THE APPLICATION OF TIME SERIES ANALYSIS IN THE EVALUATION OF RAIN ATTENUATION

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ABSTRACT

A new approach to the problem of estimating the rain attenuation on microwave links is introduced in this work. The application of the time series analysis to the rain rate data made it possible to benefit from the advantages of this powerful mathematical technique. The data employed is a 28-year collection of rain accumulation in Alexandria. Secular trends, seasonal and random movements of the data have been determined. Exploration of existing periodicities using power spectral density and periodogram analyses have been carried out. The correlation between the different observations in time was investigated by performing a correlogram analysis. Finally, several methods for forecasting future values have been demonstrated.

INTRODUCTION

Rain attenuation plays a dominant role in degrading terrestrial and earth-space radio communication links in the GHz frequency band. In order to meet a certain availability objective on such links, the estimation of expected values of rain attenuation is essential. According to this estimation, the transmitted power, the spacing between repeaters and diversity routes can be determined. The prediction of rain attenuation can be carried out either from prior knowledge of attenuation measured on similar links [1-3] or from available point rainfall rate data [4-6]. In either approach, prediction methods are based on the compilation of cumulative distributions of the expected attenuation on the link under study. However, such statistical analysis conceals important information like the presence of general trends, periodicities, correlation between observations, etc. This information, if available, can help to design techniques to alleviate a-priori the consequences of rain attenuation.

The approach followed in this work is to consider the rainfall data and the attenuation calculated from it as a collection of observations in time, and then to apply the time series analysis to it. The results obtained from this approach complement those of the statistical analysis to give a better picture of the behaviour of the expected rain attenuation on any link.

RAIN ATTENUATION

The estimation of rain attenuation from the knowledge of point rainfall rate measurements is a widely used technique because of the availability of rainfall data in vast areas of the world. Despite the complicated nature of this problem and its dependence on many parameters, a semi-empirical relation between the point rainfall rate R (mm/hr) and the specific attenuation (dB/km) has been obtained in the simple form [7]

$$\alpha = a R^b \quad \text{dB/km} \tag{1}$$

where a and b are coefficients that depend on the frequency, drop size distribution, drop temperature and the type of the rain.

Due to the fact that rain displays a significant spatial and temporal variations, it is not a straight forward approach to translate the calculated specific attenuation at a point into attenuation along the path. In order to solve this nonlinearity problem, two approaches are available. In the first aproach, R in equation (1) is replaced by a path average rainfall rate and the exact path length of the link is utilized. Alternatively, the measured point rainfall rate can be employed directly in equation (1) along with an effective path length for the link. The available prediction techniques differ in the manner by which they estimate either the path average rainfall rate or the effective path length used in the calculations. Some techniques even employ a combination of both.

In this work we are more interested in introducing the time series analysis as a powerful means of analyzing the attenuation data than in comparing between the different prediction techniques. Therefore, the method suggested by Lin [6] was employed in our calculations for its simplicity. In this technique the effective path length is given by

$$L_e = L/B \qquad (2)$$

$$B = 1 + L/\overline{L(R)} \qquad (3)$$

$$\overline{L(R)} = 2636 / (R - 6.2) \qquad (4)$$

where L is the actual path length.

If in equations (1) and (4) average values of rainfall rates collected over a given period of time are employed, another factor is introduced to account for the nonlinearity of the problem.

TIME SERIES ANALYSIS

The rainfall data employed in this analysis is a 28-year rainfall accumulation in Alexandria [8]. It was essential to transform this data into a suitable form for the application of the time series techniques [9]. For each month two quantities were calculated: the monthly average rain rate and the peak rain rate. The former is obtained by dividing the total rain accumulation in a given month by the total accumulation periods within that month. The latter is obtained by dividing each rain accumulation by its accumulation period and choosing the maximum value in each month. In each of these two quantities a time averaging takes place. Due to the fact that the averaging process is not uniform, the attenuation calculated from these two quantities should not be interpreted separately.

