A TWO-DIMENSIONAL DYNAMIC MODEL OF THE TIBIOFEMORAL KNEE JOINT

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ABSTRACT

A two dimensional dynamic model of the human knee joint is presented. In this model the surfaces of the tibial and femoral condyles are represented by polynomials. The major four ligaments of the knee joint are modelled as nonlinear elastic springs of realistic stiffness properties. These ligaments are medial collateral, lateral collateral, anterior cruciate, and posterior cruciate ligaments. Nonlinear equations of motion coupled with nonlinear constraints are solved numerically. The friction force is included in the mathematical model. Time derivatives are approximated by Newmark difference formula and the resulting nonlinear algebraic equations are solved by applying the Gauss elimination method.

NOMENCLATURE

A	The amplitude of the external dynamic load (N).
[A]	The coefficient matrix.
{D}	The vector of known values (at certain time).
$f_1(x)$	A function representing the equation of the femoral articulating surface.
$f_2(x')$	A function representing the equation of the tibial articulating surface.
I_z	The moment of inertia of the leg (Nms^2).
$(\hat{I}, \hat{j})$	The unit vectors along the X & Y directions respectively.
$(\hat{i}', \hat{j}')$	The unit vectors along the X' & Y' directions respectively.
M	The mass of the leg (kg).
M_e	The external moment (Nm).
$\hat{n}_1$	The unit normal to the femoral surface.
$\hat{n}_2$	The unit normal to the tibial surface.
$\bar{r}_c$	The position vector of the contact point in the fixed coordinate system.
$\bar{r}_m$	The position vector of the attachment point of the m ligament in the fixed coordinate system.
t	The time elapsed from the start of motion (s).
t_o	The load time duration (s).
[T]	The orthogonal transformation matrix.
(X,Y)	The fixed coordinate system.
(X',Y')	The moving coordinate system.
x_o	The distance between the coordinate systems in the X direction (m).
y_o	The distance between the coordinate systems in the Y direction (m).
α	The angle of rotation between the moving and fixed coordinate systems.
$\{\Delta\}$	The vector of incremental quantities.
$\hat{\lambda}_m$	The unit vector along the ligament m.
μ	The coefficient of friction between the tibial and femoral articulating surfaces.
$\bar{\rho}'_c$	The position vector of the contact point c in the moving coordinate system.
$\bar{\rho}'_m$	The position vector of the attachment point of the m ligament in the moving coordinate system.

INTRODUCTION

The knee joint is the largest and the most structurally complicated joint in the human body. This is due to the fact that it connects the largest levers of the lower limb (the femur and leg bones) which are characterized by the widest range of movements during walking [1]. The knee joint consists of two main joints namely, the tibiofemoral joint and the patellofemoral joint. However in the present work, special attention is given to the tibiofemoral joint. This is agreed with Van Eijden et al. [2] who mentioned that, with respect to the human knee it is better to describe mathematical models which are limited to the tibiofemoral part of the knee.

The major forces which act on the tibiofemoral joint are the contact forces between the articulating surfaces of the tibia and the femur, the external forces and moments as functions of time, the inertia forces and moments and the ligament forces.

One of the most important model of human knee joint was proposed by R. Crowninshield et al. [3], which was based upon mathematical modelling and, in vivo, measurements of ligament lengths. This model was accounted for the geometry, characteristic of motion, and the material properties of the knee. He pointed out that the stability of the knee joint resulted from the ligamentous structure of the knee and did not include the effect of muscular activity. In the model, the cruciate, collateral, and capsular ligaments were represented by thirteen elements. The coordinates of the attachment sites and the dimensions of the ligaments are found by, in vivo, and, in vitro, measurements. This model was presented as a three dimensional static model and the theoretical results were compared with the experimental results. To simplify this model, the relation between ligament force and ligament strain was assumed as a linear relation. Neither external dynamic loads nor body weight were considered in the analysis. The effect of contact conditions, friction, nonlinearity of ligamentous stress strain relationship, articulating surfaces equations, external moments, and tibial length, were absent as well.

