ASPECTS OF THE ATMOSPHERIC REFRACTION IN PRECISE LEVELING

Mohamed Nabil A. Shoukry
Transportation Engineering Department, Faculty of Engineering,
Alexandria University, Alexandria, Egypt.

ABSTRACT

The problem of the proper refraction correction to be used in reduction of precise level notes is discussed. The relation of the sign and amount of such a correction to be applied as a function of the micro-climate conditions prevalent when the leveling is run, and which is a must, is examined in detail. A theoretical derivation of the applicable equations and the effect on accuracy for various classes of leveling is included. The effect of systematic error due to atmospheric vertical refraction can be eliminated totally or partially.

INTRODUCTION

A marginally significant problem in surveying observations, especially in geodetic ones, is the meteorological refraction and its coefficient. Several investigations through the recent decades have been made to furnish empirical determinations for the refractive index in order to control its effect on the observations. The majority of these investigations delved towards the refraction in trigonometric leveling of long lines through which the sight lines pass more or less through the layers which are quite close to the ground surface. The problem of terrestrial atmospheric refraction in precise leveling has not been thoroughly investigated, depending on the possibility of placing the level midway between the backsight and foresight so as to avoid the effect of earth curvature and atmospheric refraction; this procedure of course cannot always be fulfilled for all cases. Several formulae have been forwarded, relating the refractive index and the various meteorological parameters. But mostly these formulae rarely considered the micro-climatic conditions within a few meters of the terrain. The climatic effects in this micro-region are certainly most recognisable in precise leveling observations and have to be taken into consideration.

Thus, these theoretical approaches cannot be universally applied confidently due to the considerable variations in the local circumstances, and accordingly, no quantitative procedure of determining the refractive index yet advanced adequately represents all the cases involved. A plausible value for the refractive index for the reduction of precise leveling is sought so as to cope with a wide variety of conditions.

In his explicit study, and after extensive observations, R. Eder reported that the refractive index value can be considered as 0.7 on condition that there always be a quite sufficient layer of air below the line of sight. For winter observations in the micro-region, and especially in spring, this value may increase several times, and there is no empirical value which may have a general form. In another report, Eder stated that the refractive index in India varies from -0.9 to + 1.21, and again in a further report, stated that for the Tobo triangulation net at Ketsh in India the value varies from - 0.01 to - 1.0.

The intention of the investigation presented in this paper is to inter-relate the various meteorological parameters and to assess and interpret their effect on the reduction of precise leveling.

OBSERVATIONS

It is indeed cumbersome to accurately determine the refractive index with an error of as much as 15 % or more, and any research study attempted to improve this percentage is of value and appreciated. In the present work, various empirical equations relating the index of refraction with the ambient meteorological factors (temperature, relative humidity, atmospheric pressure and diurnal variation of refractive index) are treated with. Also the effect of the height of instrument above ground surface is taken into consideration. The velocity of the wind is referred to as well. It is recommended that future analysis should include other factors such as the density of the dust in the air and the magnitude and source of illumination. A prime factor which should also be recommended for, is the nature of the terrain itself, weather grass, asphalt... etc., where the sun's radiation is absorbed and heats the air close to the ground surface.

The observations, although taken over a relatively short period of one year, do shed some new light on the problem and assist in interpreting an accurate value for the refractive index, as being proved herein after.

Two different distances were taken, namely, 100 ms. and 80 ms. All observations were performed in the shade for the former distance, whereas all the observations for the latter distance were taken in the sun. In each case, the precise level was always placed very close to one of the staffs so as to diminish the effect of curvature and refraction on its readings, and that all the effect be imposed on the readings of the farthest staff. As for the staffs, they were accurately placed on their iron bases fixed in position.

The observations commenced from the 1st of September to the end of August of the following year and covered a total of 1400 *readings. They were taken at different times* throughout the day and under all climate conditions, from calm to stormy, clear to cloudy, dry to rainy, sunny to shady and from a temperature ranging from approximately 10° C. to 45°. The lapse rate was not included in the observations since a thermometer reading to an accuracy of 1/1000, or at least 1/100 of a degree is required, which is impracticable.