The year 1949 was taken as a base year and Lin's prediction technique was applied for a hypothetical terrestrial path of 20 km at 10 GHz. It should be pointed out that Lin's method is based on 5 min. average rain rate. Since a majority of the accumulation periods in the data is larger than this, the calculated attenuation will tend to underestimate the correct values. Furthermore, although this method was based on measurements conducted in the USA, it is a common practice to adopt such methods worldwide.

The least square method was used to estimate the attenuation trend line for each month of the rainy season that starts in October and ends in March. The results are given in the form

$$A(t) = c + m.t \quad dB \qquad (5)$$

where t is the number of years starting from 1949 (at which $t = 0$). The coefficients c and m for both the monthly average and peak values are given in Table (1) and samples of the data are presented in Figures (1) and (2). The results indicate that the highest monthly average attenuation occurs in November while the highest peak value occurs in January. The average monthly attenuation has a positive rate of increase for most of the months while the peak attenuations behave oppositely.

Table 1. The coefficient of trend lines of the monthly average and the peak rain attenuations.

	Monthly average		Monthly peak	
	c	m	c	m
October	2.542	0.018	4.025	0.024
November	4.938	-0.093	6.959	-0.069
December	2.514	-0.029	6.777	-0.014
January	1.912	0.003	7.455	-0.103
February	2.192	0.005	4.778	-0.022
March	1.524	0.038	2.914	0.015

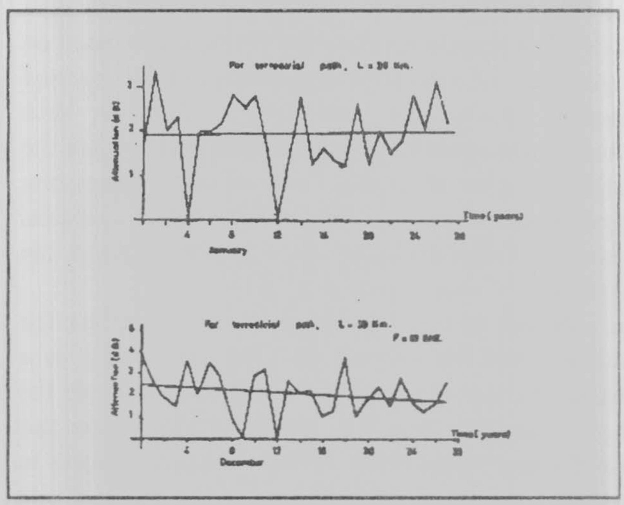

Figure 1. Rain attenuation due to average rain rate. Original data and trend lines.

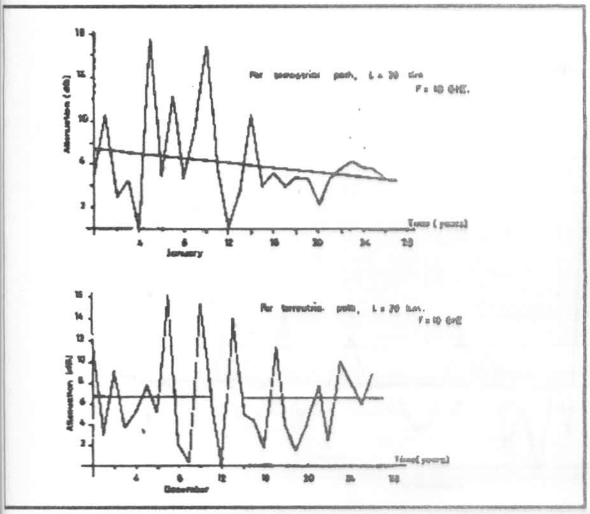

Figure 2. Rain attenuation due to peak rain rate. Original data and trend lines.

In order to estimate the seasonal fluctuations in the data, the average percentage method is applied to calculate the seasonal index. In this method, the attenuation value for each month is expressed as a percentage of the yearly average, and the percentages for the corresponding month of different years are averaged. The results are indicated in Table (2). They show that the attenuation due to average rain rate has its highest value during November while that due to peak rain rate occurs in December. These results are almost similar to those obtained from the trend analysis.

Table 2. Rain attenuation seasonal index.