M. H. Pope, et al. [4], presented a dynamic in vivo study about knee joint. They conclude that the knee behaves as a single degree of freedom spring -mass- damper system. T.P. Andriacchi, et al., [5] presented a three-dimensional mathematical model of the ligamentous knee joint. the bony portions of the model were represented by rigid bodies while soft tissue structures were represented by springs. Studies with this model indicated that the geometric type nonlinearities contribute to the overall non-linear response of the knee joint. They used an incremental linearization procedure for the geometric and material nonlinearities.

M. Moeinzadeh, et al. [6], proposed a mathematical dynamic model of the two dimensional representation of the knee joint. Using a two dimensional digitizing technique the profiles of the joint surfaces were determined. The ligaments were modeled as non-linear elastic springs. Non-linear equations of motion coupled with non-linear constraint conditions were solved numerically by Newton Raphson iteration scheme, after approximating the time derivatives by Newmark difference formulae. The friction force between articulating surfaces, and effect of ligament laxation or rupture were not considered.

In the present work, a two-dimensional dynamic model of the tibiofemoral joint is presented, in which the femur is fixed and the leg is extended applying impuls dynamic load. The non-linear geometry and ligament forces are considered. The difference between this work and the previous models, is that the effect of friction force generated between the articulating surfaces of the femur and the tibia is presented. In addition, effect of body weight on the ligaments and cartilages is also considered.

FORMULATION

The knee joint will be modelled as two rigid bodies connected by nonlinear elastic elements simulating the ligaments. It is assumed that the femur is rigidly fixed and the tibia is undergoing a general plane motion relative to the femur as shown in Figure (1).

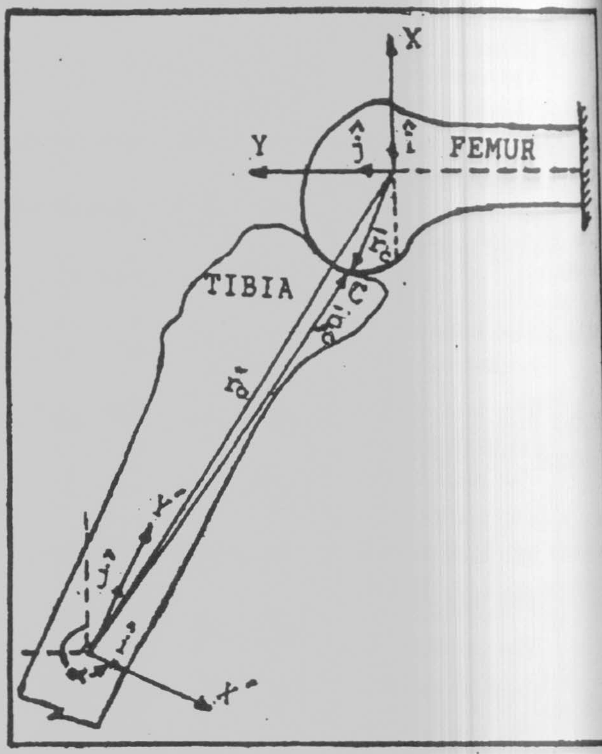

Figure 1. Coordinate systems locations and relative positions of the tibia and femur are shown for the two dimensional dynamic model of the knee joint.

Due to the two dimensional nature of the model, proper components of the ligament forces in the plane of motion are considered. An expression for evaluating the value of the friction force as a function of the contact force is introduced in the mathematical model, although the coefficient of friction between the articulating surfaces, owing to the presence of the synovial fluid, is known to be very low [7]; and was neglected in the model given by Moienzadeh et al [6].

The position of the tibia relative to the femur is described by two coordinate systems as shown in Figure (1). A moving coordinate system (X',Y') with the origin coinciding with the centre of mass of the tibia and the Y'-axis is directed along the longitudinal axis of the tibia. The other coordinate system (X,Y) is fixed to the femur with the X-axis directed along the posterior-anterior direction, and the Y-axis coinciding with the femoral longitudinal axis. The positions of the origins of both coordinate systems are obtained knowing the equations of the articulating surfaces of both the tibia and the femur. The position vector of the origin of the moving system relative to the fixed one is given by

$$\bar{r}_o = x_o\hat{i} + y_o\hat{j} \tag{1}$$

Assuming rigid body contact between the tibia and the femur at point C as shown in Figure (1),then the contact surfaces may be represented by the mathematical functions

$$y = f_1(x) \tag{2}$$

and,

$$y' = f_2(x') \tag{3}$$

where $f_1(x)$ and $f_2(x')$ are two functions representing the tibial and femoral articulating surfaces.