1. *Temperature Effect*

To appreciate the extent of the refractive index variation with the temperature variation, each of the two groups of observations taken in the sun and shade was divided into temperature ranges, table (1). The average effect of refraction was obtained in each range by calculating the curvature effect and subtracting it from the total effect of curvature and refraction together, for each case. Also, due to the difference in distance of the sun and shade observations, and for the sake of comparison, the refraction effect in the sun observations of distance 80 ms. was converted to it's effect in the case of a distance equal to that of the shade observations, 100 ms.

Figure (1) portrays a plot of log temperature versus log refraction effect, for each group. The figure reveals that there is no marked difference between the refraction effect in the sun observations and that in the shade observations, with the variation in temperature.

Table 1

Shade observations		Sun observations	
temperature	no. of obs.	temperature	no. of obs.
9°-13°	62	10°-16°	55
13°-17°	98	16°-20°	118
17°-22°	171	20°-29°	90
22°-28°	115	23°-26°	69
28°-32°	99	26°-30°	30
32°-36°	61	30°-33°	38
36°-45°	42	33°-36°	25
		36°-39°	40
		39°-45°	27
	648		492

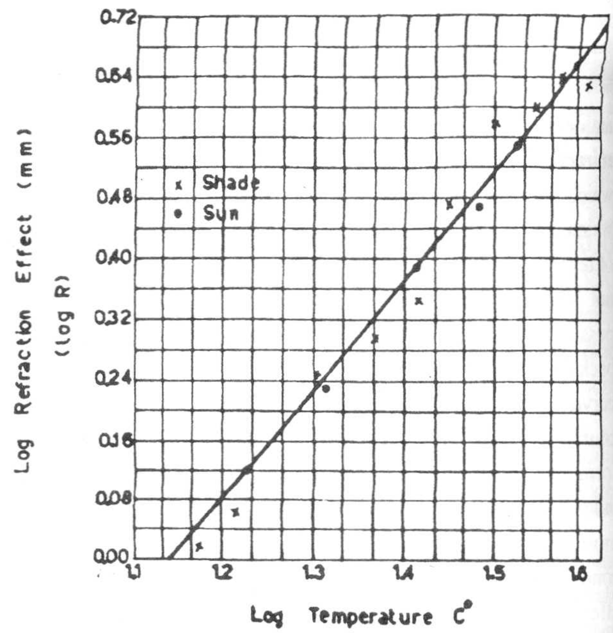

Figure 1.

From the figure, the plot conforms to the equation:

$$y = 1.440x - 1.648$$

and

$$\log R_{100} = 1.440 \log T - \log 44.46$$

$$R_{100} = \frac{T^{1.44}}{44.46}$$

R_{100} (distance 100 ms.) $= 0.0225\ T^{1.44}$

∴ The refraction effect for any distance D is given by:

$$R = 0.0225\ T^{1.44}\ D^2/(0.1)^2$$

$$= 2.25\ T^{1.44}\ D^2$$

$$= 0.945\ T^{1.427}\ D^2$$

where

R in mms.
D in Kms.
T in °C.

On the other hand,

$$R(mms.) = \frac{2KD^2 \times 1000 \times 1000}{2R_o} = \frac{KD^2}{R_o} \times 10^6$$

where R_o is the radius of the earth in Kms, and K is the refractive index. Equalizing the two equations:

$$2.25\,T^{1.44}\,D^2 = \frac{KD^2}{R_o} \times 10^6$$

renders

$$K = 2250\ T^{1.44}\ R_o \times 10^{-9},$$

substituting $R_o = 6370$ Kms.

$$\therefore K = 0.0143\ T^{1.44}$$

This equation cannot be fully considered valid for all temperatures since there is no sufficient observational information of temperature very close to 0°C. or below. Anyhow, observations in Egypt in very low temperatures or higher than 40°C are very rare and dispensable.

2. *Relative humidity effect*

Table (2) sets out the relative humidity observations divided into ranges. Here also, and as in the temperature observations, the average effect of refraction was determined for each range by the same means as previously mentioned, and for each case. The refraction effect in the sun was again (as in temperature) converted to its effect of 100 ms. (shade observations). A plot, Figure (2), was made between log relative humidity versus refraction effect , for each group. No obvious difference was apparent between the refraction effect, either in the sun or shade, with the variation in relative humidity.

