio $h_{b} / h_{c}$ for rectangular channel

| Theoretical |  |  | Experimental | Reference |
| :---: | :---: | :---: | :---: | :---: |
| Momentum | Free <br> vortex | Energy |  |  |
| 0.667 | 0.673 | - | 0.715 | $[4]$ |
| - | - | - | 0.780 | $[8]$ |
| -667 | - | - | 0.714 | $[9]$ |
| 0.667 | 0.673 | - | 0.700 | $[10]$ |
| 0.667 | 0.673 | - | 0.705 | $[12]$ |
| - | - | 0.667 | 0.703 | $[14]$ |
|  |  | $\left(c_{\mathrm{v}}=1.0\right.$ |  | The present |
|  |  |  |  |  |

The average measured value of $h_{b} / h_{c}$ in the present study is found to be 0.72 which is close to the value 0.715 given by Rouse [2] with a deviation of $0.7 \%$.
For different values of the pressure coefficient, $\mathrm{K}_{\mathrm{p}}$ used in the momentum equation, the ratio $h_{b} / h_{c}$ according to references $[23,24]$ is plotted in figure (8) and compared with that calculated using Equation (21). It is clear that the values of $h_{b} / h_{c}$ obtained according to the energy equation are less than the value generated by the momentum equation.


Figure 8. Comparison between momentum and energy equation for evaluating $h_{b} / h_{c}$.

## CONCLUSION

On the basis of the present theoretical and experimental ades it can be concluded that the specific energy pation is more suitable to be applied to the parallel flow ut broad crested weirs. It gives good results and the grement with the experiments is remarkable over a wide use of both relative height and length of weir with apect to its head.
In this study a group of simple theoretical relationships we developed and they can be easily used in the hlowing:
Estimation of flow depth over the weir in relation to the velocity coefficient $C_{v}$, and the total head of the weir $\mathrm{H}_{0}$.
Evaluation of brink depth with respect to both total head $\mathrm{H}_{0}$ and the critical depth. Four relationships were developed to get the brink depth ratio depending on velocity coefficient and Froude number of the approaching flow.
Determination of both velocity and discharge coefficients, $\mathrm{C}_{\mathrm{v}}$ and $\mathrm{C}_{\mathrm{d}}$, in terms of critical depth related to the total head.
4 Calculation of the discharge of free overfall as a function of brink depth.
Finally, it was found that the measured brink depth ratio bas an average value of 0.415 and 0.72 related to the total head and the critical depth respectively.

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Table 1. Experimental measurements and comparison between theoretical and experimental results.

