STRENGTH AND BEHAVIOUR OF SIMPLY SUPPORTED SKEW SLAB DECK BRIDGES

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ABSTRACT

Experimental and theoretical investigation was carried out to study the behaviour and strength of the simply supported skew slab deck bridges. The major parameters practically affecting the studied structure were incorporated in the theoretical and the analytical study, namely: the skew angle of the slab, the aspect ratio of the slab (B/L), the slab deck thickness along with the concrete strength, while reinforrcement ratio and arrangement were kept constant. The test programme included four test series, including eleven reinforced concrete slab decks so designed, to cater for the major studied parameters at different stages of loading. The theoretical study involved an elastic analysis using the grillage method and an ultimate strength analysis using the well known yield-line theory. The comparison between the experimental and the theoretical studies were made and practical recommendations were therefore presented.

INTRODUCTION

After the 2nd worldwar, the great demand for modern traffic routing skew bridges were increasingly specified in the course of reconstruction of new bridges.

There have been considerable amount of research carried out in the field of bridge structures subjected to concentrated loads. Load distribution analysis, and design methods based upon these analysis, have been developed and confirmed by experimental work for right bridges of various types. Further simplified design procedures have been put forward for right slab bridges to abnormal loading [1].

It was untill recently, if, in exceptional case, a skew bridge was specified, it would be analysed by methods for right bridges. However, such methods of analysis are not applicable to bridges with skews of more than 20°.

In 1958 Homberg. H and Max. R [2], Published the first book on skew slabs. This was based mainly on experimental measurements from model investigation, for single span skew slab decks with two free and two simply supported edges.

Neilsen [3], was the first to investigate the bending moment in skew slab decks on large scale using the finite difference method with a parallowgram mesh. Serious inaccuracies in his results, were obtained, becuase he did not take proper account of the boundary conditions and the assumed mesh was too course. Satisfactory results were later obtained by WÜnch [4]. Reliable, specific finite difference solution to the governing differential equation has been obtained also for orthotropic skew bridges.

Ultimate load analysis based upon yield-line theory which extensively developed by Johansen [5,6] was successively employed by Granholm and Rowe [7] for the study of the ultimate load of simply supported skew slab bridges subjected to abnormal loading. It would be obvious from literature study that, in the case of skew bridges, a method of analysis based upon ultimate load would offer an easily understood and quickly applied tool that, would be of considerable use in design especially if coupled with a check on deflections under working load conditions.

The present investigation gives a thourough investigation for the study of the behaviour and the strength of simply supported skew deck bridges. The analysis included elastic and ultimate load approaches suitably covers all the stages of loading up to failure. The new code of Practice [9] was applied in this context. Based on the study and the comparison between the analytical and the experimental findings, practical design recommendations were suggested.

TEST PROGRAMME

This investigation included testing of eleven one-span simply supported skew slab decks. One side of the test slabs was simply supported on fixed rollers and the other side was simply supported on a movable rollers. The