Table 2

Shade observations		Sun observations	
0- 5	5	0- 5	5
5-10	10	5-10	18
10-15	14	10-15	14
15-25	40	15-25	31
25-35	30	25-35	25
35-45	85	35-45	66
45-50	78	45-50	70
50-60	117	50-60	98
60-65	112	60-65	80
65-75	82	65-75	103
75-85	80	75-85	60
85-100	35	85-100	16
	690		586

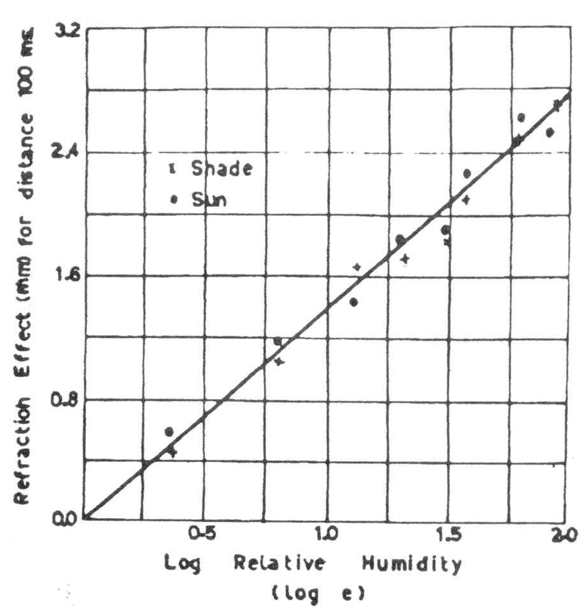

Figure 2.

From the figure:

$$y = 1.41\,x$$
$$R_{100} = 1.41 \log e$$
$$R_D = 1.41 \log e \times D^2/(0.1)^2$$
$$= 1.41\,D^2 \log e$$

where R is in mms. and D in Kms.

$$141\,D^2 \log e = \frac{KD^2}{R_o} \times 10^6$$

and $K = 141 \log e \times R_o \times 10^{-6}$
Substituting for R_o = 6370 Kms.

$$\therefore K = 0.898 \log e$$

For temp. and rel. humidity, the previous plotted illustrations in a log form yield to a convenient comparative study between the several items.

3. *Atmospheric pressure effect*

The variation in the atmospheric pressure has shown to be of insignificance. The observations recorded a minimum of 745 mm. Hg. to a maximum of 767 mm. Hg. They were divided into two groups (sun and shade), with each group being also arranged into ranges of 5 mm. Hg., except the last range of 7 mm. Hg. Table (3) and Figure (3) represent the variation of refraction effect with atmospheric pressure variation.

From Figure (3):

$$y = -0.1283\,x + 98.75$$
$$R_{100} = -0.1283\,P + 98.75$$
$$R_D = (-0.1283\,P + 98.75)\,D^2 / (0.1)^2$$
$$= (-12.83\,P + 98.75)\,D^2$$

where

D in Kms.
R in mms.
P in mm. Hg.

But $(-0.1283\,P + 98.75)\,D^2 = (KD^2/R_o) \times 10^6$

$$\therefore K = (-12.83\,P + 98.75)\,R_o \times 10^{-6}$$

Substituting for R_o = 6370 Kms.

$$K = -0.083\,P + 62.90$$

Table 3

Shade observations		Sun observations		
no. of obs.	av. refrac. effect (mm)	no. of obs.	av. refrac. effect (mm)	pressure
83	2.911	81	2.599	745-750
256	2.201	127	1.943	750-755
102	1.610	41	1.981	755-760
18	0.995	17	0.772	760-765
465		266		

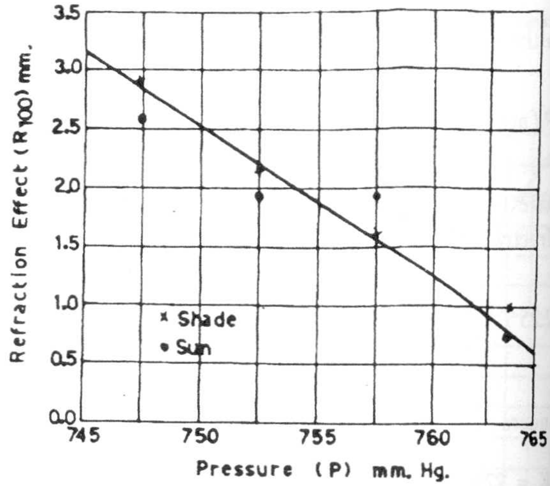

Figure 3.

