# TRANSMISSION OF NONLINEAR CHIRPED OPTICAL PULSES IN $GeO_2$-DOPED FIBERS: THE CONCEPT OF PLANCK, AND THE EFFECT OF THERMAL FIELD

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ABSTRACT

Light soliton transmission of chirped picosecond puleses in germania doped waveguide fibers is parametrically studied over a wide range of interest of the affecting parameters, where the time-dependent wavelength for semiconductor film lasers is considered. The concept of "Planck" is introduced to account for the product of power and pulse width square. Temperature-dependent chromatic dispersion is considered to investigate the effects of thermal fields. Basically two typical chirped optical sources are employed in the analysis. It is found that chirped pulses posses lower product compared to unchirped pulse. Variations of the product have positive correlations regarding variations in thermal field, while possessing negative correlations with germania percentage. Different types of correlations are found against the variation of the waveguide radius. Theoretical conditions required to compress the optical pulse are investigated and a reduction of about 86% of the initial width is obtained.

## INTRODUCTION

Due to the effect of free carriers on the refractive index of the semiconductor laser active material, the laser radiation possesses a time-dependent wavelength. Such sweep in the wavelength (chirping) is attributed to the change in free-carrier concentration during the pulse generation which in turn contributes to the value of the refractive index of the semiconductor material [1-5]. Such dynamical single mode semiconductor lasers, that operate under rapid direct modulation, have been developed for high capacity and long distance single mode fiber communication systems.

The transmission properties of a chirped pulse through a single mode fiber have been studied theoretically and experimentally for its significant practical importance [1-7] and it was pointed out that the dynamic wavelength shift caused a power penalty in the transmission at bit rates greater than 1 Gbit/sec [7].

If a chirped or nonchirped pulse is transmitted through a dispersive nonlinear medium (fibers or planar guides) it undergoes a solitary motion in which the dispersion effects result in the broadening of the pulse while nonlinearity tends to sharpen it [8-10]. Hasegawa and Kodama [11] analyzed the signal transmission by optical solutions where a transmission rate of about 1 T bit/sec. per 30 km could be achieved using envelop solutions with a peak power of about 10 W(0.1 Tbit/sec. with 10 mW) in monomode fibers. The product $\Gamma = P\tau^2$, where P is the peak power of the solitary pulse and $\tau$ is its duration has its significant practical importance when dealing with high data rate optical communication systems where it must be minimized. The parameters affecting this product are i) dispersion characteristics ii) chirping characteristics, iii) fiber nonlinearity, and iv) thermal filed. The present paper presents a mathematical model for the soliton propagation of nonlinear optical pulses through germania doped single mode optical waveguides subjected to a thermal field. A measure or unit called "Planck" is introduced to account for the power × pulse width squared product.

## WHAT IS MEANT BY A "PLANCK"

The product $\Gamma$ is defined as:

$$\Gamma = P\,\tau^2 \text{ Watt. sec}^2. \qquad (1)$$

The Planck constant is given as [9]

$$h = 6.62517 \times 10^{-34} \text{ Joule. sec} = \text{Watt. sec}^2. \quad (2)$$

The product Γ has thus the dimensions of Planck's constant h; moreover Γ assumes very small values in the order of magnitude of h. Therefore we can introduce the dimensionless product $\Gamma_P$ as the product referred to Planck's constant h as:

$$\Gamma_P = \Gamma/h, \text{ Plancks} \quad (3)$$

## MATHEMATICAL MODEL AND ANALYSIS

Experimentally [1], the chirping of film lasers is measured by optical upconversion sampling of the laser pulse followed by spectral filtering and a mathematical model is casted and employed to extract the instantaneous time-dependent laser wavelength λ (t) from the measured data for two In GaAsP film lasers where

$$\lambda (t) = \lambda_o + \Delta\lambda_1 \, t \, \Delta\lambda_2 \, t^2 \quad (4)$$

$\lambda_o$ , $\Delta\lambda_1$ , and $\Delta\lambda_2$ are, respectively, the unchirped wavelength, linear chirping parameter, and nonlinear chirping parameter (See table 1).

Table 1.

| | First Source | Second Source |
|---|---|---|
| $\lambda_o$, μm | 1.151 | 1.261 |
| $\Delta\lambda_1$ , μm/sec. | 940± 150 | 100±22 |
| $\Delta\lambda_2$, μm/sec$^2$ | (-80±12)x10$^{12}$ | (-50±25)x10$^{12}$ |

If a chirped pulse is passed through a dispersive medium, the pulse may lengthen or shorten depending on the relative sign of the dispersion. If the radian frequency of the chirp is expressed as:

$$\omega (t) = \omega_o + \Delta\omega_1 t + \Delta\omega_2 t^2 \quad (5)$$

The parameters $\Delta\omega_1$ and $\Delta\omega_2$ can be related to the corresponding parameters $\lambda_o$ , $\Delta\lambda_1$, and $\Delta\lambda_2$ of equation (4) as

$$\Delta\omega_1 = -\frac{2\pi c}{\lambda_o^2} \Delta\lambda_1 \quad (6)$$

and

$$\Delta\omega_2 = \frac{2\pi c}{\lambda_o^2} [- \Delta\lambda_2 + \frac{(\Delta\lambda_1)^2}{\lambda_o}] \quad (7)$$

References [1,3] reported the following equation for the rms width σ of the pulse of an initial rms width $\sigma_o$ as

$$\frac{\sigma^2}{\sigma_o^2} = [(1 + k''L\Delta\omega_1)^2 + (\frac{4k''L}{T_e^2}) + (\tfrac{1}{2} \, T_e \, k''L\Delta\omega_2)^2 \quad (8)$$

where
k" = second derivative of k w.r.t ω,

$$k = \frac{n\omega}{c} ,$$

L = waveguide length, and
$T_e$ = full width at the $e^{-1}$ points.

In many theoretical calculations of fiber dispersion the choice of Sellmeier coefficients is made such that it is capable of handling arbitrarily the concentration of $GeO_2$. Fleming [13] fitted the index measurements to a three-term relation in the form

$$n^2 = 1 + \sum_{i=1}^{3} \frac{A_i \lambda^2}{\lambda^2 - \lambda_i^2} \quad (9)$$

where n is the refractive index, $A_i$ is the oscillator strength, and $\lambda_i$ is the oscillator wavelength. both $A_{i,}$ and $\lambda_{i,s}$ are functions of temperature T and the mole fraction of germania x.