	Monthly average	Monthly peak
October	105.401	88.122
November	126.143	105.152
December	95.974	130.930
January	94.945	119.495
February	95.139	93.380
March	82.392	62.918

In order to get a better insight to the probability model that generated the data point x_t, it is useful to compute the set of autocorrelation coefficients r_k. These give a measure of the correlation between observations at different time lags k, and can be obtained from

$$r_k = c_k / c_o \qquad k = 1, 2, ..., m \tag{6}$$

where m < N (the number of observation points) and c_k is defined as

$$c_k = 1/N \sum_{t=1}^{N-k} (x_t - \bar{x})(x_{t+k} - \bar{x}) \tag{7}$$

$$\bar{x} = \sum_{t=1}^{N} x_t / N = \text{overall mean} \tag{8}$$

The correlograms obtained for the monthly attenuation data are shown in Figures (3) and (4). From Figure (3), we can consider that the time series for all the six months is almost random since most of r_k are within $\pm 2/\sqrt{N}$ (where N = 28 in our case). The time series is alternating for February, March and November. It shows a hidden periodicity of 4 years in October. The results for the attenuation peaks, shown in Figure (4), also indicate that they represent an almost random time series. It is alternating for February, March, October and November. However, more hidden periodicities can be detected compared to the results in Figure (3). It ranges from 3 to 5 years in January, 7 for February, 4 for March, 2 to 4 for October, 5 for November and 3 for December. Similar results were obtained when a periodogram and a power spectral analyses were performed on the data.

May be the most important feature of the time series analysis is its ability to forecast future values of the observed variable. This forecasting can be carried out by either obtaining stochastic models that fit the observed time series or by applying proper forecasting procedures.

Three types of stochastic models were attempted:

a- The autoregressive process: where the present value can be expressed as a function of the past p values and a purely random process z_t with zero mean, in the form

$$x_t - \bar{x} = \alpha_1 (x_{t-1} - \bar{x}) + ... + \alpha_p (x_{t-p} - \bar{x}) + z_t \tag{9}$$

where the coefficients α are functions of the autocorrelation coefficients r_k. As examples, the results for the rain rate peaks in January for a first and second order models are

$$x_t = 15.9 - 0.143 (x_{t-1} - 15.9) + z_t \tag{10}$$

$$x_t = 15.9 - 0.144 (x_{t-1} - 15.9) - 0.012(x_{t-2} - 15.9) + z_t \tag{11}$$

b- The moving average process: where x_t can be expressed as a function of a purely random process z_t of zero