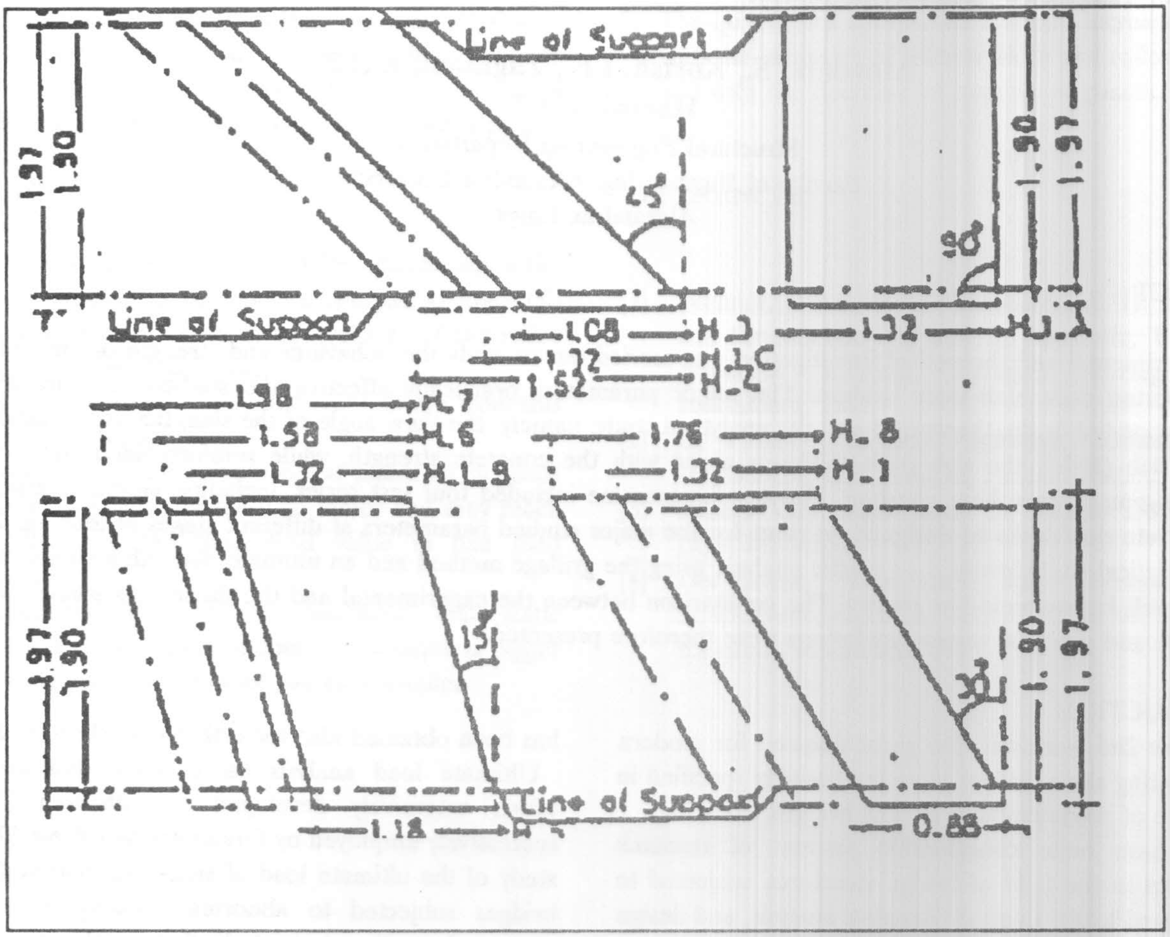

Figure 1. Dimensions of test slab decks and the skew angles.

supported edges were prevented from lifting up, as provided by a hard rubber strip between the top surfaces of the supporting edges and a fixed supporting element. The tests were performed at the Reinforced Concrete Laboratory of the Faculty of Engineering, Alexandria University.

The major parameters included in this investigation were, the skew angle of the slab deck, the aspect ratio of the slab deck B/L and the aspect ratio, B/L, against the skew angle of the slab deck, ϕ. Other minor variables considered, in the test programme, were slab deck thickness and concrete strength. The parameters included were those which were thought to have the major effect on the strength and behaviour of the skew slab deck bridges.

Detailed dimensions and skew angles of the test slabs are given in Figure (1). They were grouped into four groups to serve the purpose of the study as demonstrated in Table (1).

The reinforcement arrangement of the test slab decks shown in Figure (2) was constant throughout the tests. The steel bars were 13 mm diameter of ordinary mild steel with average yield stress of 2400 kg/cm^2.

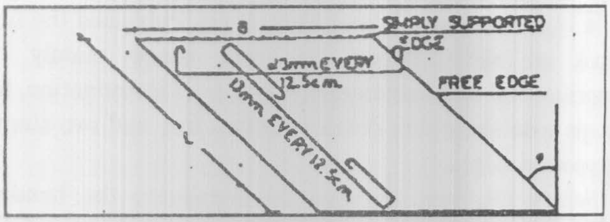

Figure 2. Detail Reinforcement of test slab decks.

LOADING AND TEST PROCEDURE

All the test slab decks were tested under the effect of

Table 1. Test Results.

Test Series	Skew angle	Slab Nº	Aspect ratio (B/L)	Age at test (days)	Cube strength (Kg/cm²)	Slab thicknes (cm)	Effective depth (cm)	Design Ultimate load (tons)	Ultimate Load (tons)	Ult. Load / Design load
I	45°	H-3	0.40	42	278	12	10.5	8.01	8.5	1.06
		H-1-C	0.49	14	220	12	10.5	8.45	9.0	1.07
		H-4	0.60	28	250	12	10.5	11.91	11.0	0.92
II	30°	H-2	0.40	28	270	12	10.5	7.96	6.25	0.78
		H-1	0.60	42	300	12	10.5	12.03	10.25	0.87
		H-8	0.80	18	205	10	8.5	10.46	11.50	1.10
III	15°	H-5	0.60	35	300	12	10.5	12.00	13.00	1.08
		H-1-B	0.67	25	260	12	10.5	13.207	14.25	1.08
		H-6	0.80	28	260	12	10.5	15.94•	14.00	——
		H-7	1.00	21	224	10	8.5	13.29	15.50	1.17
IV	0	H-1-A	0.695	17	290	12	10.5	13.78	15.75	1.14
	15°	H-1-B	0.67	25	260	12	10.5	13.21	14.25	1.08
	30°	H-1	0.60	42	300	12	10.5	12.03	10.25	0.87
	45°	H-1-C	0.49	14	220	12	10.5	8.45	9.00	1.07
	45°	H-4	0.60	28	250	12	10.5	11.91	11.00	0.92

• Slab (H-6) did not loaded to failure
Right span for all the test specimens l=1.90m and the right length = 1.97m

uniformly distributed load. The load was applied by means of 16 tons capacity hydraulic jack. The jack load was distributed equally through distributing beam system, capable of rotating at their ends through ball bearings to prevent arching effects of any kind through 32 square plates 10x10 cm each, see Figure (3).