The thermal effects are considered through the dependence of both n and the guide radius R [14] where

$$A_i = A_{io} < f (T) > \quad (10)$$

$$f (T,\lambda) = a_o (\lambda) + a_1 (\lambda) T \quad (11)$$

$$\lambda_i = \lambda_{io} (T_o/T) \quad (12)$$

where T is the temperature and $T_o$ is the room value. Both $a_o$ (λ) and $a_1$ (λ) are calculated at different wavelengths and averaged to be

$$< a_o > = 0.93821 \quad (13)$$

$$< a_1 > = 2.0857 \times 10^{-4} \quad (14)$$

Both $A_{io}$& $\lambda_{io}$ take the same definitions and the same functional relationships as in Ref. [13].
The temperature-dependent core radius R is modelled in the form [15]

$$R = R_o [1 + \in (T - T_o)] \tag{15}$$

where $\in$ is reported in [16] as

$$\in = 7.067 \times 10^{-6} + 2.11 \times 10^{-8} T \tag{16}$$

The effect of chirping is considered through the averaging of k, k', and k" over the chirping length ($\omega_2$ - $\omega_1$) where $\omega_1$ is the radian frequency at the beginning of the pulse duration which will be considered equal to $\sqrt{2}T_e$, while $\omega_2$ is the radian frequency at the end of the duration. Thus we have

$$< k > = (k(\omega_2) + k(\omega_1))/2 \tag{17}$$

$$< k'> = (k(\omega_2) - k(\omega_1))/(\omega_2 - \omega_1) \tag{18}$$

$$< k"> = (k'(\omega_2) - k'(\omega_1))/(\omega_2 - \omega_1) \tag{19}$$

In the case of no chirping we have

$$k = \frac{\omega n}{c}\Big|_{\omega=\omega_1} \tag{20}$$

$$k' = (1/c)[n - \omega n']\Big|_{\omega=\omega_1} \tag{21}$$

$$k" = (1/c)[n - \omega n"]\Big|_{\omega=\omega_1} \tag{22}$$

The product of the peak power and the square of the pulse duration is given as [8.11]

$$\Gamma = 0.5 \in_o v_g n^2 S \psi^2 \tag{23}$$

where $\epsilon_o$ = 8.854 x $10^{-12}$ F/m, $v_g$ = c/n, and S is the cross-sectional area of the guide. reference [8] expressed the function $\phi^2$ in the form

$$\psi^2 = \frac{-[1-1/kR_e]kk" + (1/kRe)k'^2}{(n_2/n)k^2} \tag{24}$$

where $n_2$ is the nonlinear index coefficient, $n_2$ = 1.22x$10^{-22}$ and R is the equivalent guide radius,

$$Re = \sqrt{\text{guide area}/\pi} \tag{25}$$

In fact $\Psi$, represents the peak of light soliton that propagates through the guide with velocity $v_g$ [8]. equation (23) is rewritten as

$$\Gamma = 1.39134 \times 10^{-11} v_g n^2 R_e^2 \Psi^2 \tag{26}$$

which terminates the suggested model

## IV. RESULTS AND DISCUSSION

The model casted above, is employed to investigate the variations of the product for two cases i) case of no chirping and ii) case of chirping, where in each case, the thermal effects are either considered or not considered.
Two measuring criteria, namely, $D_\Gamma$ and $R_\Gamma$ are to be defined and employed to measure both the thermal and chirping effects where

$$D_\Gamma = \Gamma(T) - \Gamma(300\ ^oK) \tag{27}$$

and

$$R_\Gamma = \Gamma_{chirping} / \Gamma_{no\ chirping} \tag{28}$$

The variations of the variables $\Gamma$, $D_\Gamma$, and $R_\Gamma$ against the variations of the independent set of variables [$R_e$,x,T, Te] are studied over wide range of variation.

### I. VARIATIONS OF $\Gamma$:

Variations of $\Gamma$ as portrayed in units of Joule.sec, as given by Eq. (26), or as casted in units of "Plancks", as given by Eq. (3), are displayed in Figures (1-6). Data in Figs. 1 and 2 clarify that:

1- $\Gamma$ and T are in positive correlation at any value of R and x as higher values of T yield higher values of n and thus higher values of $\Gamma$ as given by Eq. (26).
2- $\Gamma$ and x are in negative correlation at any value of R and T as higher values of x produce dispersions that reduce the quantity $\Psi^2$ as given by Eq. (24).
3- Chirping yields lower values for the product.
4- Based on table I, the first source is more suitable as employed to give lower values of $\Gamma$.

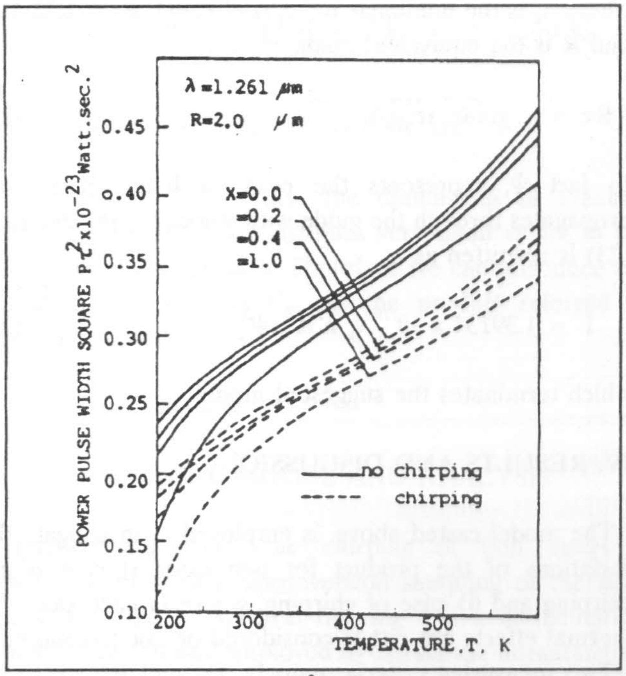


Figure 1. Variations of $P\tau^2$ against variations of T.

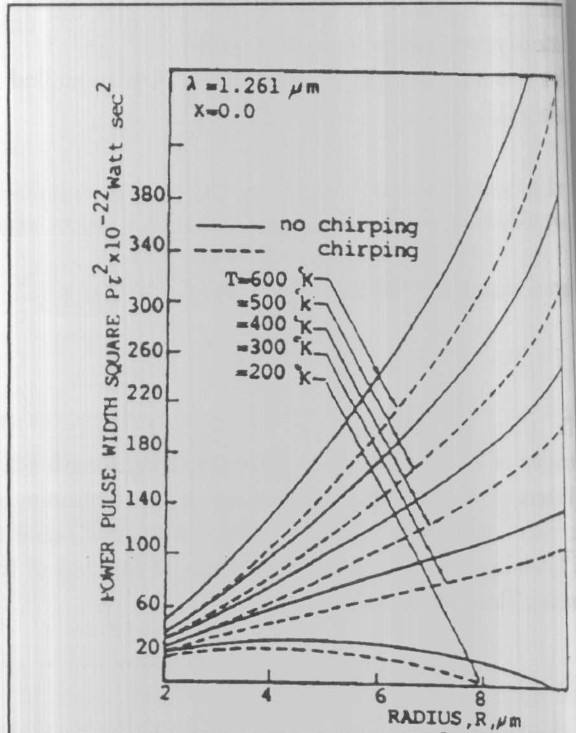


Figure 3. Variations of $P\tau^2$ against variations of R.