Correlation of Parameters Influencing the Coefficient of Refraction

In according to the foregoing, the total effect of a certain temperature, relative humidity and atmospheric pressure upon the index of refraction for precise leveling may be summed up and furnished in the following form:

$$K = a\,T^{1.44} + b \log e + cP + d$$

where a,b,c,d are constants determined by forming observational equations from the available observations and applying the theory of least squares. The values of these constants were obtained as shown in the eq. :

$$K = 5.9\times10^{-4} T^{1.44} + 1.3\times10^{-6} \log e - 0.0814\,P + 62.417$$

EFFECT OF INSTRUMENT HEIGHT

To investigate the effect of refraction variation with the change of the height of instrument above ground upon precise leveling, two fixed points were chosen at distances of 80 ms. and 8 ms. respectively from the level. The latter distance was chosen so that there would be no effect on the rod readings because of its closeness. Two sets of readings were observed. The instrument height was 1.55 ms. to 1.65 ms. for the first set and 0.81 ms. for the second group.

The measurements lasted from January to June of the same year and were taken throughout the day and under variable weather conditions giving approx. 120 readings for each group. The temperature, relative humidity and atmospheric pressure were recorded each time. The temperature varied from 12°C to 45°C, the relative humidity from 5% to 96 %, while the atmospheric pressure ranged from 747 to 753 mm. Hg., except for three observations which reached 756 mm. Hg. Accordingly, the variation in atmospheric pressure was insignificant especially since most of the readings were enclosed between 749 and 751 mm. Hg.

The refraction effect on the levels was calculated (in both cases of instrument height) after subtracting the curvature effect. The ratio was then obtained between the refraction effect at lower and higher instrument heights. To demonstrate the individual effect of temperature and relative humidity, the observations were once ordered into ranges of 10% (w.r.t. relative humidity), and again into ranges of approximately 8°C. (w.r.t. temp.). Tables (4), (5) show these divisions.

The average effect of lowering the instrument in all the observations = 1.85.

The relative humidity and temperature versus the effect of instrument height on refraction were plotted, Figures (4), (5). The plots designate that the lowering of the instrument from 1.60 to 0.81 ms. (i.e. approx. to the half), will averagely increase the refraction effect twice. From the relative humidity plot, it can be readily seen that the effect of the refraction due to the lowering of the instrument, is relatively proportional to the increasing degree of the relative humidity. Also, it is withdrawn that up to approximately 30 to 35 % relative humidity, a low instrument height will have practically no effect on refraction. As a matter of fact, the installation of the device at a low height will decrease the effect of refraction, whereas it increases thrice at 85 to 90 %. Anyhow, and in general, the refraction effect increases with a low instrument height, and even reaches in some rare cases, to as high as 12 times (N.B. these cases were omitted when taking the average).

Table 4

Relative humidity from-to	no. of obs.	Effect of refrac. with av. instr. height 0.81 ms. / Effect of refrac. with av. instr. height 1.60 ms.
0-10	7	0.59
10-20	10	1.62
20-30	7	0.80
30-40	8	1.08
40-50	14	1.79
50-60	17	2.34
60-70	20	2.33
70-80	15	2.39
80-90	11	3.10
90-100	9	2.88
	118	

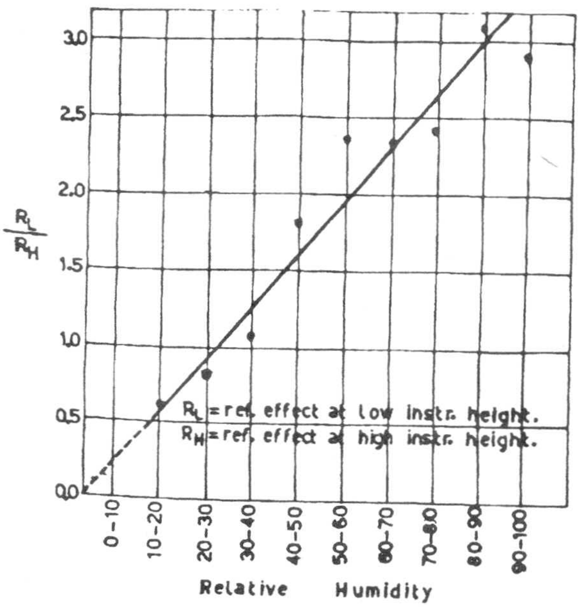

Figure 4.