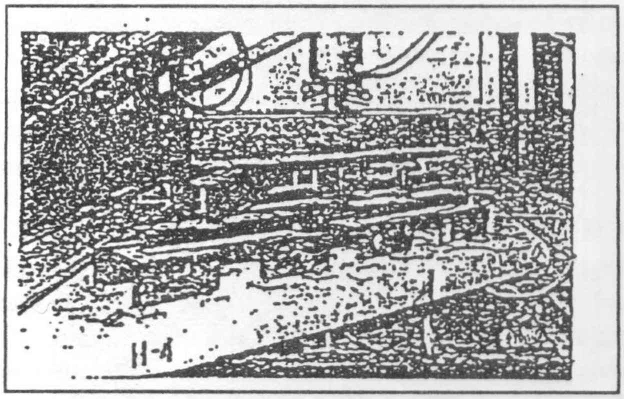

Figure 3. Loading set.

The load was applied in increments up to failure of the test slab decks. Before and after each loading increment the recording of the applied load, strain in the reinforcing bars at the selected points, the demec strain gauges fixed at the top surface of the test slab decks and the deflectometers readings mounted at the bottom surface of the test slabs were recorded. The measuring of the crack width and marking of the progress of the crack pattern were also made after each loading increment.

TEST RESULTS

a. General behaviour and mode of failure.

In most of the test slabs, the cracks appeared first at the tension side in the middle region of the slab in a direction inclined to the mid centre-line of the slabs, parallel to the support edges with an angle approximately equal to half the skew angle of the test slab deck, for slab decks of high aspect ratio B/L equal or greater than 0.80. For smaller slab decks aspect ratio the angle of inclination of the cracks from the mid centre-line of the slab decks was equal to the skew angle. When the load increased the cracks appeared at the two free edges at a certain position between the mid-point of the free edges and the obtuse corrner of the slab decks. With additional increments of the load, cracks at the middle region progressed towards the cracks at the free edges until they joined together forming, at higher loads, one major yield-line extended from

Table 2.

Test Series	Skew angle ϕ	Slab No.	Aspect ratio (B/L)	% of load causing middle cracks / The ultimate load	% of load causing edge cracks / The ultimate load	% of load causing vertical cracks / The ultimate load
I	45°	H-3	0.40	30	41	47
		H-1-C	0.49	30	30	40
		H-4	0.60	36	45	50
II	30°	H-2	0.40	25	40	48
		H-1	0.60	30	38	52
		H-8	0.80	35	43	45
III	15°	H-5	0.60	30	30	40
		H-1-B	0.67	32	32	46
		H-6*	0.80	28	50	70
		H-7	1.00	32	45	52
IV	0	H-1-A	0.695	25	40	47
	15°	H-1-B	0.670	32	23	48
	30°	H-1	0.60	30	38	52
	45°	H-1-C	0.49	30	30	40
	45°	H-4	0.60	36	45	50

* Slab (H-6) did not reach to the failure load

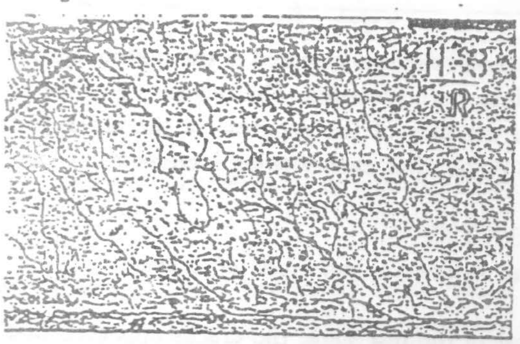

Figure 4. Yield-liner and crack pattern at failure, (H-3), B/L=0.5, $\phi=45^{\circ}$.

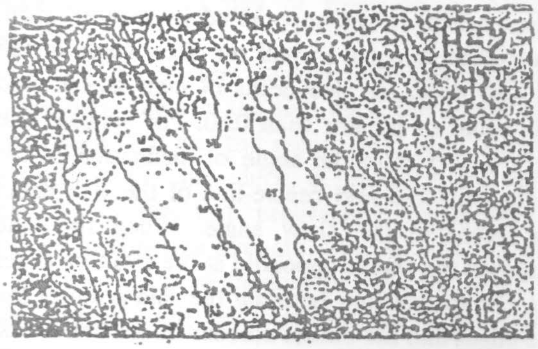

Figure 5. Yield-line and crack pattern at failure (H-3)B/L = 4.0, ϕ = 30°.

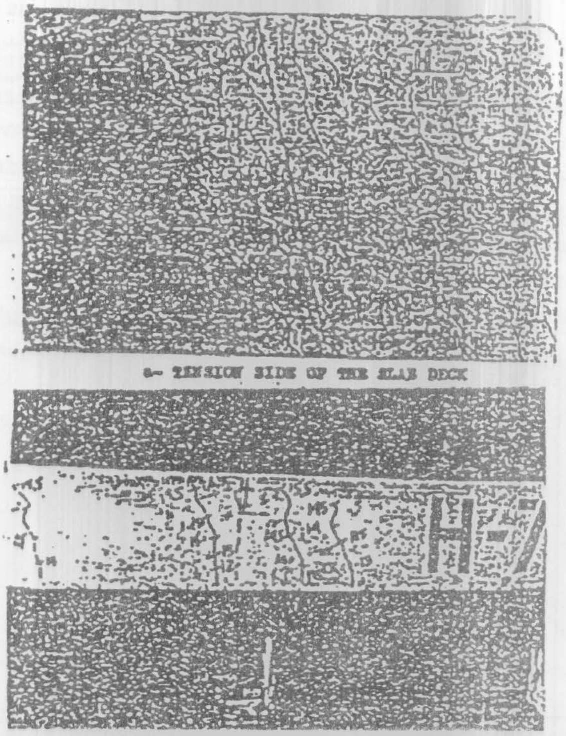

Figure 6. Yield-line and crack pattern at failure (H-7) B/L = 1.0, ϕ = 15°.

one edge to the other, see Table (2). At the same time many secondary families of cracks appeared at various stages of loading parallel to that of the major cracks.

