STEADY FLOW IN COLLAPSIBLE TUBES

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ABSTRACT

The steady flow of an incompressible fluid in a thin-walled compliant tube was studied both theoretically and experimentallly. The flow parameters included friction effect, body forces other than gravity and pressure variation along the tube. The study of such flow is important in understanding physiologic flow in the venous and pulmonary systems. Governing equations, for the steady one-dimensional flow, were coupled with the equation representing the tube mechanics to yield a set of differential equations representing the dependent variables as functions of the initial conditions: the tube properties, the external pressure and the applied body forces. A general numerical solution was obtained for these equations. The numerical results showed distinct curves for the dependent variables along the tube length with the possibility of chocking within the tube length. A laboratory experiment was designed and carried out to test some of the flow characteristics obtained from the numerical simulation. A single latex rubber tube of 42 cm length and 9 mm inner diameter was used in this experiment. The tube was subjected to variable external pressure. Measurements of differential pressure and area change were obtained. Experimental results confirmed the numerical results and offered explanations to some of the physiologic phenomena.

NOTATIONS

- A tube cross-sectional area, m^2
- A_0 neutral cross-sectional, m^2
- c wave speed, m/s
- D_0 neutral tube diameter, m
- f friction factor
- K flow function defined by Eq (14)
- n tube law index
- P static pressure, kN/m^2
- P_e external pressure, kN/m^2
- Q volumetric flow rate, cm^3/s
- S speed index
- u one-dimensional velocity, m/s
- W tube perimeter, m
- x distance along the tube, m

Greek Symbols

- α area ratio, (A/A_0)
- ρ fluid density, kg/m^3
- μ fluid dynamic viscosity, $N.S/m^2$
- τ wall shear stress kN/m^2
- ε circumferential bending stiffness, kN/m^2

INTRODUCTION

The flow through rigid circular tubes have been widely analyzed by physicists, mathematicians and hydraulic engineers, and have found wide applications in industry. In contrast, it is only during recent years that studies have been carried out concerning the flow through thin walled compliant tubes [1-16], and these have been few in comparison to the studies of flow through rigid tubes. It is noted that flow in compliant tubes differs from that in rigid tubes in that these tubes collapse when the transmural pressure (internal minus external pressure) becomes negative. Moreover, the mechanics of the flow is closely coupled to the structural mechanics of the tube.

The early research works on the problem of flow in compliant tubes were carried out by phycisits and physiologists. These works have principally been motivated by the need of better understanding of the nature of flow in the vessels found in the human body. Holt [1], Rodbard et al [2], Katz et al [3] Olsen [4] and Kececioglu et al [5] have carried out experimental studies of flow in thin walled rubber tubes representative of the human veins in a simple laboratory apparatus introduced first by Holt [1].

Lambert et al [6], Conrad [7], Brower et al [8] and McClurken et al. [9] have attempted theoretical analysis for flow in collapsible tubes and concluded that the flow is a function of the multi-value of the pressure drop.

Griffiths [10] presented a theory for the flow of an incompressible invicid fluid through an elastic tube with uniform elastic properties. Later, Griffiths extended the work [11] and developed one dimensional theory for steady flow through a tube with nonuniform elastic properties. From the theory, he predicted that supercritical flow could only be developed by smooth transition through critical velocity.

Shapiro [12] was the first to offer a general theory for the steady one dimensional flow in a thin-walled partially collapsed tube. The theory includes the effects of friction, lengthwise variation in external pressure, variation in elevation, neutral area, wall stiffness and mechanical properties of the tube. The flow of air in the respiratory system exhibits many of the characteristics of flow in compliant conduits. Dawson et al. [13] described the mechanism of forced expiratory flow limitation based on a collapsible tube theory. Their results confirmed that the shapes predicted for the maximum flow static recoil curves depend only upon the nature of the pressure-area curve at the choke point.

Recent developments included the work by Kamm et al [14] on the unsteady one-dimensional flow in a collapsible tube, Cancelli and Pedley [15] on the collapsible-tube oscillations using a separate-flow model and Dardel [16] on the wave propagation in elastic tubes.

The objectives of the present work are:

To develop a theoretical model to explain the features of the flow through collapsible thin-walled tubes that have not been yet understood.

To design an experiment to study the phenomena exhibited by that type of flow and test the theoretical model proposed in this study.