Table 5

Temp. from-to	no. of obs.	Effect of refrac. with av. instr. height 0.81 ms. / Effect of refrac. with av. instr. height 1.60 ms.
12-20	18	1.09
20-28	26	1.30
28-35	53	1.91
35-45	21	2.69
	118	

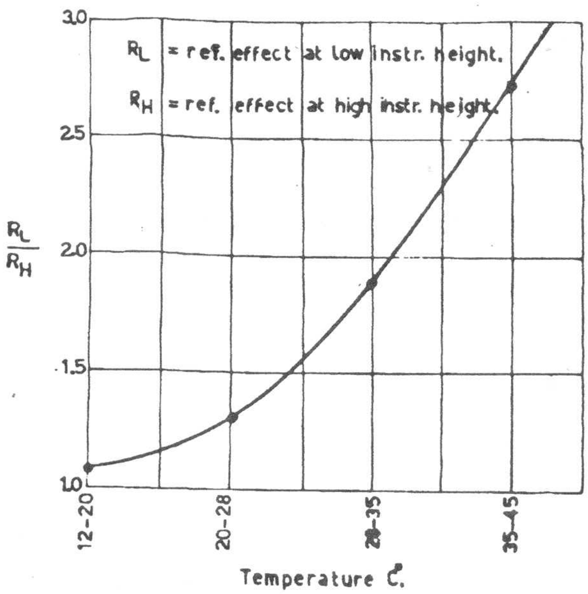

Figure 5.

DIURNAL VARIATION OF REFRACTIVE INDEX

Over 800 observations were taken at different hours of the day throughout a whole year. These observations were thoroughly analyzed in order to investigate the average effect of time during the day on the refractive index.

To refer time to a certain datum, it was assumed that:

T_o = time of observation
T_n = noontime
T_s = time of sunset

On this basis we assumed that the observation datum is T, where:

$$T = \frac{T_o - T_n}{T_s - T_n}$$

On this datum the value of T for sunrise = -1 and for sunset = + 1. The observations were listed according to the value of T as shown in table (6). Also, the spring and summer observations were arranged together since the inclination of the sun does not vary much at these periods of the year. The same arrangement was performed for the autumn and winter observations together. The table also shows the average refraction effect at the different periods of T and plotted in figs. (6), (7).

It is obviously apparent from both figures, that the refraction effect gradually increases towards sunrise and sunset. This increase however, and as expected, is less during winter. Also, the refraction effect, and accordingly the refractive index, is minimum at approximately T = + 0.15 in spring and summer, and at approximately T = + 0.28 in winter.

EFFECT OF WIND

It is interesting in this aspect to examine the effect of wind on the magnitude of the atmospheric refraction and accordingly, the refractive index. Some observations were performed at different weathering conditions; calm status, moderate weather and a stormy one. The behaviour of the atmospheric refraction was recognised all over these cases. Because of the lack of some observations, no definite significant proof could be assured but some valuable remarks could be edited, namely:

The effect of the atmospheric refraction and the amount of refractive index in the calm status seem to be steady and slightly high especially at noontime. This is due to the certainty of the laminarity of the atmosphere layers near the terrain surface. On the other hand, in windy weather turbulent fluctuations of the refractive index are quite obvious and a decrease in its amount is apparent. This is due to the turbulent behaviour of the atmospheric layers which affect the values of the refractive index and accordingly, the magnitudes of the observations. In addition, the value of the refractive index seems to be stable during cloudy weather. This assures the relation between the atmospheric temperature, the temperature gradient versus the refractive index.

Table 6

T	Summer & Spring		T	Autumn & Winter	
	refr. effect (mm)	no. of obs.		refr. effect (mm)	no. of obs.
- 0.8 to - 0.6	1.92	60	- 0.8 to - 0.6	2.10	72
- 0.6 to - 0.5	1.89	44	- 0.6 to - 0.4	1.78	27
- 0.5 to - 0.3	1.65	16	- 0.4 to - 0.2	1.51	39
- 0.3 to - 0.1	1.51	48	- 0.2 to - 0.1	1.41	40
- 0.1 to 0.0	1.35	34	- 0.1 to 0.1	1.48	31
0.0 to 0.1	1.29	26	0.1 to 0.2	1.42	42
0.1 to 0.2	1.15	16	0.2 to 0.4	1.48	43
0.2 to 0.4	1.45	36	0.4 to 0.6	1.35	41
0.4 to 0.6	1.55	32	0.6 to 0.8	1.49	36
0.6 to 0.8	1.58	30	0.8 to 1.0	1.69	45
0.8 to 0.9	1.89	13	1.0 to 1.2	1.90	19
0.9 to 1.0	2.08	19			
		374			435