Since the contact point C lies on each of the profiles, then:

$$\bar{r}_c = x_c\hat{i} + y_c\hat{j} \tag{4}$$

and,

$$\bar{\rho}'_c = x'_c\hat{i}' + y'_c\hat{j}' \tag{5}$$

The relation connecting the articulating surface profiles $f_1(x)$, $f_2(x')$ and the angle of rotation (α) is found from the condition of geometric compatibility of the tibial and the femoral surfaces and proved to be,

$$\{\bar{r}_c\} = \{\bar{r}_o\} + [T]\{\bar{\rho}'_c\} \tag{6}$$

Where [T] is an orthogonal transformation matrix given by:

$$[T] = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \tag{7}$$

At the point of contact, both normals to the surfaces of the tibia and the femur must be collinear. Let $\hat{n}_1$ and $\hat{n}_2$ be the unit normals to the femoral and the tibial surfaces respectively and directed toward their centers of curvature [6], then $\hat{n}_1$ and $\hat{n}_2$ relative to the fixed coordinate system, are given by:

$$\hat{n}_1 = \frac{\dfrac{d^2f_1}{dx^2}}{\left|\dfrac{d^2f_1}{dx^2}\right|}\left[1+\left(\frac{df_1}{dx^2}\right)^2\right]^{-0.5}\left[-\left(\frac{df_1}{dx}\right)\hat{i}+\hat{j}\right] \tag{8}$$

and,

$$\hat{n}_2 = \frac{\dfrac{d^2f_1}{dx'^2}}{\left|\dfrac{d^2f_1}{dx'^2}\right|}\left[1+\left(\frac{df_1}{dx'^2}\right)^2\right]^{0.5}\left[-\left(\left(\frac{df_1}{dx'}\right)\cos\alpha+\sin\alpha\right)\hat{i}\right] + \left[\left(\cos\alpha-\left(\frac{df_2}{dx'}\right)\sin\alpha\right)\hat{j}\right] \tag{9}$$

The colinearity of the normals at contact point requires that

$$\hat{n}_1 * \hat{n}_2 = 0 \text{ at } x = x_c,\ x' = x'_c$$

Then the contact condition is given by:

$$\tan\alpha = \frac{\left[\left(\dfrac{df_1}{dx}\right)_{x=x_c} - \left(\dfrac{df_2}{dx'}\right)_{x'=x'_c}\right]}{\left[1+\left(\dfrac{df_1}{dx}\right)_{x=x_c} - \left(\dfrac{df_2}{dx'}\right)_{x'=x'_c}\right]} \tag{10}$$

The two-dimensional profiles $f_1(x)$ and $f_2(x')$ of the femoral and tibial articulating surfaces are given by M. Moienzadeh et al. [6], as follows:

$$f_1(x) = 0.04014 - 0.247621\,x - 6.889185\,x^2$$

$$- 270.4456\, x^3 - 8589.942\, x^4 \quad (11)$$

And

$$f_2(x') = 0.213373 - 0.0456051\, x' + 1.073446\, x'^2 \quad (12)$$

FORCE ANALYSIS

The tibiofemoral joint is subjected to several forces. These forces are external and internal forces. The external forces may be static or impulse forces. The internal forces are those forces exerted on ligaments and the contact force between the articulating surfaces of the femoral and tibial condyles. The four major ligaments of the tibiofemoral joint are the lateral collateral ligament (LC), the medial collateral ligament (MC), the anterior cruciate ligament (AC), the posterior cruciate ligament (PC). Due to the structure and anatomy of ligaments they will be modelled as nonlinear elastic springs.