| experimental data |  |  |  | values of depth h |  |  | brink depth, $\mathrm{h}_{\mathrm{b}}$ |  |  |  |  |  |  |  |  | values of discharge coefficient $c_{d}$. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. | P cm | $\begin{aligned} & \mathrm{q} \\ & \mathrm{l} / \mathrm{s} / \mathrm{c} \\ & \mathrm{~m} \end{aligned}$ | $\begin{aligned} & \mathrm{H} \\ & \mathrm{~cm} \end{aligned}$ | $\begin{aligned} & \mathrm{h}_{\mathrm{me}} \\ & \text { as } \\ & \mathrm{cm} \end{aligned}$ | $\begin{aligned} & \mathrm{b}_{\mathrm{cal}} \text { using } \\ & \mathrm{Eq}(3-\mathrm{b}) \\ & \mathrm{cm} \end{aligned}$ | $\begin{aligned} & \text { \% } \\ & \text { deviatio } \\ & n \end{aligned}$ | $\begin{aligned} & \mathrm{b}_{\mathrm{b}} \\ & \text { meas } \mathrm{cm} \end{aligned}$ | $h_{b}$ cal using <br> Eq <br> (16) | $\begin{aligned} & \text { \% } \\ & \text { devia- } \\ & \text { tion } \end{aligned}$ | $b_{b}$ cal using Eq (17) | \% <br> devia- <br> tion | $b_{b} \mathrm{cal}$ using <br> Eq <br> (20) | \% <br> devia- <br> tion | $b_{b} \mathrm{ca}$ ! using <br> Eq <br> (21) | \% <br> devia- <br> tion | ${ }^{c_{\mathrm{d}}} \begin{aligned} & \text { exp. } \end{aligned}$ | ${ }^{c} d$ cal. | \% <br> devia- <br> tion |
| 1 | 11.4 | 0.133 | 4.75 | 207 | 216 | +4.3 | 1.84 | 1.86 | +1.0 | 1.87 | +1.6 | 1.92 | +4.4 | 1.93 | +4.9 | 0.431 | 0.427 | -0.93 |
| 2 | 9.6 | 0.137 | 4.68 | 2.16 | 222 | +28 | 1.90 | 1.86 | -21 | 1.87 | -1.6 | 1.93 | +1.6 | 1.95 | +26 | 0.451 | 0.447 | -0.89 |
| 3 | 9.0 | 0.138 | 4.83 | 232 | 221 | -4.7 | 2.01 | 203 | $+1.0$ | 202 | $+1.0$ | 1.98 | -1.5 | 1.98 | -1.5 | 0.434 | 0.437 | $+0.70$ |
| 4 | 12.0 | 0.138 | 4.74 | 240 | 226 | -5.8 | 205 | 207 | $+1.0$ | 208 | $+1.5$ | 1.95 | -4.9 | 1.96 | -4.4 | 0.450 | 0.447 | -0.67 |
| 5 | 4.8 | 0.147 | 4.71 | 2.48 | 233 | -0.6 | 210 | 210 | 0.0 | 211 | 0.0 | 200 | -4.76 | 202 | -3.8 | 0.470 | 0.468 | -0.43 |
| 6 | 7.2 | 0.160 | 5.14 | 2.49 | 2.64 | +6.0 | 212 | 215 | $+1.4$ | 214 | $+0.94$ | 2.15 | +1.4 | 215 | +1.4 | 0.453 | 0.547 | +0.88 |
| 7 | 11.4 | 0.184 | 5.80 | 2.60 | 270 | +3.8 | 227 | 225 | -0.9 | 226 | -0.44 | 237 | +4.4 | 238 | $+4.8$ | 0.440 | 0.437 | -0.68 |
| 8 | 12.0 | 0.191 | 5.82 | 266 | 277 | +4.1 | 235 | 227 | -3.4 | 232 | -1.3 | 238 | +1.3 | 245 | $+4.3$ | 0.454 | 0.437 | -3.70 |
| 9 | 9.0 | 0.192 | 5.89 | 270 | 275 | +1.9 | 239 | 234 | -2.1 | 233 | -2.5 | 2.44 | +2.1 | 244 | $+21$ | 0.446 | 0.447 | $+0.22$ |
| 10 | 4.8 | 0.196 | 5.64 | 280 | 286 | $+21$ | 2.48 | 238 | -4.0 | 2.37 | -4.4 | 2.43 | -2.0 | 2.42 | -24 | 0.473 | 0.478 | $+1.00$ |
| 11 | 9.6 | 0.197 | 5.92 | 2.83 | 283 | 0.0 | 2.50 | 246 | -1.6 | 241 | -3.6 | 249 | 0.0 | 2.46 | -1.6 | 0.454 | 0.468 | +3.10 |
| 12 | 6.8 | 0.205 | 5.89 | 3.03 | 291 | -4.0 | 252 | 2.57 | $+20$ | 2.58 | $+24$ | 249 | -1.2 | 252 | 0.0 | 0.470 | 0.468 | -0.43 |
| 13 | 4.7 | 0.207 | 5.84 | 3.15 | 2.97 | -5.7 | 261 | 266 | +1.9 | 268 | $+27$ | 250 | -4.2 | 253 | -3.0 | 0.472 | 0.468 | -0.85 |
| 14 | 7.2 | 0.218 | 6.23 | 3.04 | 3.06 | +0.7 | 264 | 261 | -1.1 | 262 | -0.75 | 262 | -0.8 | 264 | 0.0 | 0.460 | 0.457 | -0.65 |
| 15 | 6.8 | 0.223 | 6.35 | 3.17 | 3.04 | -4.1 | 280 | 273 | -25 | 272 | -2.9 | 267 | -4.6 | 269 | -3.9 | 0.460 | 0.457 | $+0.22$ |
| 16 | 11.4 | 0.230 | 6.74 | 3.20 | 3.10 | -3.1 | 283 | 278 | -1.8 | 279 | -1.4 | 2.76 | -2.5 | 274 | -3.2 | 0.438 | 0.437 | -0.23 |