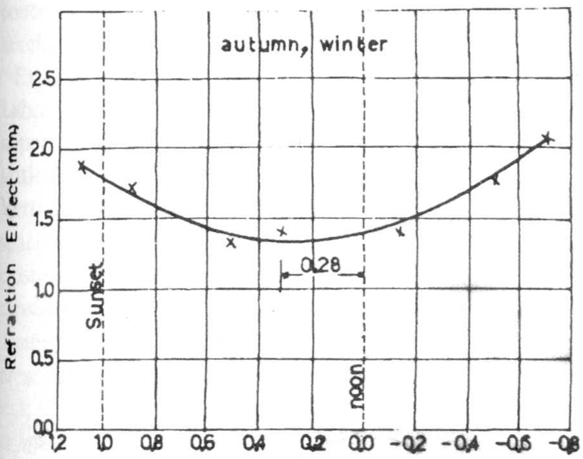

Figure 6.

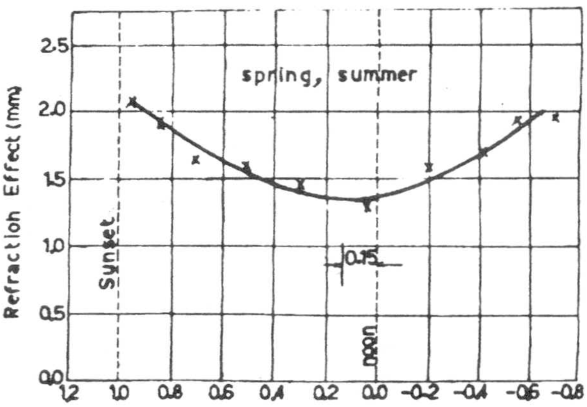

Figure 7.

Finally, it is felt that further intensive investigations concerning the wind factor effect upon the refractive index should be carried out in order to clarify this correlation.

CONCLUSIONS

From the foregoing discussions and results of over 1400 observations covering the different days of the year and different hours of the day, and in interpreting the effect of the systematic errors due to refraction on precise levels, we may conclude the following:

1. The refractive index in precise leveling, where the line of sight passes through the layers of air very close to the ground surface, differs greatly from the refractive index in trigonometric leveling where the line of sight mostly passes through the layers of the free air.
2. The refractive index in precise leveling varies widely from approximately - 2.0 to + 2.0. These values are unusual in trigonometric leveling, and as a matter of fact, the negative values are also unusual.
3. Concerning our aspect, it seems to be that no adequate equation can be confidently advanced for the accurate determination of the refractive index, but a general form may be furnished so as to properly evaluate and shed light on it. This or these equations may be applicable only on the local conditions, and a universal equation to be applied at all places cannot be forwarded.
4. The refractive index is directly proportional to the temperature as well as to the relative humidity, while it is inversely proportional to the atmospheric pressure.
5. By applying the principle of least squares, a relationship is introduced correlating the refractive index with the meteorological parameters, namely, temperature, relative humidity and atmospheric pressure.
6. The error of terrestrial refraction in precise leveling can be reduced by a careful choice for a location of a reasonable height for the line of sight above the ground. The lowering of the instrument to half the height will averagely double the refraction effect, and may even increase 7 times in individual cases. A height of 1.2 to 1.5 meters is favourably suggested.
7. The amount of refraction is minimum slightly afternoon, varying from 0.15 (from noon to sunset) in spring and summer, to 0.28 (at the same period) in autumn and winter. This value increases towards sunrise and sunset. Performance of leveling under a cloudy sky is highly recommended, as the refraction problem could be ignored with excellent theoretical results.
8. Emphasis should be made on the following factors to interpret the extent of each of their effect on the refractive index:
   - nature of terrain, weather grass, rock, asphalt ... etc., and the effect of its reflection number
   - the magnitude and source of light.

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