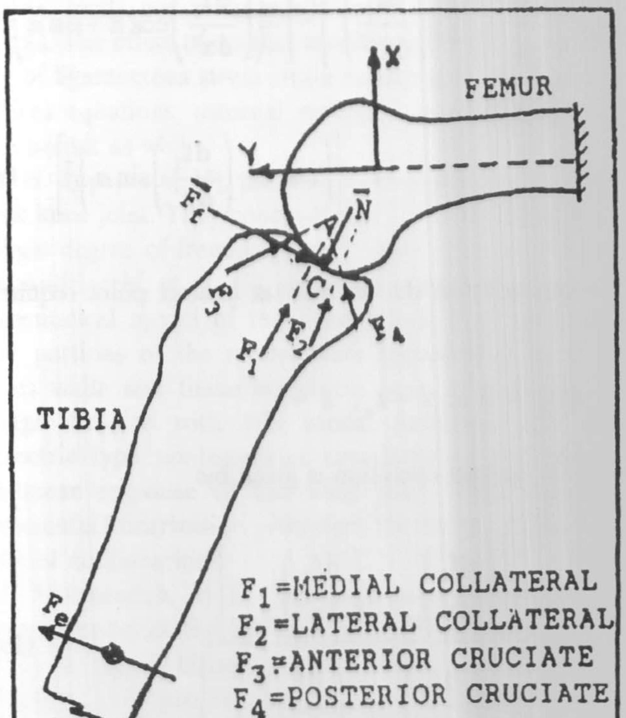

Figure 2. Forces acting on the moving tibia are shown for the two dimensional model of the knee joint.

The directions of the ligament forces are given in Figure (2) and according to Moienzadeh [6], the following force-elongation relationship is assumed for each ligament (m),

$$F_m = K_m (L_m - l_m)^2, \quad \text{for } L_m > l_m$$

Where, K_m is the force coefficient of the ligament spring constant, L_m is the current length of the liga at certain position and l_m is the original ligament le So F_m is the tensile force in the m^{th} ligament. assumed that the ligaments cannot carry any compre force; accordingly;

$$F_m = 0.0 \quad \text{for } L_m < l_m$$

The force coefficient values, K_m, are estimated accor to the data available in the literature [8] and [9], and values used here are given in table I.

Table I. Ligament force coefficient

Ligament	K_m (N mm^{-2})
MC	15
AC	30
PC	35
LC	15

Initial strains in the ligaments are taken equal since, at present, there is no accurate data availa these strains as a function of flexion angle [6]. Thi cepted because the variation in the original le wide, and if an appropriate starting angle un external load is chosen [6]. The ligament is consid a straight line with length L_m and by simple ge using the transformation matrix and substitute values of the position vector of the contact point to both fixed and moving coordinate systems, L_m by;

$$L_m = \sqrt{[(\bar{r}_m) - (\bar{r}_o) - T(\bar{\rho}_m')] * [(\bar{r}_m) - (\bar{r}_o) -}$$

And since the ligaments are connecting the tibia femur, so the unit vector $(\hat{\lambda}_m)$ along the liga directed from the tibia to the femur is

$$\hat{\lambda}_m = \frac{(\bar{r}_m) - (\bar{r}_o) - T(\bar{\rho}_m')}{L_m}$$

We can realize that $(\hat{\lambda}_m) = \pm 1$, to indicate the direction of ligament force with respect to the origin of the base of coordinates (x,y).

Thus the axial force in the ligament m, in its vectorial form becomes;

$$\bar{F}_m = F_m \hat{\lambda}_m \tag{17}$$

The positions of the insertion points of the ligaments in both the tibia and the femur used in this work are listed in Table II.

Table II. Coordinate values of the insertions of the ligaments in meters.

Ligament	Tibia		Femur	
	x'_m	y'_m	x_m	y_m
MC	0.008	0.163	0.023	0.014
LC	0.025	0.478	0.025	0.019
AC	-0.005	0.213	-0.023	0.019
PC	0.025	0.208	-0.032	0.024

The contact force $\bar{N}$ acting in the direction of the normal to the surfaces of the tibia and femur at the point of contact;

$$\bar{N} = \gamma N \hat{n}_1 \quad \text{at } x = x_c \tag{18}$$

Where N is the magnitude of the contact force, and γ is either +1 or -1 to ensure the correct direction of the contact force [6].

$$\gamma = \pm 1 \quad \text{at } x = x_c \tag{19}$$

For the friction force and although it was neglected in the previous models because the coefficient of friction between the healthy cartilages of the human joints is too small due to the presence of the synovial fluid, [7], it will be considered here as a function of the contact force to account for its effect on elderly and disabled people. On the other hand, the coefficient of friction will be considered constant, during the range of motion, due to the complex system of joint lubrication of the knee. Thus the friction force will take the form

$$F_f = \mu \bar{N}(\hat{n}_1)$$

The external force acting on the mass-center of the tibia has a general form;

$$\bar{F}_e = (F_e)\hat{i} + (F_e)\hat{j} \tag{20}$$

This external force may be caused by knocks or by a car crash or any type of accidents.