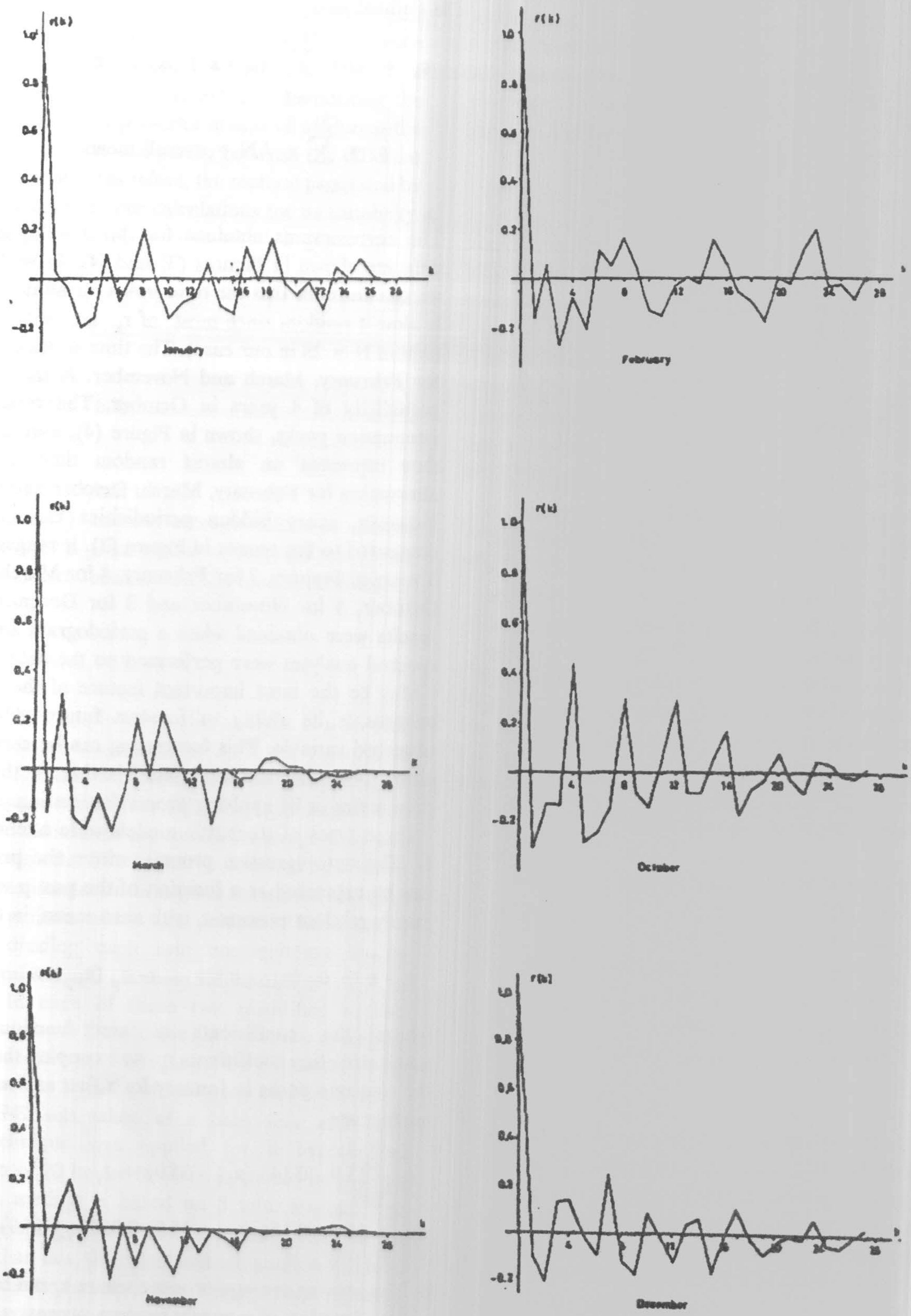

Figure 3. Correlogram of the monthly averaged rain rate and attenuation.