THEORETICAL ANALYSIS

Steady, incompressible, one-dimensional flow in relatively long, thin-walled compliant tube is considered in this analysis. The tube is assumed to have uniform properties with longitudinal uniformaties represented by the constant neutral area (A_o) and the constant circumferential bending stiffness (ε). The mechanics of the tube, involving the load due to transmural pressure, is represented [12] by, the equation:

$$P - P_e = \varepsilon\ (1 - \alpha^{-n}) \qquad (1)$$

where n is a constant to be deterimined by experiment.

The flow through an infinitismal control volume consisting of a section of the tube of length dx, given in Figure (1), is governed by the continuity equation:

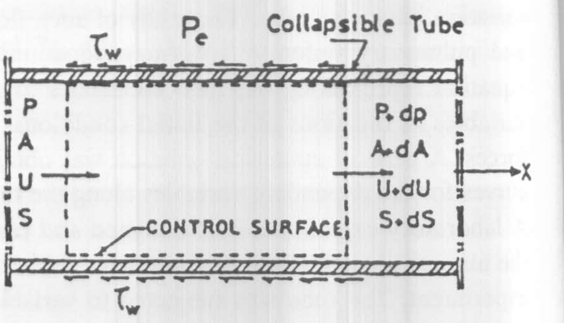

Figure 1. Infinitismal control volume in the tube.

$$\frac{dA}{A} + \frac{du}{u} = 0 \qquad (2)$$

and the momentum equation for a horizontal tube:

$$-d(P-P_e) - \tau \times \frac{W}{A}\ dx = \rho u du \qquad (3)$$

where A,P and u representing the area, the pressure and the flow velocity are assumed to be continous functions of x. P_e is the external pressure surrounding the tube and τ is the wall shear stress which equals $1/2\ f\rho u^2$ where f is the friction coefficient.

If P_e is hold constant over the entire length of the tube, then differentiation of equation (1) yields:

$$\frac{dP}{dx} = \frac{n\varepsilon}{\alpha^{n+1}} \frac{d\alpha}{dx} \qquad (4)$$

Since, (A_o), the mentral area is constant, then equation (2) gives:

$$\frac{du}{dx} = - \frac{u}{\alpha} \frac{d\alpha}{dx} \qquad (5)$$

and the definition of $\alpha = A/A_o$ gives:

$$\frac{d\alpha}{dx} = \frac{1}{A_o} \frac{dA}{dx} \qquad (6)$$

The pressure wave in the tube is considered as a propagated wave travelling with a speed c given by:

$$c^2 = \frac{A}{\rho}\frac{d(P-Pe)}{dA} \qquad (7)$$

substituting by equations (1) and (6) into (7) gives

$$c^2 = \frac{n\varepsilon}{\rho}\, a^{-n} \qquad (8)$$

To relate the wave speed c with the flow speed u a speed index S is defined as

$$S = u/c \qquad (9)$$

i.e.

$$\frac{dS}{S} = \frac{du}{u} - \frac{dc}{c} \qquad (10)$$

After straight-forward algebraic manipulation of equations (3), (4), (5), (6) and (10), the result is a set of three differential equations in the following form :

$$\frac{d\alpha}{dx} = \frac{fKW/2A_o}{K - n\alpha^{2-n}} \qquad (11)$$

$$\frac{d(P-P_e)}{dx} = \frac{nfKW/2A_o}{K-n\alpha^{2-n}} \times \frac{\varepsilon\,\alpha^{-(n+1)}}{K-n\,\alpha^{2-n}} \qquad (12)$$

$$\frac{dS}{dx} = \frac{fKW/2A_o}{K-n\,\alpha^{2-n}} - \frac{S(n-2)}{2\alpha} \qquad (13)$$

$$\text{where } K = \frac{\rho}{\varepsilon}\left(\frac{Q}{A_o}\right)^2 \qquad (14)$$

The function K is referred to as the flow function. The value of this function determines the effect of the friction force on the flow as compared to other forces.

The above set of equations (11-13) relate independent parameters such as the flow rate, the tube initial dimensions and the fluid and tube properties to the dependent parameters such as the area ratio, transmural pressure and the speed index. The equations are of the first order non-linear differential equation type and may not be solved on a closed form. Therefore, numerical integration methods are the only practical alternative to obtain a general solution. This solution is in the form of family of curves for area ratio, transmural pressure and speed index as functions of the dimensionless axial distance (x/D_o). The numerical integration requires that the independent parameters be defined at the start of the solution (i.e at the tube entrance).