One of the most realistic forcing function used as a typical representation of the dynamic load was given by Engin et al. [12], as:

$$F_e(t) = A e^{4.73(t/t_o)^2} \sin(\pi t/t_o) \tag{21}$$

THE EQUATIONS OF MOTION

The equations governing the forced motion of the tibia with respect to the femur are as follows:

In the x direction,

$$(F_e)_x + \gamma N(\hat{n}_1)_x + \delta \mu N(\hat{n}_1)_y + \sum_{m=1}^{m=4} F_m(\hat{\lambda}_m)_x = M\ddot{x}_o \tag{22}$$

In the y direction,

$$(F_e)_y + \gamma N(\hat{n}_1)_y + \delta \mu N(\hat{n}_1)_x + \sum_{m=1}^{m=4} F_m(\hat{\lambda}_m)_y = M\ddot{y}_o \tag{23}$$

where $\delta = \pm\gamma$ for extension and flexion respectively.

Equating the inertia force with the summation of external moments gives:

$$M_e + (T\bar{\rho}'_c)(\gamma N\hat{n}_1) + (T^{-1}\bar{\rho}'_c)(\delta \mu N\hat{n}_1) + \sum_{m=1}^{m=4} (T\bar{\rho}'_m)(F_m\hat{\lambda}_m) = I_z\ddot{\alpha} \tag{24}$$

In which the subscripts x and y denote the components of the related quantities in the x and the y-directions respectively. The dots denote derivatives with respect to time t.

For these equations, the description is completed by assigning the initial conditions which are,

$\dot{x}_o = \dot{y}_o = \dot{\alpha} = 0.0$.

Solving these equations of motion with the contact condition equation (10), and the two geometric compatibility conditions (6) numerically, one can get the values of the unknowns for each time station.

The six unknowns here are (α, x_o, y_o, x_c, x'_c ,N), knowing that these variables affect the ligament forces because the current length of each ligament depends on (α, x_o, y_o, x_c, x'_c)

NUMERICAL PROCEDURE

The numerical solution of the three equations of motions, the contact conditions and the two geometric compatibility conditions is as follows:

1- Newmark operators [13] are used to replace the time derivatives with temporal operators.

$$\dot{X}^t = \frac{2}{\Delta t}(X^t - X^{t-\Delta t}) - \dot{X}^{t-\Delta t}$$

$$\ddot{X}^t = \frac{4}{(\Delta t)^2}(X^t - X^{t-\Delta t}) - \frac{4}{\Delta t}\dot{X}^{t-\Delta t} - \ddot{X}^{t-\Delta t}$$

Where Δt is a time interval and $\dot{X}$, $\ddot{X}$ are the velocity and acceleration in the X-direction respectively.

Similar expressions for Y and α are found. The superscripts of the above equation denote the time station under consideration.

2- The variations in the variables during the time interval Δt is small. Δ^2 and higher orders are neglected.

3- Using steps 1 and 2, equations (6), (10), (22), (23), and (24) take the form

$$[A]_n \{\Delta\}_n = \{D\}_n \qquad (25)$$

where n is the number of the equation.

4- The resultant equations can be arranged in the matrix form as follows:

$$[A_{i,j}] \{\Delta_j\} = \{D_i\} \qquad (26)$$

where i is the number of row, j is the number of column.

These equations are solved using a computer progr applying Gauss-elimination method with scaling factor a inverse matrix subroutines.The solution of delta quanti at certain time is used to get the new values of t elements of the known values vector of the next tim station. This step is repeated till the minimum possib flexion angle.