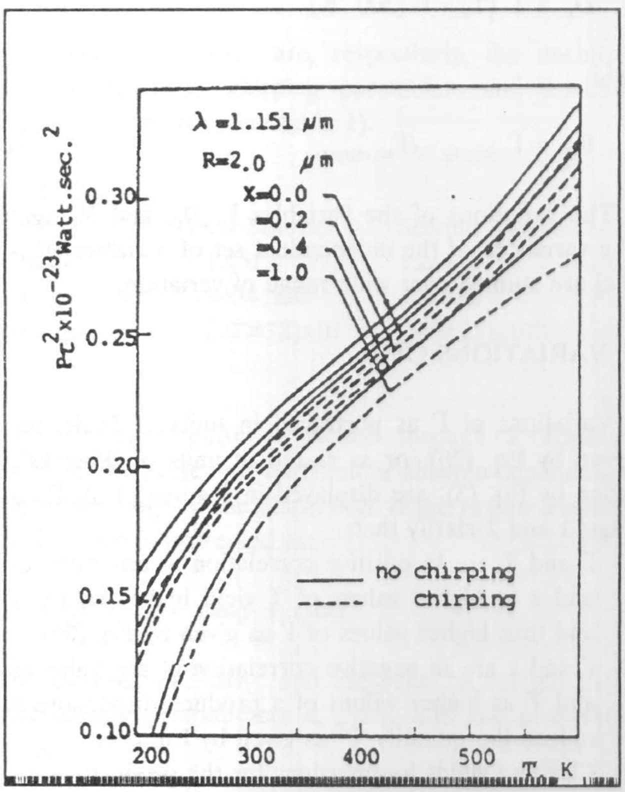


Figure 2. As in Figure 1, $\lambda = 1.151$ $\mu$m.

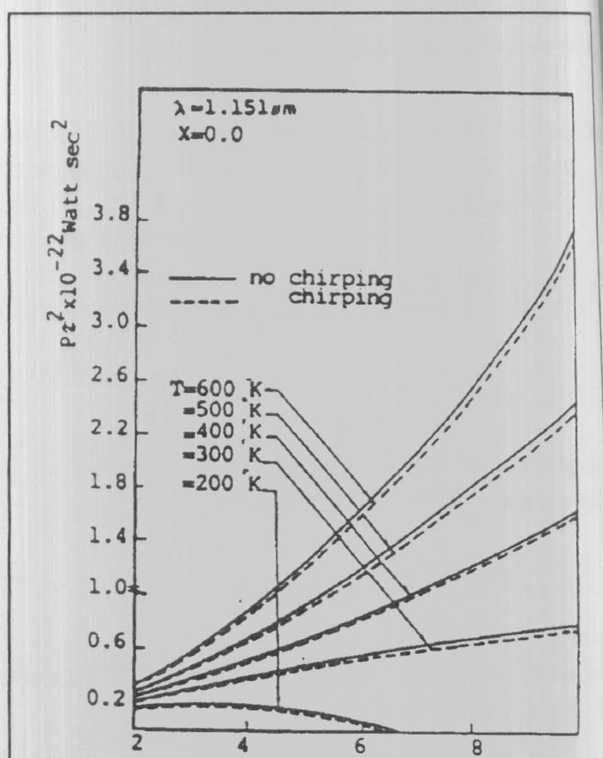


Figure 4. As in Figure 3, $\lambda = 1.151$ $\mu$m.

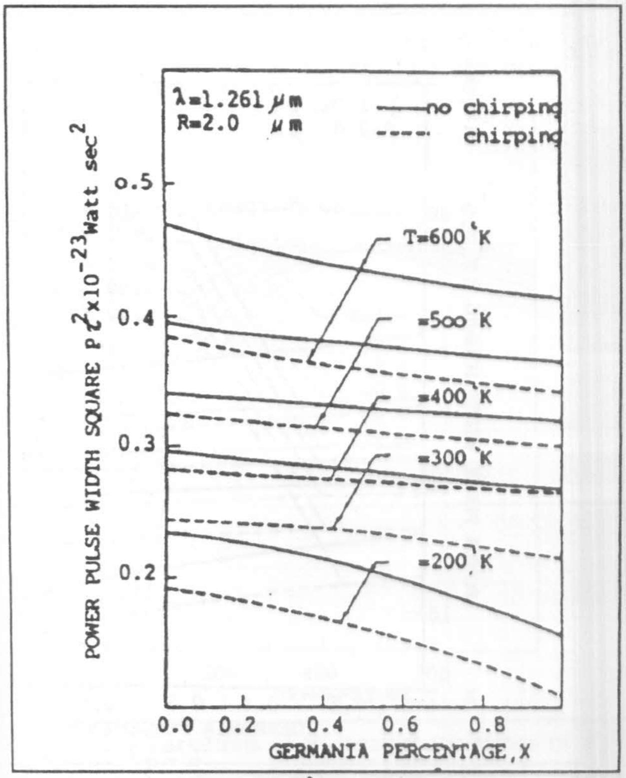


Figure 5. Variations of $P\tau^2$ against variations of X.

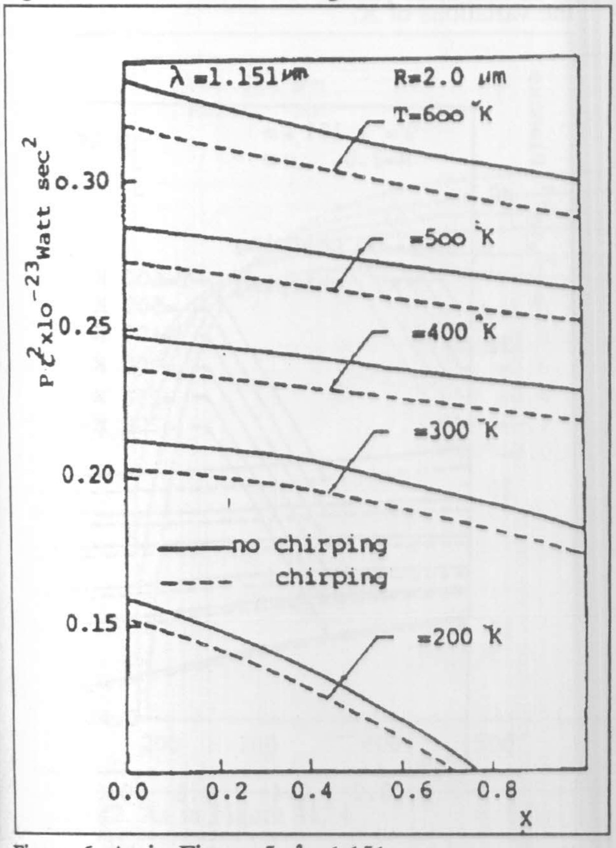


Figure 6. As in Figure 5. $\lambda = 1.151\ \mu m$.

Chirped pulses of wavelength $\lambda$ (t), where

$$\lambda(t) = \lambda_o + \Delta\lambda_1(t) + \Delta\lambda_2(t),$$

possess lower product $\Gamma$ compared to unchirped pulses of wave length $\lambda_o$ because at $<\lambda(t)>$ [which is the average chirped wavelength and is larger than $\lambda_o$], the chirped pulses possess dispersive characteristics lower than the unchirped pulses, which in turns produces $\Psi^2$ of less value and consequently a product $\Gamma$ of lower values

Data in Figure (3) and (4) indicate that :

1- Above the room temperature (to = 300°K), $\Gamma$ and R are of positive correlation, while for T < To, the correlation is negative.