Initial Conditions

An important and decisive independent flow parameter is the area ratio at the tube entrance (α_1). It is a function of the external pressure at tube entrance which may be adjusted artificially. In general, the speed index S_1 is related to the area ratio (α_1) by using equations (8), (9) and (14) to give:

$$S_1 = \sqrt{K} \times \sqrt{\frac{\alpha_1^{n-2}}{n}} \qquad (15)$$

The value of the flow function K depends on the reference flow and pipe paameters ρ, Q, ϵ and A_o.

Equation (15) is plotted in Figure (2) for a constant value of n = 0.5 and different entrance area ratios (α_1). The plotted relationship enables the determination of the speed index at entrance (S_1) when both the flow function (K) and the entrance area ratio (α_1) are known. For a certain value of K, the flow may be either subcritical S < 1 or supercritical (S > 1) depending on the value of the area ratio at entrance (α_1). This phenomena has many physiological implications such as: the use of pressurizing cuff, sudden change of external pressure, change of heart rate and anastemosis of a vein.

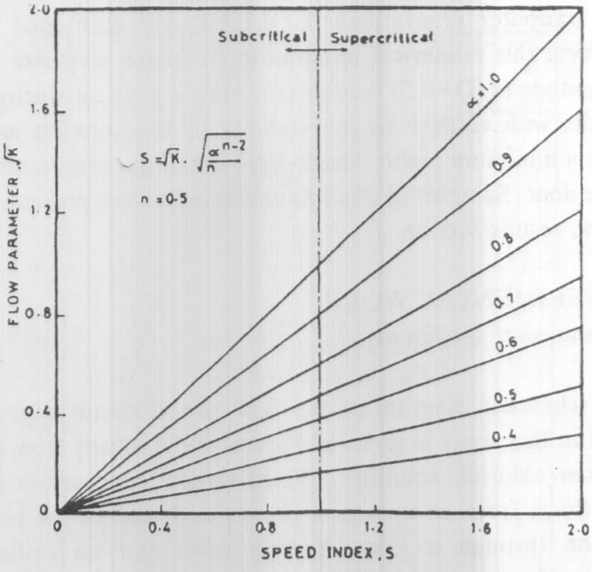

Figure 2. Relationship between the speed index (S) and the flow function (K) for different values of area ratio (α).

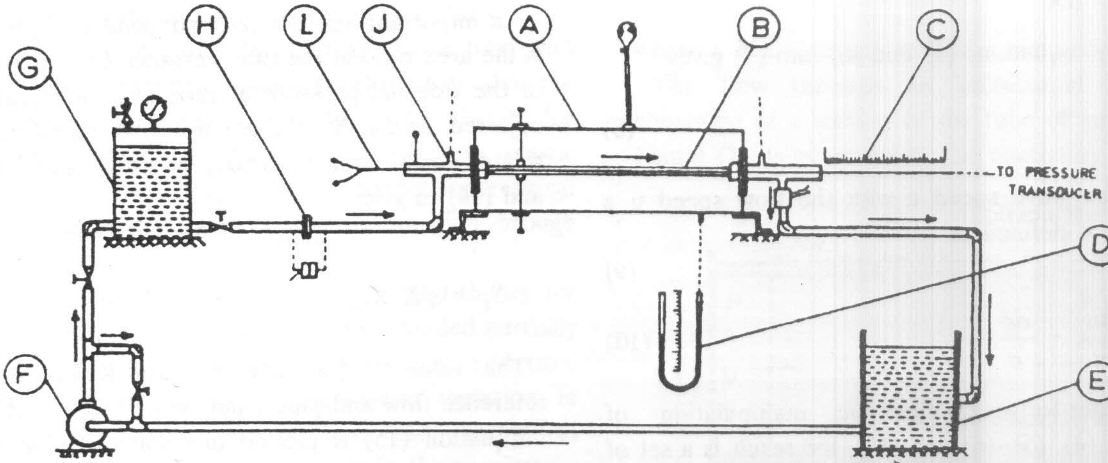

Figure 3. Schematic diagram for the experimental apparatus; A- Test chamber; B- Latex tube; C- Scale; D- Manometer; E- Downstream tank; F- Pump; G- Pressurized tank; H- Orifice meter; I- Area measurement electrodes; J- Inlet pressure.