RESULTS AND DISCUSSION:

The model presented is general and can be suitable i a wide range of anatomical conditions, as leg length an variable external impacts. The results have to b considered as a qualitative measure of the considere problem due to the fact that the mechanical properties c ligaments are not the same for all persons, the shape an formulae of articulating surfaces are variable, and th weight distribution due to style abnormality also affec the results.The different diseases of joints as osteoarthriti joint degeneration, rheumatoid, osteo-arthrosis or oth diseases cause the increase in the coefficient of friction the joint as well as changing the shape of the conta contour of the articulating surfaces. Since the change the contour formulae is different from person to pers and requires very sophisticated equipments to acquire [1 therefore the effect of the coefficient of friction restricted to healthy subjects. Although, it is very small, effect on the lateral collateral ligament has to considered.

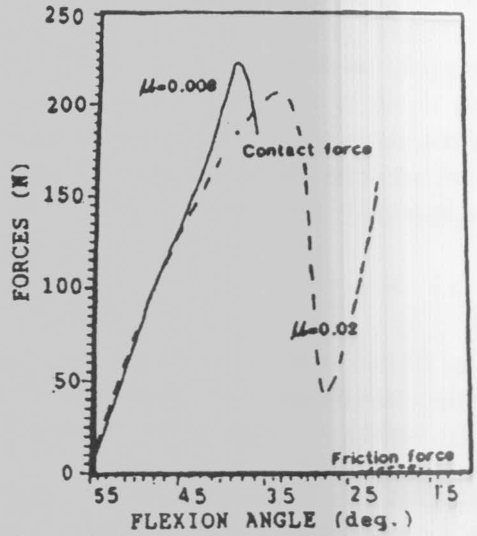

Figure 3. The contact and friction forces plotted against the flexion angle.

Figure (3) shows the effect of the coefficient of friction on both the contact and friction forces. The two values of the coefficient of friction used are 0.008 and 0.02. the results presented in the figure are obtained for average body weight of 70 Kg, tibial length of 420 mm and the peak of pulse load is 60 N. The dashed line near the horizontal axis is the friction force. From the figure, it is clear that increasing the coefficient of friction decreases the contact force developed between the articulating surfaces. Also, it can be realized that the magnitude of the friction force is too small to be considered.

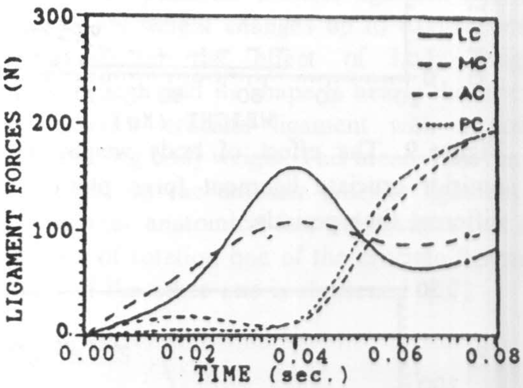

Figure 4. The ligament forces for a coefficient of friction of 0.008 plotted against the time elapsed after the impact.

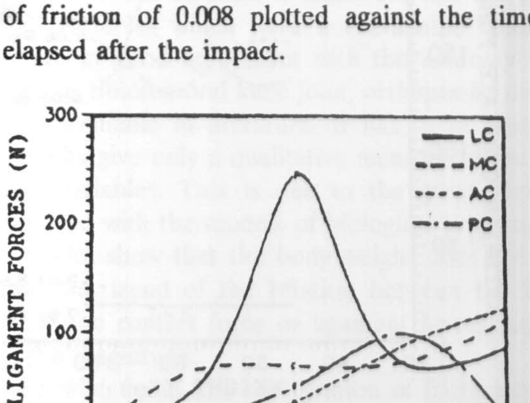

Figure 5. The ligament forces for a coefficient of friction of 0.02 plotted against the time elapsed after the impact.

Figures (4) and (5) present the relation between ligament forces and time elapsed from the impact for low and high coefficients of friction respectively for the same conditions as Figure (3). From figures it is clear that the lateral collateral ligament force increases with 50% of its value for low coefficient of friction. On the other hand the peak value of the medial collateral ligament force in the case of high coefficient of friction is decreased by about 36% than that of low coefficient of friction. This means that the load is shared, in a way or another, between the collateral ligaments. Not only the cruciate ligament forces generally decrease with the increase of the coefficient of friction, but also the shape of the curves is changed. For low coefficient of friction the value of cruciate ligament forces increases up to 0.02 second, then decreases till 0.04 second, after that it increases sharply. But for high coefficient of friction the anterior cruciate ligament force increases gradually without any point of inflection. For the posterior cruciate ligament force a point of inflection takes place after 0.06 second from the start of motion.