   Variation of the product $\Gamma$ have positive thermal correlations because as T increases n and the dispersive characteristics increases and consequently $\Gamma$ increases. Without loss of consequenc $\Gamma$ increases. Without loss of generality, the increase of X lowers the dispersive characteristic and consequently the produced $\Gamma$ decreases.

2- Here, also, the chirping gives lower values for $\Gamma$ and the first source is better.

Data in Figs. (5) and (6) illustrates:

1- At any value of R and T, $\Gamma$ and X possess negative correlation.

2- Data casted here assure the pervious data (the same feature and the same correlations).

II. VARIATIONS OF $\Gamma_P$

Variations of $\Gamma_p$ against the variations of one or more of the set of parameters, $\{R_e, X, T\}$ are depicted on Figures (7-10). Without loss of generality, $\Gamma_p$ possess the same sort of variations of $\Gamma$ but in a modified unit namely the "Planck".

III. VARIATIONS OF $R_\Gamma$

These variations are displayed in Figures (11-16) where the main features are:

1- Variations possess correlation with the independent variable up to a certain value where the correlation reverses its sign.

2- At any set of the affecting parameters, $\{R_e, T, X, R_\Gamma\}$ is less than unity.

3- The first typical source yields values of $R_\Gamma$ smaller than that obtained form the second typical source.

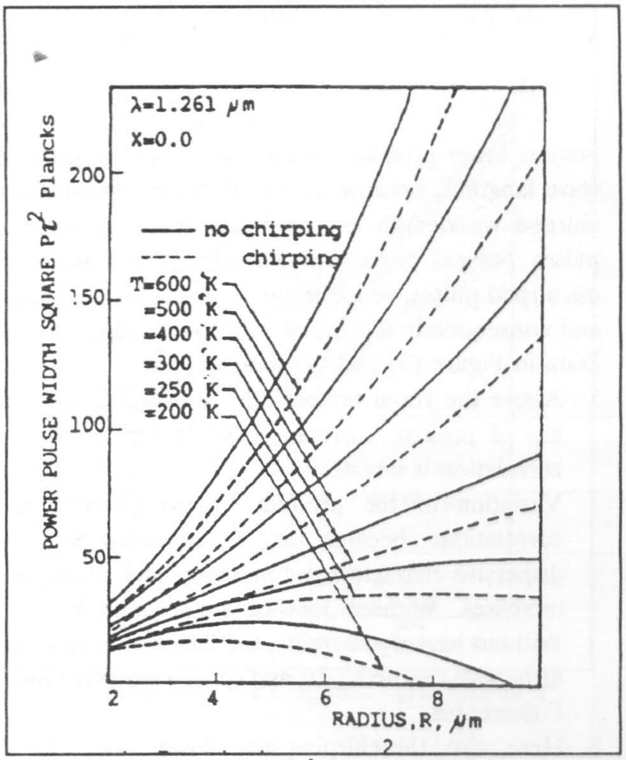


Figure 7. Variations of $P\tau^2$ against variations of R.

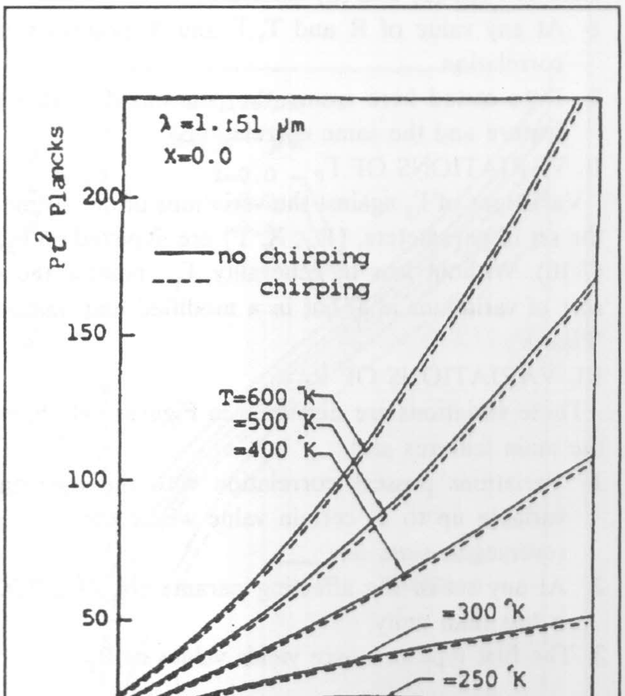


Figure 8. As in Figure 7, $\lambda = 1.151\ \mu m$.

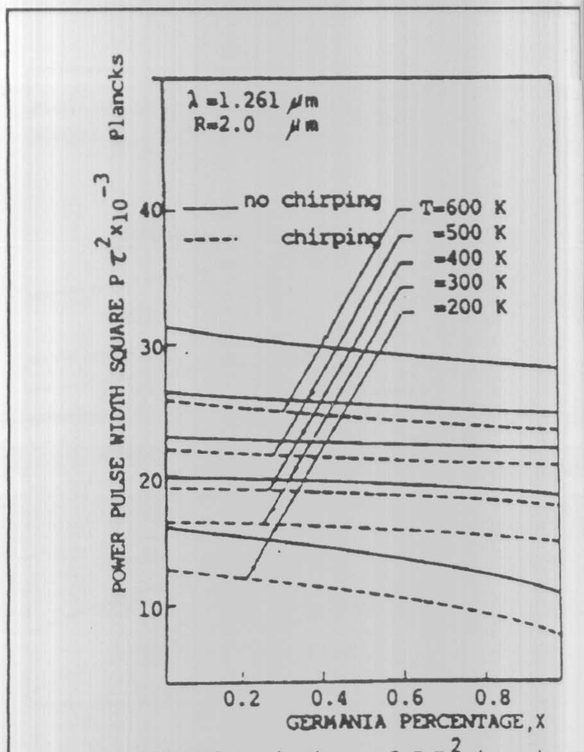


Figure 9. Variations of $P\tau^2$ in units of plancks against the variations of X.

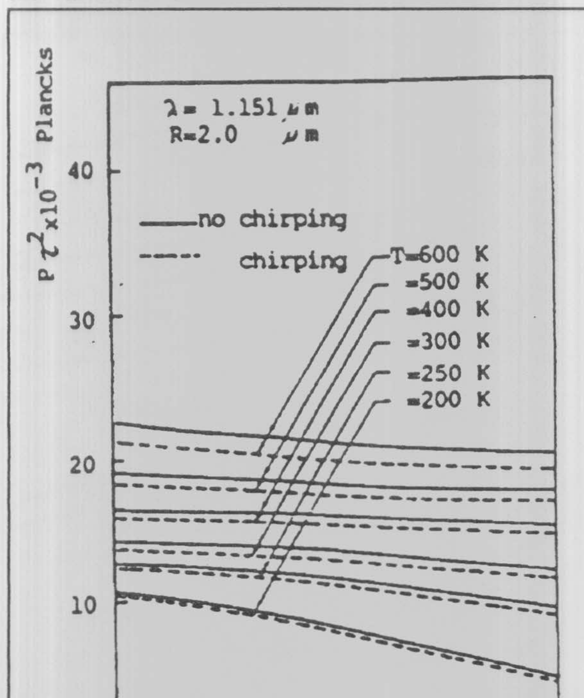


Figure 10. As in Figure 9, $\lambda = 1.151\ \mu m$.