At higher stages of loading the cracks that started at the edges of the slabs progressed deeply within the slab thickness towards the loaded surafce; see Table (2). With the increase of loading the crack width at the tension side increased and reached to about 0.3 mm width at a measured load very close to the failure load.

Figure 4,5, 6 and 7 demonstrate the typical crack patterns that occured during the tests and the major yield-line region and its position and direction with respect to the centre-line of the slab parallel to the line of supports for each of the first three main groups. All the other slab decks had similar crack patterns and mode of failure.

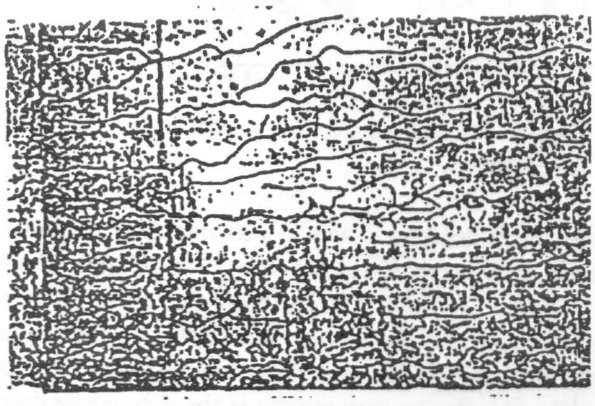

Figure 7. Yield-line and crack pattern at failure, (H-1-A) B/L = 0.695, =0.

Table (2) summarizes the stages of loadings at which each of the above mentioned crack stages were developed in the test slab decks, mainly the load at which the cracks appeared across the so called major yield-line region as percentage of the ultimate failure load, that for the load causing the edge cracks and that for the load causing the vertical cracks.

The failure of all the test specimens was due to the formation of yield-lines, along which the steel crossed these lines yielded causing final failure due to crushing of the concrete at the compressed surface. Figure (8), gives an example for one slab deck from each of the test groups present in this investigation. The figure proved the formation of the yield-line by the indication of the occurence of the yielding of the steel across this line. From this figure one may notice that, in most cases, the steel bars nearer to the free edges of the slab deeck began to yield before similar bars located closer to the centre point of the slab deeck. This observation was confirmed throughout the tests from the concrete strain measurements at the compressed surface of the slabs at different stages of loading across centre-line of the test slab decks. Also from the measured concrete strains at the middle point of the free edges at different stages of loading, the strains were larger than concrete strains measured at the slab deck's middle points see Figs. 10 and 12.

The concrete strain measurements across any other line apart from the centre-line and located towards the supporting lines, were smaller than the concrete strains measured across that centre-line, see Figures 9 and 10.

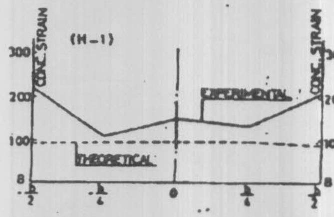

Figure 10. Distribution of concrete strains across line (B-B).

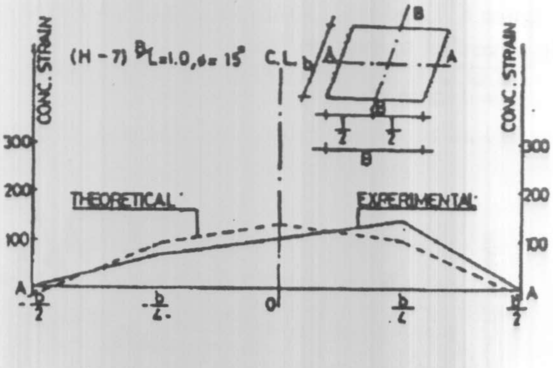

Figure 11. Distribution of concrete strains across line (A-A).

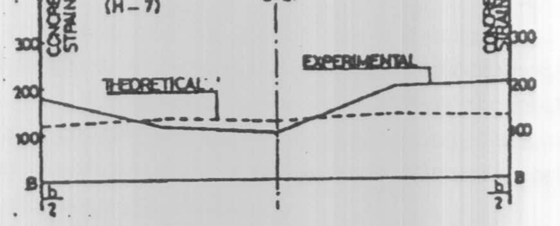

Figure 12. Distribution of concrete strains across line (B-B).