Then the increase of the coefficient of friction changes the behaviour of the cruciate ligaments but decreases the maximum forces developed in them and increases the load in the lateral collateral ligament.

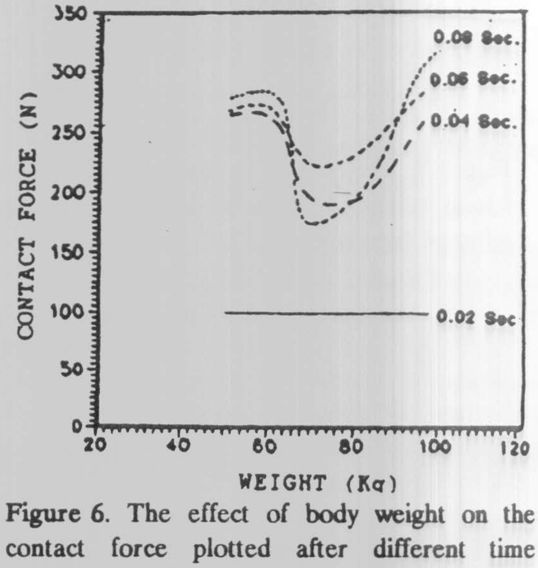

Figure 6. The effect of body weight on the contact force plotted after different time periods.

On the other hand, the effect of body weight on the contact force and ligament forces is found. In Figure (6), the relation between the contact force generated between the articulating surfaces and body weight is represented for successive time intervals after impact. After 0.02 second the contact force seems to be constant with respect to body weight. From 0.04 second and up to 0.08 second, it is clear that the contact force is larger for persons under 70 Kg and over 90 Kg than that for average persons. This result is in good agreement with the medical observations

[15]. On the other hand, the number of overweighed patients suffering from degenerative osteoarthritis is no greater than the number of underweight patients who are similarly affected [16]. In the present model the high contact force of slim persons may exist due to the small inertia of the thin leg which permits a wide flexion angle after short time. This yields that the shock load achieves its peak value after a short time.

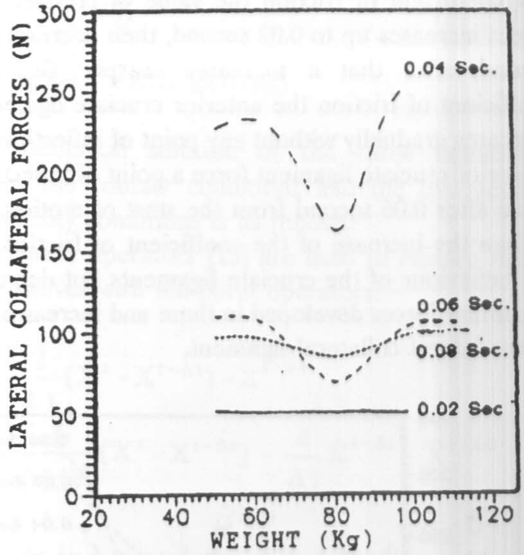

Figure 7. The effect of body weight on the lateral collateral ligament force plotted after different time periods.

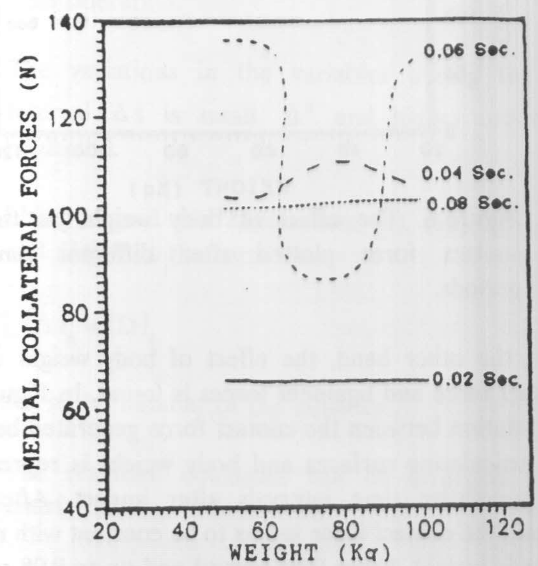

Figure 8. The effect of body weight on the medial colleteral ligament force plotted after different time periods.