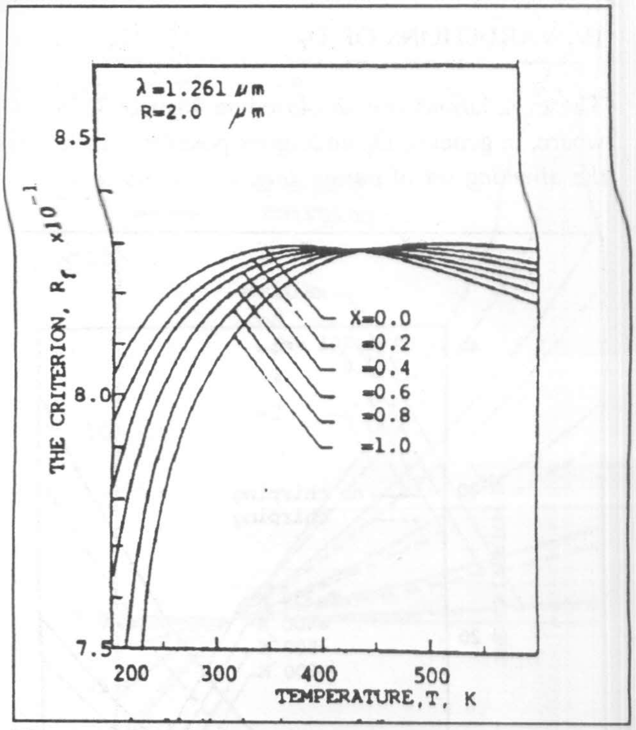


Figure 11. Variations of $R_r$ against variations of T.

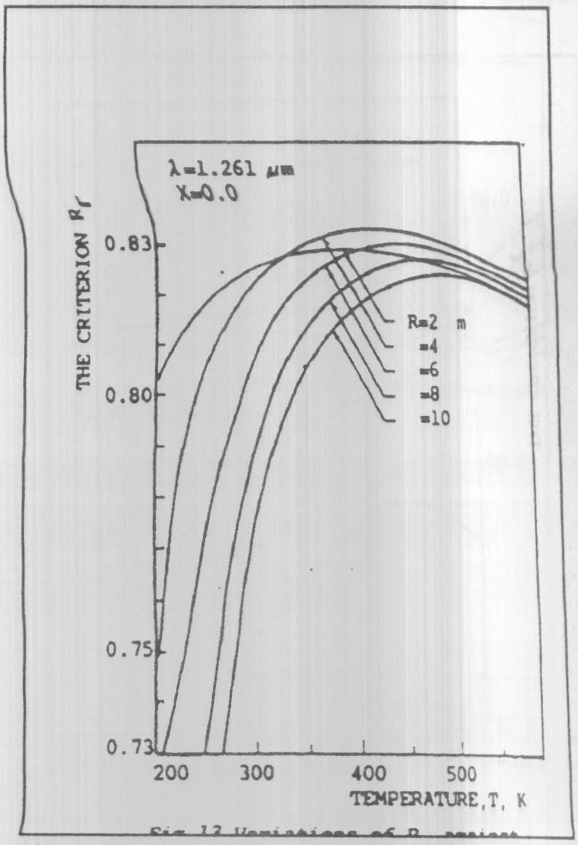


Figure 13. Variations of $R_r$ against variations of T.

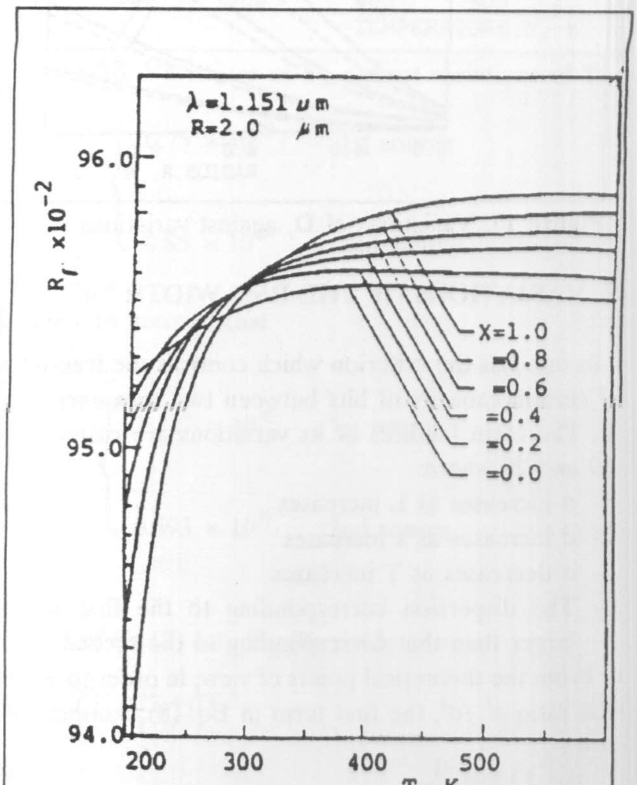


Figure 12. As in Figure 11, $\lambda$ =1.151 $\mu$m.

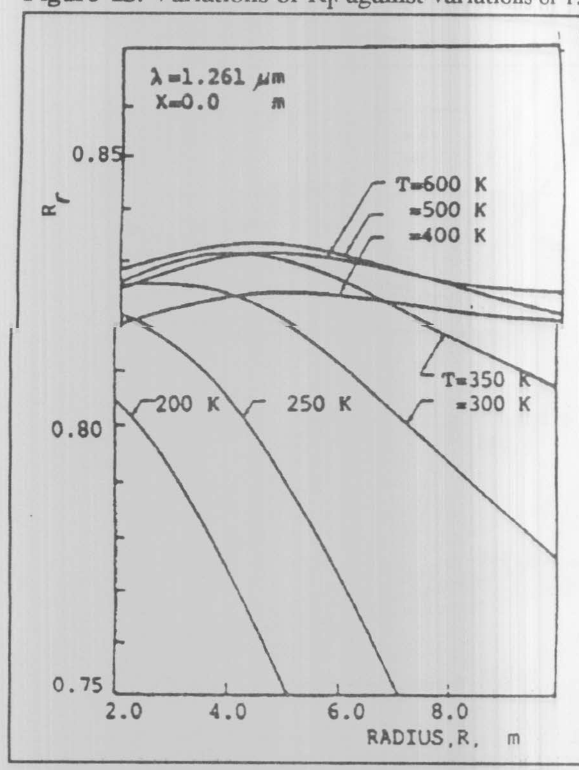


Figure 14. Variations of $R_r$ against variations of R.