| 17 | 4.8 | 0.238 | 6.34 | 3.25 | 3.24 | 0.0 | 287 | 2.75 | -4.2 | 275 | -4.2 | 274 | -4.5 | 276 | -3.8 | 0.480 | 0.478 | -0.42 |
| 18 | 9.0 | 0.241 | 6.89 | 3.30 | 3.22 | -24 | 277 | 286 | $+3.2$ | 287 | $+3.6$ | 284 | +2.5 | 287 | +3.6 | 0.441 | 0.437 | -0.91 |
| 19 | 12.0 | 0.257 | 7.11 | 3.35 | 3.39 | +1.2 | 283 | 288 | +1.8 | 290 | +2.5 | 294 | +3.9 | 297 | +4.9 | 0.451 | 0.447 | -0.89 |
| 20 | 9.6 | 0.263 | 7.13 | 3.50 | 3.41 | -26 | 290 | 3.00 | +3.4 | 3.00 | +3.4 | 298 | +275 | 299 | +3.1 | 0.456 | 0.457 | +0.22 |
| 21 | 11.4 | 0.276 | 7.61 | 3.65 | 3.50 | -4.1 | 3.20 | 3.09 | -3.4 | 3.18 | -0.63 | 3.12 | -2.5 | 3.14 | -1.9 | 0.437 | 0.437 | 0.0 |
| 22 | 7.2 | 0.282 | 7.34 | 3.75 | 3.62 | -3.5 | 3.24 | 3.19 | -1.54 | 3.22 | -0.62 | 3.10 | -4.3 | 3.13 | -3.4 | 0.362 | 0.457 | -1.10 |
| 23 | 6.8 | 0.284 | 7.44 | 3.74 | 3.60 | -3.7 | 3.30 | 3.22 | -24 | 3.21 | -273 | 3.14 | -4.8 | 3.18 | -3.6 | 0.456 | 0.457 | $+0.22$ |
| 24 | 4.8 | 0.295 | 7.22 | 3.68 | 3.77 | +24 | 3.06 | 3.10 | +1.3 | 3.09 | $+1.0$ | 3.16 | +3.3 | 3.17 | +3.6 | 0.485 | 0.489 | $+0.82$ |
| 25 | 9.0 | 0.302 | 7.87 | 3.65 | 3.73 | +22 | 3.14 | 3.14 | 0.0 | 3.15 | 0.0 | 3.28 | +4.5 | 3.29 | +4.8 | 0.450 | 0.447 | -0.67 |
| 26 | 4.7 | 0.308 | 7.42 | 4.06 | 3.86 | -4.9 | 3.35 | 3.43 | +2. 4 | 3.40 | +1.5 | 3.26 | -2.7 | 3.26 | -27 | 0.484 | 0.489 | $+1.00$ |
| 27 | 120 | 0.323 | 8.11 | 3.85 | 3.99 | +3.6 | 3.40 | 3.49 | $+26$ | 3.30 | -29 | 3.39 | 0.0 | 3.43 | $+0.9$ | 0.463 | 0.457 | $-1.30$ |
| 28 | 11.4 | 0.334 | 8.53 | 4.08 | 4.00 | -2.0 | 3.45 | 3.54 | +26 | 3.53 | +2.3 | 3.54 | $+26$ | 3.54 | $+26$ | 0.443 | 0.447 | $+0.90$ |
| 29 | 7.2 | 0.340 | 8.30 | 4.10 | 4.11 | 0.0 | 3.50 | 3.50 | 0.0 | 3.51 | 0.0 | 3.52 | +0.6 | 3.55 | +1.4 | 0.461 | 0.457 | -0.87 |
| 30 | 9.6 | 0.347 | 8.33 | 4.00 | 4.18 | +4.5 | 3.58 | 3.53 | -1.4 | 3.56 | -0.56 | 3.53 | -1.4 | 3.58 | 0.0 | 0.474 | 0.468 | $-1.30$ |
| 31 | 4.8 | 0.367 | 8.15 | 4.31 | 4.39 | +1.9 | 3.60 | 3.60 | 0.0 | 3.59 | 0.0 | 3.62 | +0.6 | 3.63 | $+0.83$ | 0.496 | 0.499 | $+0.60$ |
| 32 | 6.8 | 0.382 | 8.55 | 4.47 | 4.47 | 0.0 | 3.75 | 3.74 | 0.0 | 3.74 | 0.0 | 3.72 | -1.1 | 3.75 | 0.0 | 0.491 | 0.489 | -0.41 |
| 33 | 9.0 | 0.388 | 9.16 | 4.40 | 4.43 | +0.7 | 3.80 | 3.78 | -0.5 | 3.78 | -0.53 | 3.87 | +1.8 | 3.88 | $+21$ | 0.456 | 0.457 | $+0.22$ |
| 34 | 12.0 | 0.400 | 9.31 | 4.66 | 4.56 | -2.1 | 3.92 | 3.99 | $+1.8$ | 3.97 | $+1.3$ | 3.93 | $+0.3$ | 3.94 | $+0.5$ | 0.464 | 0.468 | +0.86 |
| 35 | 4.7 | 0.410 | 9.28 | 4.82 | 4.67 | -3.1 | 4.03 | 4.16 | +3.2 | 4.11 | +20 | 4.02 | 0.0 | 4.00 | -0.75 | 0.460 | 0.468 | $+1.70$ |
| 36 | 7.2 | 0.418 | 9.36 | 4.90 | 4.68 | -4.5 | 4.10 | 4.16 | +1.5 | 4.17 | +1.7 | 4.01 | -22 | 4.05 | -1.2 | 0.470 | 0.468 | -0.43 |
| 37 | 6.8 | 0.423 | 9.42 | 4.95 | 4.71 | -4.8 | 4.23 | 4.20 | -0.7 | 4.21 | -0.5 | 4.05 | -4.2 | 4.08 | -2.9 | 0.470 | 0.468 | -0.43 |
| 38 | 11.4 | 0.424 | 9.75 | 4.70 | 4.70 | 0.0 | 4.14 | 4.03 | -27 | 4.03 | -27 | 4.09 | -1.2 | 4.11 | -0.72 | 0.458 | 0.457 | -0.22 |
| 39 | 9.6 | 0.439 | 9.72 | 4.75 | 4.91 | +3.4 | 4.15 | 4.16 | 0.0 | 4.18 | $+0.72$ | 4.13 | -0.5 | 4.19 | +1.0 | 0.472 | 0.468 | -0.85 |
| 40 | 12.0 | 0.464 | 10.13 | 4.95 | 5.08 | +3.3 | 4.35 | 4.23 | -28 | 4.25 | -2.3 | 4.29 | -1.4 | 4.34 | 0.0 | 0.472 | 0.468 | -0.85 |
| 41 | 4.8 | 0.484 | 9.45 | 5.18 | 5.39 | +4.1 | 4.41 | 4.27 | -3.2 | 4.30 | -25 | 4.28 | -0.5 | 4.33 | -1.8 | 0.515 | 0.510 | -0.97 |