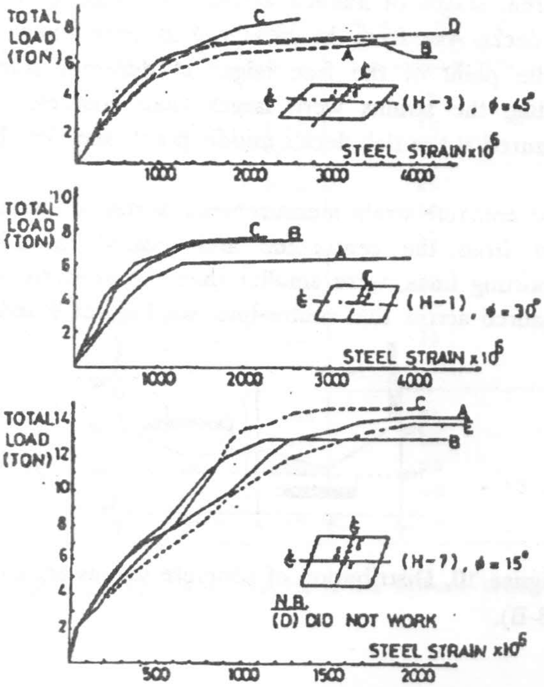

Figure 8. Load-steel strain relationships across centre line parallel to support lines.

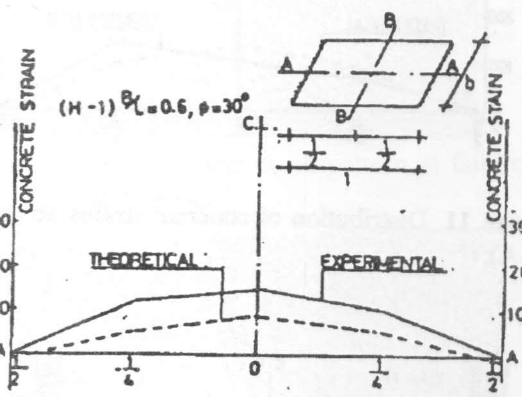

Figure 9. Distribution of concrete strains across line (A-A).

b. Strength and stiffness of the test slab deck.

The study of the effect of the major parameters included in this investigation on the ultimate strength and the stiffness was greatly sound.

Table (1) and Figures 13 and 14 demonstrate in a self explained way the effect of the skew angle ϕ, the aspect ratio (B/L), and the slab thickness of the slab deck on the ultimate carrying capacity of the test specimen. The effect of the concrete strength at the age of loading was also obvious.

The ultimate failure load increased as the skew angle of

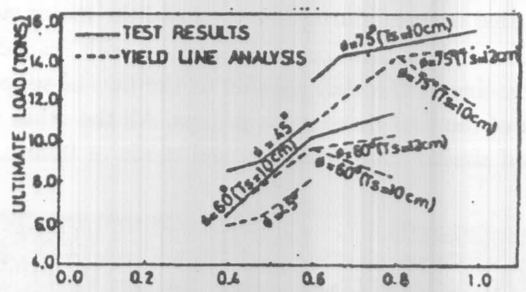

Figure 13. Effect of the aspect ratio on ultimate strength.

slab deck decreased and it also increased as the aspect ratio, B/L, increased, see Table (1) and Figure 12 and 13.

Figures (14) and (15) clearly indicate the effect of the skew anglė and the aspect ratio (B/L), on the stiffness of the test slab decks. In general, the test slab deck overall stiffness increases very rapidly as the skew angle decreases and the skew angle effect on the stiffness may be neglected for slab deck of an skew angle of 15° or less.

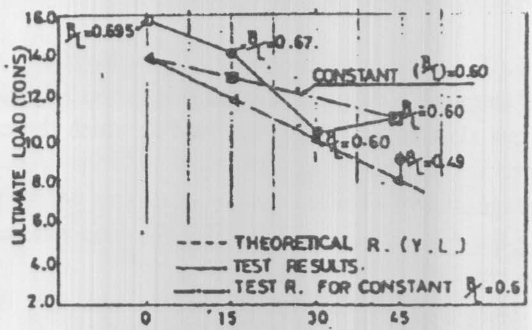

Figure 14. Effect of skew angle on the ultimate strength.

ANALYTICAL STUDIES

Two methods of analysis were applied in studying the effect of the major parameters affecting the behaviour and the strength of the simply supported skew slab bridge decks. The first method is the "Grillage method" which is based on an elastic approach. The second method is an ultimate approach using the yield-line theory, which was first established by Johansen [5,6] and has been developed in the last 45 years. It is a well recommended powerful method for estimating the ultimate load of the under reinforced slabs. This method was adopted for the analysis of skew slab bridges under the effect of concentrated loads to cater for the British Ministry of Transport Abnormal Loading [7].