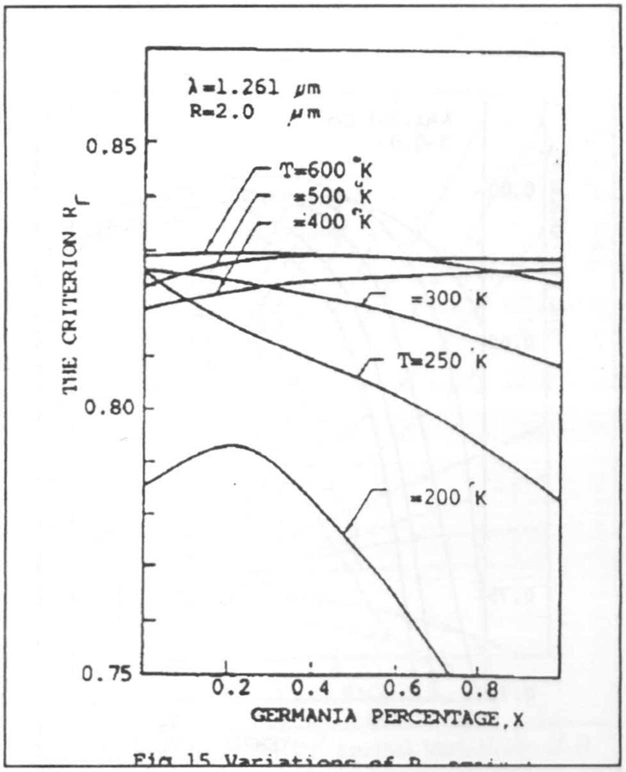


Figure 15. Variations of $R_r$ against variations of X.

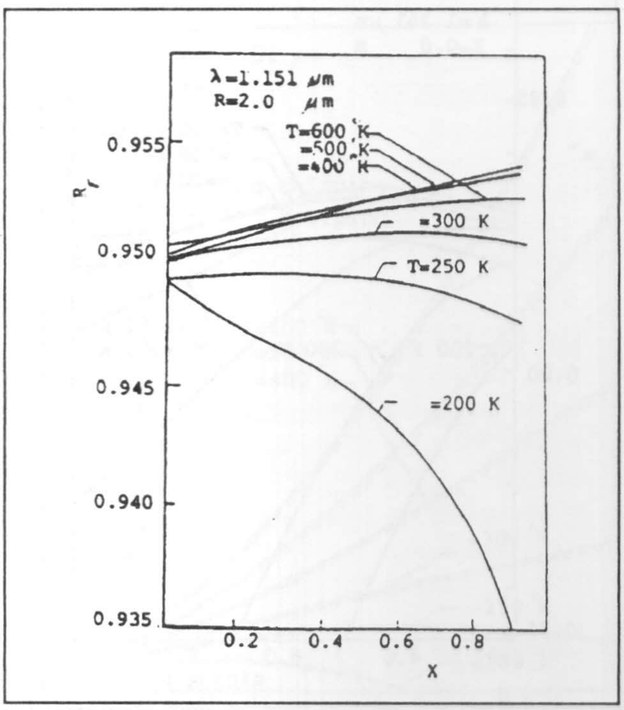


Figure 16. As in Figure 15, $\lambda = 1.151\ \mu$m.

## IV. VARIATIONS OF $D_r$

These variations are displayed in Figures (17) and (1 where, in general, $D_r$ undergoes positive correlations with the affecting set of parameters.

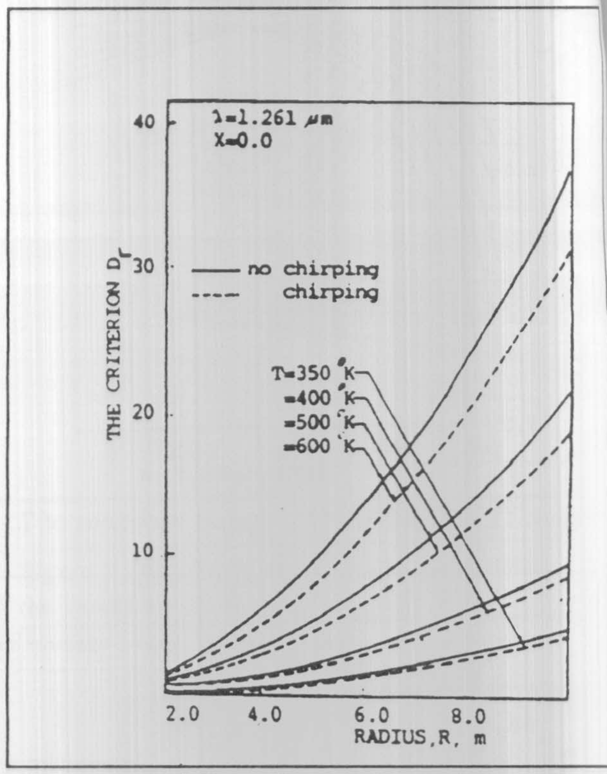


Figure 17. Variations of $D_r$ against variations of R.

## V. VARIATIONS OF THE RMS WIDTH " $\sigma$"

In fact $\sigma$ is the criterion which controls the transmission of certain capacity of bits between two repeaters of span L. The main features of its variations are casted in Figs. 19 and 20 where:

1- $\sigma$ increases as L increases
2- $\sigma$ increases as x increases
3- $\sigma$ decreases as T increases
4- The dispersion corresponding to the first source is larger than that corresponding to the second source.

From the theoretical points of view, in order to minimize the ratio $\sigma^2/\sigma^2$, the first term in Eq. (8) vanishes

$$\text{i.e. } Lk'' = \frac{-1}{\Delta\omega_1} \tag{29}$$

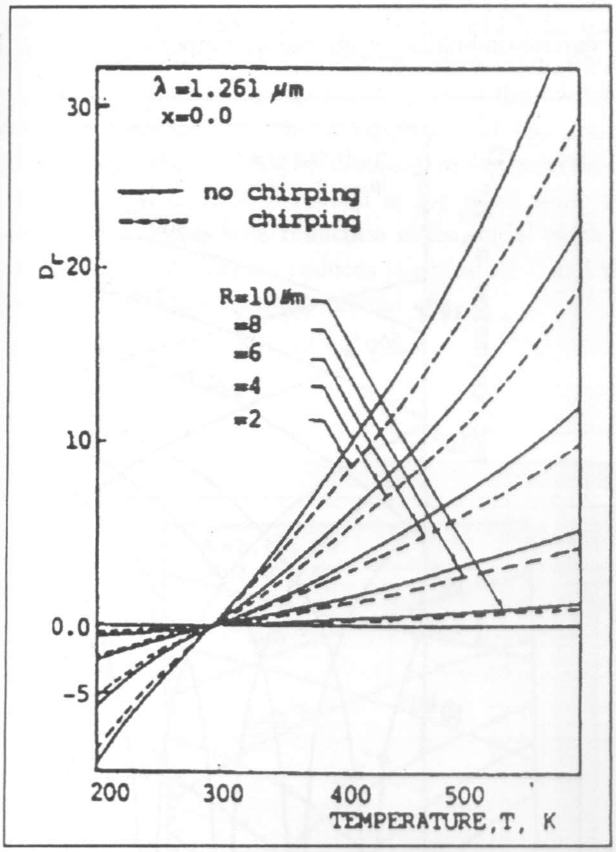


Figure 18. Variations of $D_\Gamma$ against variations of T.