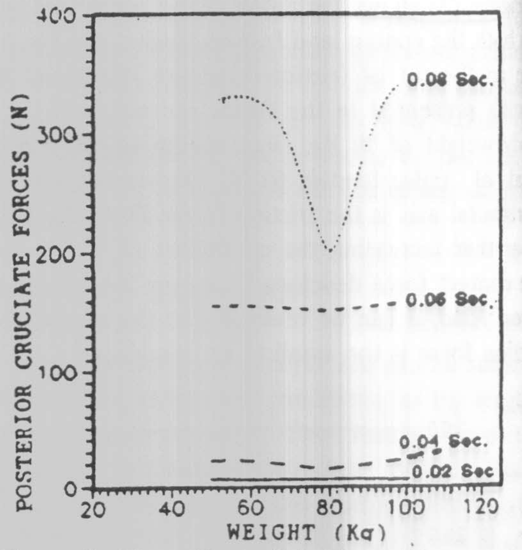

Figure 9. The effect of body weight on the antirior cruciate ligament force plotted after different time periods.

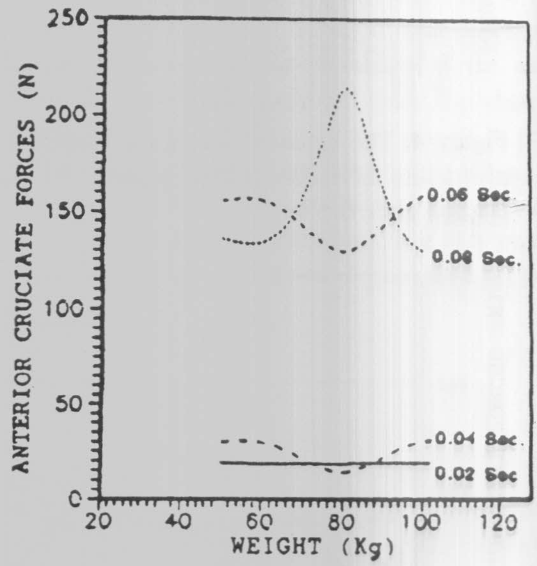

Figure 10. The effect of body weight on the posterior cruciate ligament force plotted after different time periods.

These observations are also noticed in the case of the lateral collateral Figure (7) and posterior cruciate ligaments, while the medial collateral and anterior cruciate ligaments behave differently. Figure (8) shows the relation between the force developed in the medial collateral ligament and body weight for a given time after applying the load. The figure shows that the maximum amplitude of the medial collateral ligament force occurs at 0.06

second, which shows that the effect of body weight on the medial collateral ligament force is generally as that for the other ligaments.

The relation between the anterior cruciate ligament force and body weight is shown in Figure (9) in which, the effect of body weight is similar to the other ligaments up to 0.08 second after motion is started. After that the effect is reversed giving a maximum amplitude of ligament force corresponding to a body weight of 80 Kg. This value means that the body weight affects the anterior cruciate ligament force in an opposite manner compared with the other ligament forces. Figure (10) shows that the force developed in the posterior cruciate ligament has slightly affected by body weight changes up to 0.08 second. But after 0.08 second the effect of body weight is comparatively high and its shape is nearly the inverse of that for anterior cruciate ligament with a point of inflection at 80 Kg body weight. This means that the effect of body weight on the anterior cruciate ligament force coincides with the anatomical fact which states that during some periods of rotation one of the cruciate ligaments is elongated and the other one is shortened [17].

CONCLUSIONS

The used model at hand is made suitable for a wide range of variables which were not available before. Its results are in good agreement with the anatomy of the ligamentous tibiofemoral knee joint, orthopaedic practice, and data available in literature. It has to be clear that these results give only a qualitative measure of the effect of some variables. This is due to the great variations accompanied with the models of biological subjects.

The results show that the body weight does not affect generally the trend of the relation between the flexion angle and the contact force or ligament forces, but only affects the magnitude.

On the other hand, the contribution of friction force is essential although the friction force is negligibly small. This is because the change of the coefficient of friction causes changes in the ligament forces specially the lateral collateral ligament in which the increase on the force developed in it is more than 50 % through the range of healthy subject.

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