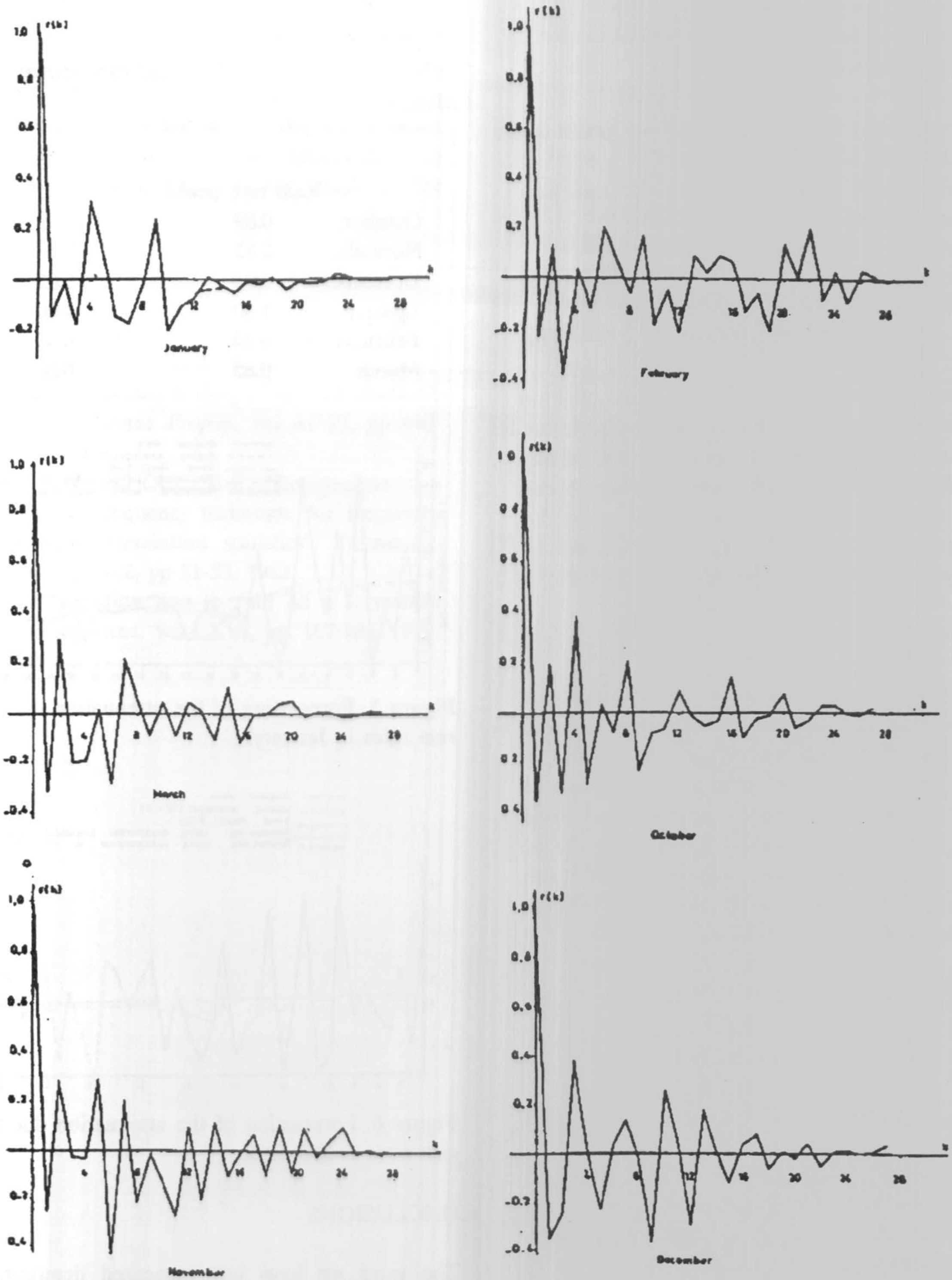

Figure 4. Correlogram of the monthly peak rain rate and attenuation.

mean, for an order q this is given by

$$x_t = \bar{x} + z_t + \beta_1 z_{t-1} + \ldots + \beta_q z_{t-q} \qquad (12)$$

where the coefficients β are chosen to give a minimum value for Σz_t^2.

The first and second order models for the rain rate peaks in January are given by

$$x_t = 15.9 + z_t - 0.55\, z_{t-1} \qquad (13)$$

$$x_t = 15.9 + z_t - 0.1\, z_{t-1} - 0.1\, z_{t-2} \qquad (14)$$

c - The (p,q) mixed models is formed from a combination of p autoregressive terms and q moving average terms. The importance of these types of models is that oftenly stationary time series can be described by fewer terms than in the case of either moving average or autoregressive models alone. The January data can be represented in the form of a (1,1) process by

$$x_t = 15.9 - 0.7\,(x_{t-1} - 15.9) + z_t + 0.6\, z_{t-1} \qquad (15)$$

As for the forecasting procedures, the simplest and least expensive is the one that depends on the extrapolation of the trends lines as those given in equation (5). Alternatively, the exponential smoothing procedure can be applied to any time series after removing from it any trend or seasonal patterns. This smoothing of the data can be accomplished by using the moving averages method with a suitable order. For N data points and order M, the smoothed data can be calculated from.

$$R_k = 1/M \sum_{i=k}^{M+k-1} x_i \qquad 1 < k < (N - M + 1) \qquad (16)$$

and then the future value can be obtained from

$$\tilde{R}(k,1) = \alpha R_k + \alpha(1-\alpha)R_{k-1} + \ldots + \alpha (1-\alpha)^{k-1} R_1 \qquad (17)$$

where α is a constant, such that $0 < \alpha < 1$, which is chosen to minimize the sum of the squared prediction errors. Finally the future value of the original data can be obtained by substituting back in equation (16).