$$= \begin{cases} 0.75 \times 10^{-24} & \text{1st source} \\ 0.85 \times 10^{-24} & \text{2nd source} \end{cases}$$

Taking into account that

$$\Delta\omega^2 = \begin{cases} 1.127 \times 10^{35} & \text{1st source} \\ 0.593 \times 10^{35} & \text{2nd source} \end{cases}$$

The use of Eq.(2) into Eq.(8) yields

$$R_\omega = \frac{\sigma^2}{\sigma_o^2} = \frac{\alpha}{T_e^2} + \beta T_e^2 \qquad (30)$$

with

$$\alpha = \left(\frac{4}{\Delta\omega_1}\right)^2$$

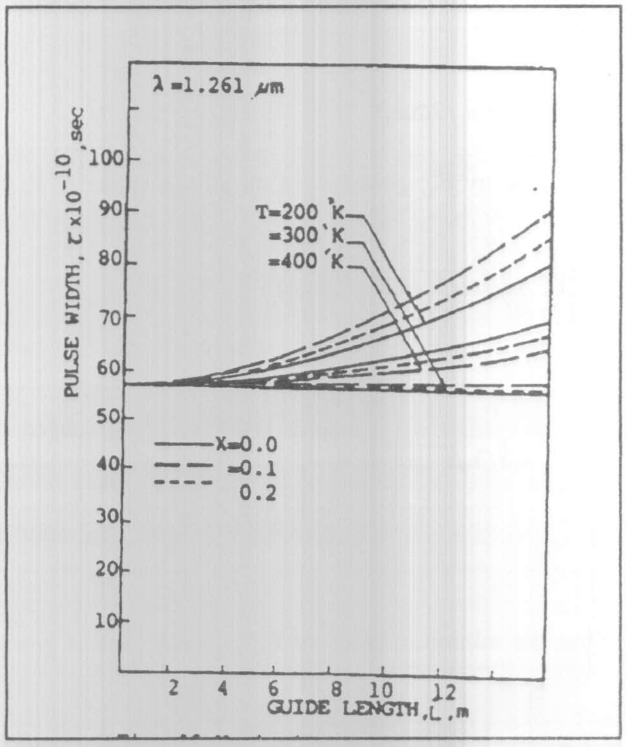


Figure 19. Variations of $\tau$ against variations of L.

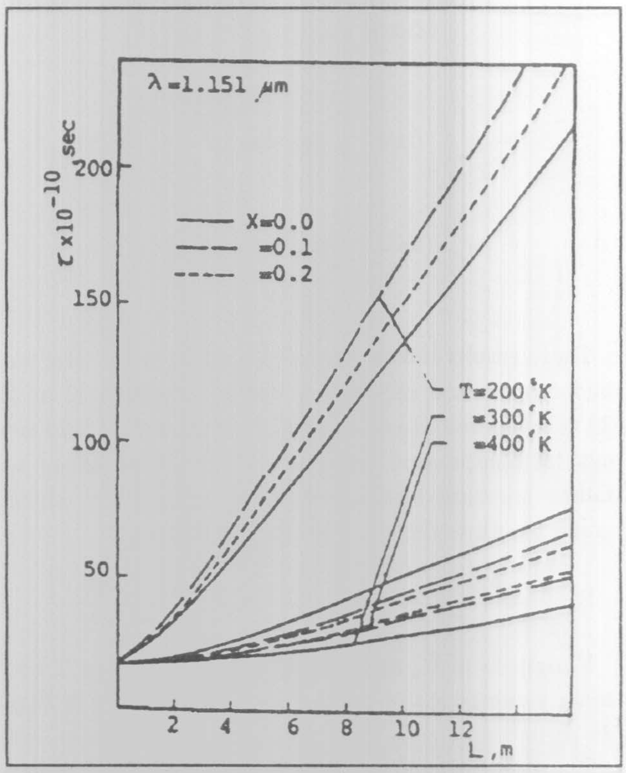


Figure 20. As in Figure 19, $\lambda = 1.151$ $\mu$m.

and

$$\beta = (\Delta\omega_2/2\Delta\omega_1)^2$$

The quantity $R_\omega$ possesses a minimum value at $T_e$ given by

$$T_e = \sqrt[6]{2\alpha/\beta} = [64/(\Delta\omega_2)^2]^{1/6}$$

$$= 2/3\sqrt{\Delta\omega_2} = \begin{cases} 4.141\times10^{-12}\text{sec.} & \text{1st source} \\ 2.564\times10^{-12}\text{sec.} & \text{2nd source} \end{cases}$$

Thus the minimum value of $R_\omega$ is $R\omega$min and is given as

$$R_{\omega.\ min} = 1.236\ (\alpha\ \beta^2)^{1/3}$$

$$= \frac{1.236}{(\Delta\omega_1)^2} \cdot (\Delta\omega_2)^{4/3}$$

$$= \begin{cases} 0.04 & \text{1st source} \\ 0.02 & \text{2nd source} \end{cases}$$

These results indicated that the pules of the first source, under optimized conditions, can be compressed to about 20 % of its initial width, while that of the second source, can be compressed to about 14 % of its initial width. Other criterion for measuring the dispersion along the guide length is the quantity $R_\sigma$ defined as:

$$R_\sigma = \frac{\sigma^2 - \sigma_o^2}{\sigma_o^2}$$

Variations of $R_\sigma$ against the variations of x or T and the other assumed set of parameters are displayed in Figs. 21-24.

$R_\sigma$ Possesses a minimum value which undergoes negative correlation with either x or T for the two sources under investigation. Thus the dispersion along the guide corresponding to an operating temperature is controll[ed]

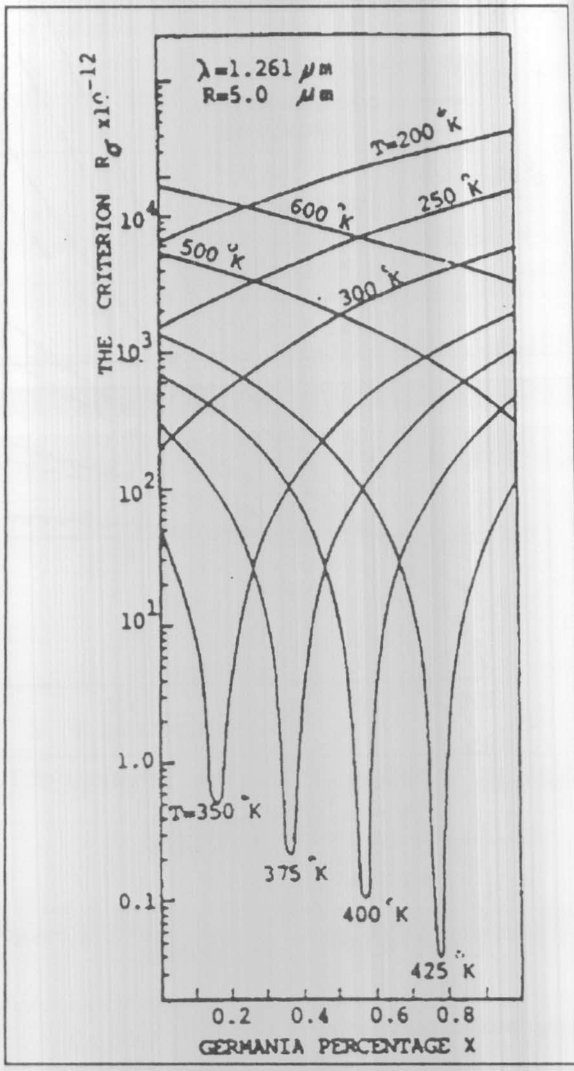


Figure 21. Variations of $R_\sigma$ against variations of X.

via the chemical structure of the guide (x % of Germinia).