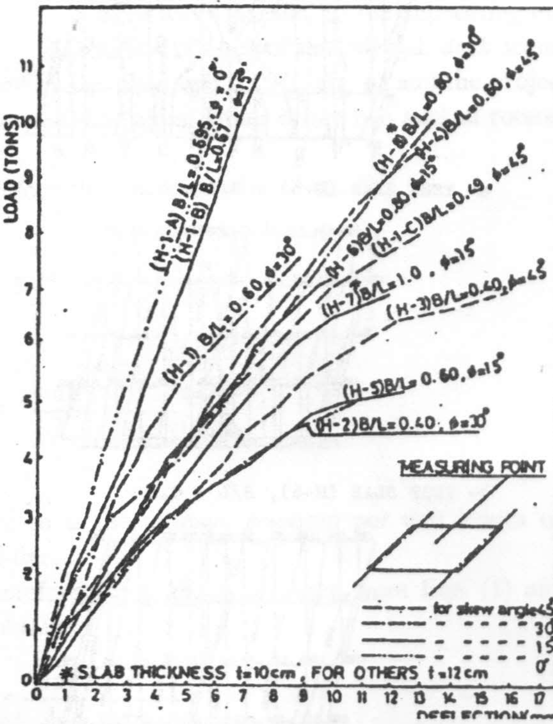

Figure 15. Load-deflection relationships.

a. The Grillage Method Results

The grillage method is an application of the stiffness method used mainly for analysis of deck bridges. In this method the slab is divided into a grillage of descrete elements mathematical model equivalent to the actual continuous structure, see for example Figure (6). This model is necessary in order to have a system with a finite number of degrees of freedom upon which matrix algebra operation can be performed. A grillage is a frame in the horizontal plane. In its simplest form it consists of a horizontal grid of straight uniform members intersect at right angles and subjected to only vertical loads. Under these conditions the grillage member has three actions at each node; shearing forces, bending moment M and twisting moment T. Also there are three corresbonding degrees of freedom, deflection and two rotations about two horizontal axes x & y. The method was used to study the distribution of strains and the bending moments at initial stages of loading of the skew slab decks which are

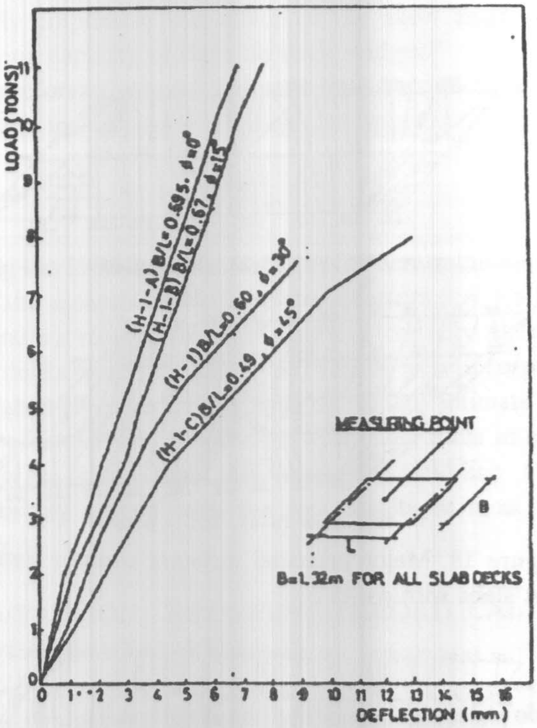

Figure 16. Load-deflectionn relationships for test series IV.

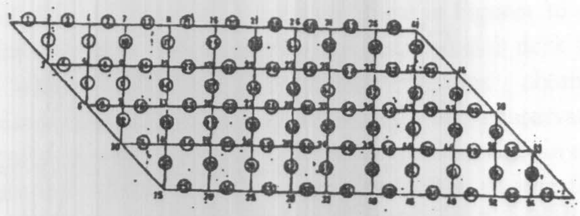

Figure 17. Details of mesh for specimen (H-4).

corresbonding to the usually known working load stages, see Figures 18 through 20. The results obtained by this method was greatly useful in indicating the position of maximum stressed points and consequently the location of the major yield-line (fracture-line) produced later on at higher stages of loading.

From the study of the distribution of the contour lines of the major principal moment, for the examined skew slab decks, Figures 18 through 20, against the two major parameters; the skew angle of the slab deck and the aspect ratio (B/L) of the skew slab decks, the following concluded remarks are quite obvious:

1. The lines of concentration of major principal bending moments lie close to the line of symmetry of slab deck parallel to the supporting edges and become quite perpendicular to the slab deck free edges as the aspect ratio (B/L) gets smaller, see Figure (18-a) and (19-a).
2. For slab decks, with small skew angle, say $\phi = 15^{\circ}$, Figure 20, the contour lines across which the major principal moment occurs becomes quite perpendicular to the slab deck free edges. This remark was confirmed exprimentally, as shown in Figures 6 and 7.

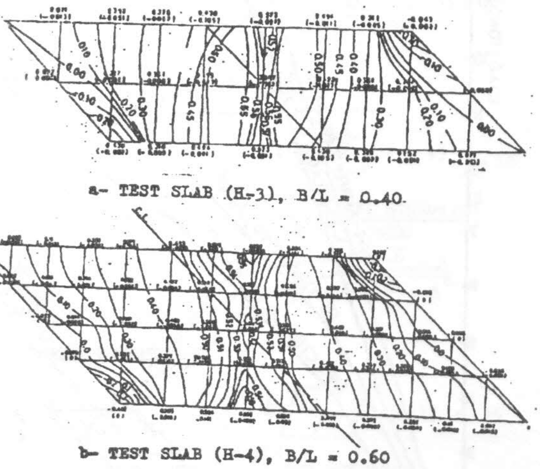

Figure 18. Major principal moment contour lines for test slabs with $\phi = 45^{\circ}$.