The values of α for the average rain rate and the rain rate peaks and their corresponding attenuations are given in Table (3). Figures (5) and (6) show the future values of the attenuation due to peak rain rates in January and December respectively for a period of 10 years.

Table 3. The constant α of the exponential smoothing technique.

	Rain rate peaks	Average rain rates
October	0.89	0.69
November	0.81	0.97
December	0.91	0.65
January	1.00	1.00
February	0.84	0.90
March	0.85	0.68

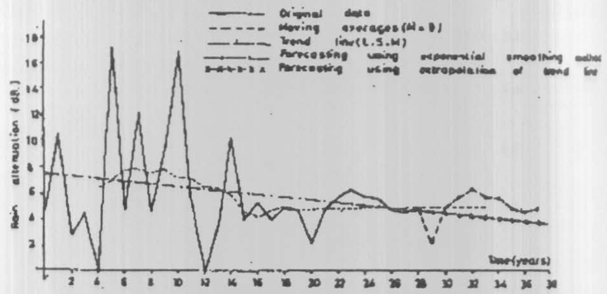

Figure 5. Forecasting of the attenuation due to peak rain rates in January.

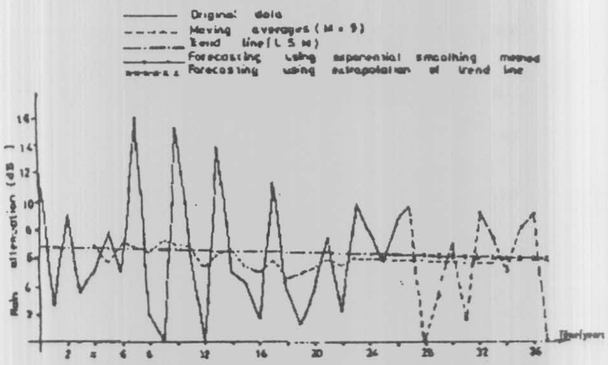

Figure 6. Forecasting of the attenuation due to peak rain rates in December.

CONCLUSIONS

The work we have just presented demonstrates the several advantages the time series analysis can provide to microwave link designers in estimating rain attenuation. It can help to improve the performance of existing links by forecasting future behaviour from the knowledge of past history. It cannot be considered as a substitute to the statistical prediction techniques based on cumulative distributions. While the former can model the available data, forecast its future values, compute the trends and the different variations in it, discover any hidden periodicities, the latter can estimate the availability of the system under the effect of rain. Therefore the two approaches can complement each other to give a better insight to the problem.

REFERENCES

[1] Hodge D.B., "Frequency scaling of rain attenuation", *IEEE Trans. Antennas Propag*, vol AP-25, pp 446-447, 1977.

[2] Kheirallah H.N. and Olsen R.L., "Comparison of a one- and a two-frequency technique for frequency scaling of rain attenuation statistics", *Electronics Letters*, vol. 18, No.2, pp 51-53, 1982.

[3] Fedi F., "Attenuation due to rain on a terrestrial path", *Alta Frequenza*, vol LXVI, pp. 167-184, 1979.

[4] Kheirallah H.N., Segal B. and Olsen R.L., "Application of synthetic storm data to evaluate simple techniques for predicting rain attenuation statistics", *Annales des Telecommunications*, vol. 35, No. 11-12, pp. 456-462, 1980.

[5] Crane R.K., "Prediction of attenuation by rain", *IEEE Trans. Commun.*, vol. COM-28, No.9, pp 1616-1733, 1980.

[6] Lin S.H., "Empirical rain attenuation model for earth satellite paths", *IEEE Trans. Com.*, vol. COM-27, No. 5, pp 812-817, 1979.

[7] Olsen R.L., Rogers D.B. and Hodge D.B.,"The aR^b relation in the calculation of rain attenuation", *IEEE Trans. Antennas Propagat.*, vol. AP-26, pp 318-329, 1978.

[8] El-Adawi M.S., "The relation between rain rates and their repetition periods", *Bulletin of the Faculty of Engineering, Alexandria University*, vol. 18, pp. 27-36, 1979.

[9] Chatfield C., "*The analysis of time series: An introduction*", Chapman and Hall, 1980.