# A COST EFFECTIVE DESIGN OF A MULTIPLE WELL SYSTEM FOR USE IN THE WADI EL-NATRUN AREA 

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## ABSTRACT

Huge losses in the groundwater resources in the Wadi-El-Natrun depression are occurring due to the evaporation from lakes and evapotranspiration from vegetation cover and the high water table near the lakes. The use of a multiple well system would intercept the inflow towards the lakes. This scheme would lower the water level in the lakes and the adjacent areas. A cost effective study procedure has been developed in order to find out the optimum well number, capacity and spacing of the well system at Wadi El-Natrun area.

CR total annual cost of individual capital investments.
CRF capital recovery factor.
hw water depth in the well.
$\mathrm{H}_{0}$ original water depth in the aquifer.
i interest rate per year.
slope of a crossflow
well casing cost.
well drilling cost.
distribution cost.
mobilization cost.
investment cost of an equipment.
well screening cost.
water supply cost.
coefficient of permeability.
economic life of an equipment in years.
Q well discharge.
$\mathrm{I}_{\mathrm{w}} \quad$ well radius.
TDH total dynamic head.

## INTRODUCTION

The Wadi-El Natrun depression receiving special attention from Water Development Agencies because of its potential as a groundwater resource [9] and [3]. Referring to the piezometric map of the West Delta area (see Figure (1)) and according to [2], [9], and [4], it can be concluded that the groundwater in Wadi El-Natrun is fed mainly from seepage of the Nile Delta and West Nubaria area. The hydraulic gradient of the crossflow towards the depression is about $2.5 \mathrm{~m} / \mathrm{km}$. The reason for the local steep gradient of the water table is mainly due to
the high rate of evaporation from El-Natrun lakes which act as a natural discharging area.


Figure 1. Piezometric map for the West Delta aquifer (after the Author (4)).