## CONCLUSIONS

The product of the peak power and the pulse width square for a nonlinear optical waveguide made of germania-doped fiber (for the possibility of theoretical treatment) is modelled, casted, and analyzed under wide ranges of controlling parameters. In order to minimize that product Γ, the effect of the set of parameters { $R_e$, X, T} is accurately studied for two typical chirped sources. Such a minimization process indicates either the work at lower power or at smaller pulse width which in turns means higher values of the system bit-rate. A simple optimization process is made to obtain the suitable operating conditions for the compression of the optical pulse through the guide. The first source is theoretically optimized to yield 80% reduction in the pulse while the second source yields 86% reduction in the initial width. It is found also that chirping reduces the product Γ and the first source is better than the second one.

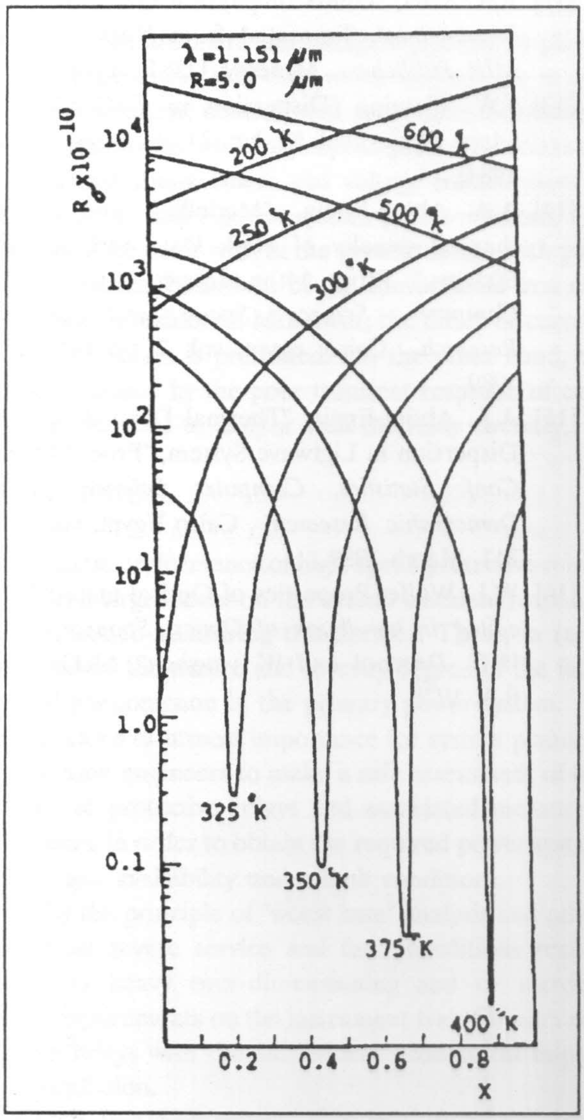


Figure 22. As in Figure 21, $\lambda = 1.151\ \mu m$.

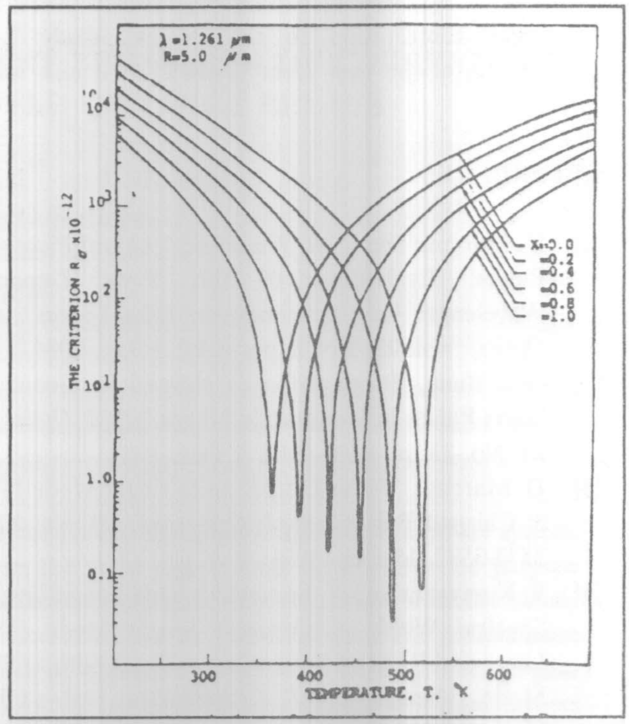


Figure 23. Variations of $R_\sigma$ agianst variations of T.

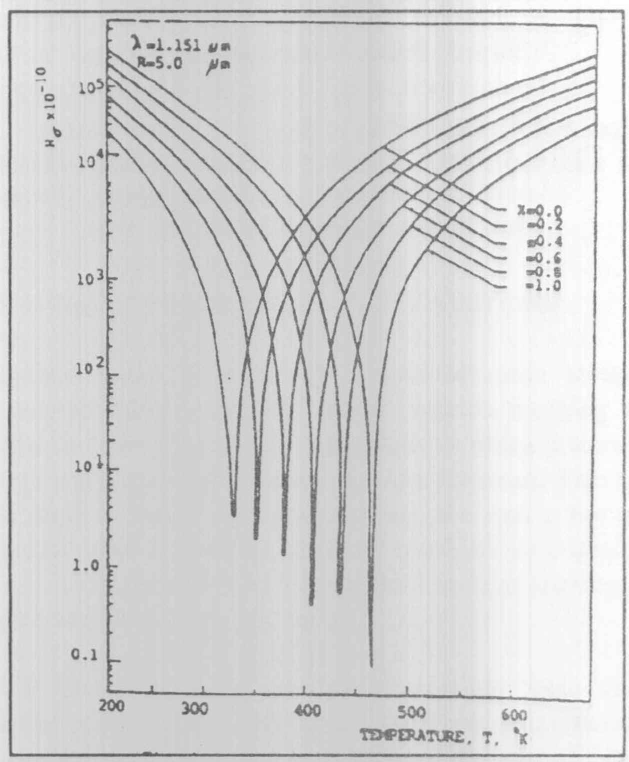


Figure 24. As in Figure 23, $\lambda = 1.151\ \mu m$.

A clear contribution in the present study is the introduction of the new term "Planck" as a unit for the product.

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