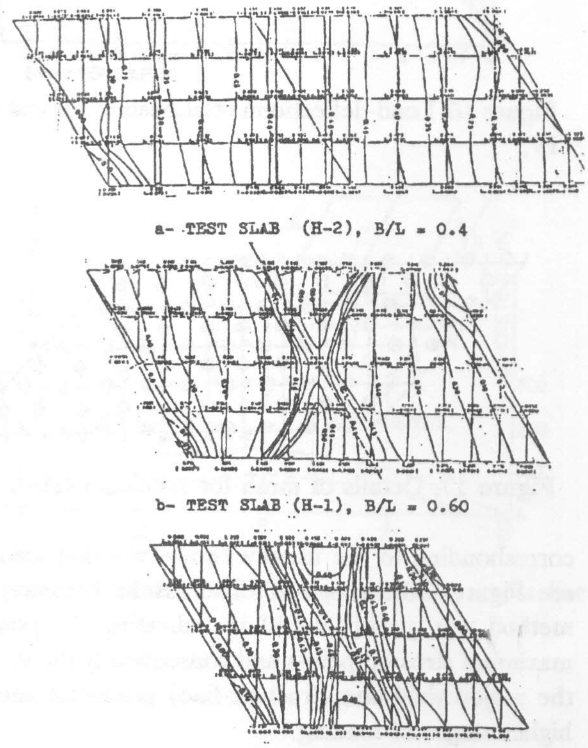

Figure 19. Major principal moment contoure lines for test slabs with $\phi = 30^{\circ}$.

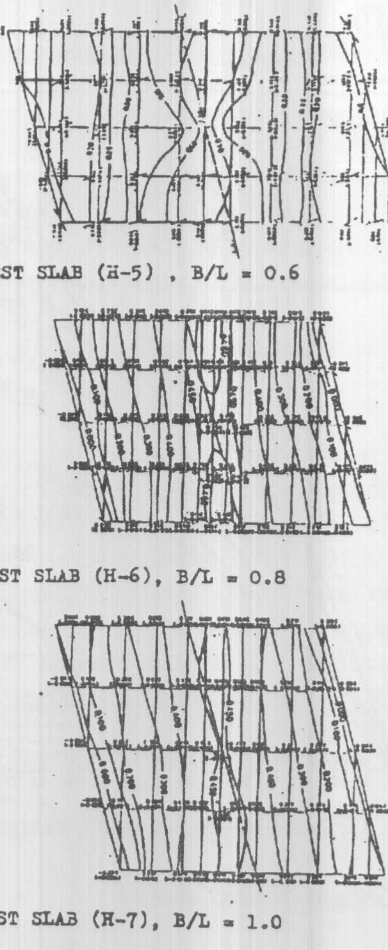

Figure 20. Major principal moment contoure lines of test slabs with $\phi = 15^{\circ}$.

b. Yield-Line Analysis Results

The yield- pattern shown in Figure 21 is selected as most possible fracture mechanism pattern, which satisfies the yield-line properties.

Using virtual work approach then, if the yield line has a virtuual displacement of the unity, the external work done,

W_e, is

$W_e = 2[B(L/2) \cos \phi \ w \ (1/2)]$ or

$$W_e = 1/2 \ BLw \cos \phi \quad (1)$$

where ,

w = the external load per unit area acting on the slab deck, that including the self-weight.

ϕ = the skew angle of the slab deck

L = length of the slab deck in the direction of the traffic.

B = slab deck width parallel to the supporting edges.

The virtual rotation of each of the two slab deck segments divided by the yield line is 2/(L cos ϕ) and the projection of the yield-line along either of the two axes of rotation is B.

Therefore the internal work done, W_i, is

$$W_i = mB \left[\frac{2}{L \cos \phi} + \frac{2}{L \cos \phi} \right]$$

$$W_i = \frac{4mB}{L} \frac{1}{\cos \phi} \quad (2)$$

where m is the ultimate moment per unit length of the yield-line.

Therefore, since $W_E = W_i$, then from Eqs. (1) and (2) we obtain:

$$\omega = \frac{8m}{L^2} \frac{1}{\cos \phi} \quad (3)$$

Then, the total ultimate load carried by the slab deck, P, is

$$P = \frac{8mB}{L} \frac{1}{\cos \phi} \quad (4)$$

In the tested slab decks the reinforcing steel was placed, to conform with the general practice of the skew slab deck bridges, parallel to the supports and the free edges of the test slab decks. In such a case since the yield-line lies in direction along the steel placed in the slab deck parallel to the supported edges, therefore, the ulimate moment per unit length of the yield-line, m, is

$$m = m_1 \cos \phi \quad (5)$$

where m_1 is the ultimate moment per unit length in a dirrection perpendicular to the steel placed parallel to the slab edges and produced by it.

Substituting for m from Eq. (5) into Eq. (4) and introducing the right span $\mathscr{L}$, where $\mathscr{L}$ = L cos ϕ, to clearly emphasize the effect of the skew angle on the carrying capacity of the slab deck bridges.

Therefore, the total ultimate load carried by the slab deck, P, is

$$P = \frac{8m_1B}{\ell} \cos \phi \quad (6)$$

Using the 1989 Egyptian code (9) for determination of m_1, with the assumption that the stress factors $\gamma_c = \gamma_s = 1.0$ for laboratory results, then the carrying capacity of the test slab decks were calculated, and results were incorporated in Table (1) under the name of "design ultimate load". These results were compared with test results in column 11. It is clear that the proposed yield-line analysis favourably agreed with the test results in most of the cases.

COMPARISON BETWEEN THEORITICAL AND EXPERIMENTAL RESULTS

The comparison between the theoretical results obtained from the analytical grillage method and the experimental results were made only for the initial stages of loading which may be in the range of the working loads. The strains obtained analytically across the slab center-line parallel to the supporting lines are given in Figures 10 and 12 for slab deck H-1 where B/L = 0.6, and slab deck (H-4) where B/L = 1.0 against strains obtained experimentally. The analytical results were conservative when compared with those obtained experimentally in case where the slab decks having aspect ratio B/L smaller than 0.6. Typical comparison is shown in Figures 9 and 10 for (H-1), as example. For test slab decks with higher aspect ratio, such as H-6, H-7 and H-8, the strains obtained experimentally and analytically were reasonably close, See Figure 11 & 12 for (H-7) as example.

As shown from Figs 17 through 20 the highly stressed points were located at points between the obtuse corners and the mid points of the free edge lines. The fracture lines (yield-lines) started to develop at those points along a line passing through the most stressed points, see Figures 17 through 20 and Figures 4 through 7. The test results showed that the angle between the yield-lines and the mid-centre-line of the slab deck, in most cases, is about half the skew angle. This concluded result was also found to be in agreement with the analytical study, Figures 17 through 20.

Table (1), Figure 13 and Figure 14 demonstrate the effect of the studied parameters on the ultimate strength of the test slab decks, and show also, the variation between the test ultimate loads and the calculated ultimate loads using the yield-lins theory.

The results obtained by the yield line analysis slightly underestimates the obtained test strength. This may be due to memberane forces produced in the plane of the test slab deck during testing. Since the difference between the experimental and theoretical yield line analysis results is on the safe side, it could be used for design purposes with confidence.

CONCLUSIONS

The results obtained from the analytical and the experimental results throughout this investigation help in verifying some important points in the design of simply supported shew slab deck bridges. The followings are the main concluded remarks.

1. For skew slabs of small aspect ratios of 0.40 to 0.6 (H-1, H-1-C, H-2, H-3, H-4 and H-5) the maximum deflections were recorded at the mid-points of the free edges. For skew slabs of aspect ratios of 0.6 to 1.0 (H-1-b, H-6, H-7, and H-8) the maximum deflections were recorded at some points existed between mid-points of free edge and obtuse corners.
2. This conclusion is also clear during the observation of the formation of the major crack at the ultimate conditions. A major yield line was formed passing at the slab mid-point and is inclined to the slab mid-centre line with an angle approximately half the skew angle for large skew angles of $\phi=45^{\circ}$ and 30°. This angle tends to be equal to zero for slab decks having smaller skew angles up to 15°.
3. The values of stresses and deflections of the slab decks of large skew angles were showed to be greater than those of smaller skew angles when they have the same width and right angles and subjected to the same load, Figure 15.
4. Slabs of large skew angles showed conservative loads compared with those of small skew angle when they have the same dimensions and reinforcement, See Table (1) and Figures 13 and 14.
5. The calculated capacity of slab decks using the ultimate load theory are always less than the maximum obtained load from the tests, this may be due to memberane forces. See Table (1).
6. The calculated slab capacities using the ultimate load theory are also less than the load which cause yielding in steel bars of the slab decks by about 5%.

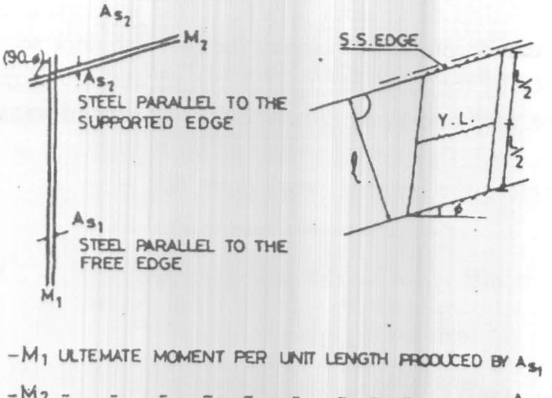

Figure 21. Yield-Line pattern produced in the slab decks at failure.

7. Under the same load, the values of stresses obtained from the elastic analysis using the grillage method, were less than the value obtained from the test results for skew slabs of small aspect ratio of 0.4 to 0.6. This variation were about 60 too 80%. For skew slabs of aspect ratio 0.6 to 1.0 the variations were about 20%. This may be due to the steel arrangement.
8. The stress distribution calculated from the grillage method, showed also that the maximum values occured at some points between the obtuse corner and the mid point of the edges of the slab deck.
9. The overall stiffness of the slab decks decreases as the skew angle (ϕ)increases, Figure 16. The effect of the skew angle on the stiffness may be neglected for slab decks with skew angles of 15° or less.

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