Numerical Solution

The standared fourth-order Runge-Kutta technique was used to integrate the system of equations (11) to (13) and to obtain a general numerical solution.

A computer programme (COLLAP) was developed to perform this numerical integration. A spatial step size of magnitude (x/D = 0.5) was chosen since it gave satisfactory results with respect to the stability of the solution and gave a minimum error. About 200 runs of the programme were done. Samples of the obtained results were presented in the results section.

EXPERIMENTAL WORK

Experimental Apparatus

A schematic diagram of the flow closed circuit system used in this study is given in Figure (3). A steady flow of sodium chloride solution (7% by-weight concentration) was flown from an upstream pressurized tank to the test section through a flow-adjusting valve and an orifice meter. The pressure in the tank was kept constant at any desired value by adjusting the compressed air pressure applied above the fluid. A centrifugal pump (0.3 kW) was used to pump back the solution to the upstream tank. The flow rate was controlled by a needle valve. The test section was a parallel-piped air tightd chamber (25 x 25 x 50 cm) made of Plexiglass sheets of 0.8 cm thickness. A thin-walled latex rubber tube (inside diameter 0.9 cm, wall thickness 0.05 cm and length 42 cm) was fastened by means of 0-rings to the rigid end tubes of the same inside diameters in order to avoid area discontinuities. One rigid end was fixed while the other could be twisted and axially moved. The former was to assure a uniform plane of collapse and the later was to adjust the longitudinal tension such that the tube is unbuckled for zero flow and zero transmural pressure. The pressure inside the test section was adjusted manually to any desired value by an air blower. The inlet area of the latex tube was adjusted by means of sphincters.

The volumetric flow rate passed in the latex tube was measured by means of an orifice-meter placed upstream of it. A differential pressure transducer (Celesco Model KP 15) was used. The pressure transducer output was connected to a reading unit (Celeco Model CD 25 C) whose output voltage was displayed on a digital voltmeter (Texktronix DM 502-Multimeter).

Pressure Measurements

The interenal static pressure was obtained by means of an axially traversing probe. The probe was formed from a length of hollow TRANSPERANT P.V.C. tubing having 0.2 cm outside diameter and 0.15 cm inside diameter. The static pressure, sensed at a side hole of 0.05 cm diameter in the P.V.C. tube, was transmitted to a differential pressure transducer (Cellesco - Model KP155). The probe was positioned against the wall of the supporting rigid tubes. Thus, the interference of the probe with tube wall was avoided. The transducer measured the difference in static pressures between the inlet of the collapsible tube and any desired location inside the tube. The transducer output was connected to the reading unit (Celesco-Model CD 25C) whose output voltage was displayed on a digital voltmeter (SOAR Multimeter 530). The tube external pressure was measured by means of a simple U-tube manometer containing water - one of its ends was attached to the tapping hole made in the Plexiglass test section while the other was opened to the atmosphere.

A manual air blower was used to maintain any desired value of the external pressure. However, the maximum value was limited to 6 kN/m^2, since the test section is made of Plexiglass sheets.

The Tube Area Measurements

The area measuring technique developed by McClurken et al [9] was proved to be the most suitable technique when applied to cross-sectional area measurements in collapsible tubes. This technique is based on the principle of electric impedence measurement of an electrolyte fluid.

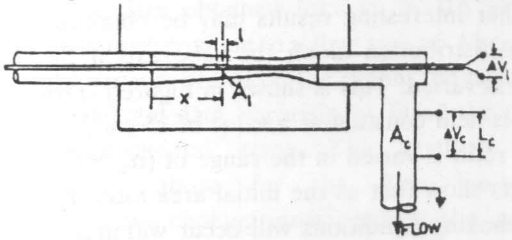

Figure 4. Electric impedence technique for area measurements (subscript c for calibrating section and subscript 1 for measured section).

Three electrode pairs were used. The first pair provided an AC voltage excitation from an electrode on the upstream rigid tube to the downstream rigid tube, and establishes an AC current flow through a salt solution. The second was used to sense the differential voltage ΔV_c at a calibrating section of a known length (L_c) and cross-sectional area (A_c). The third was used to measure the differential voltage ΔV_1 at the location where the cross-sectional area was unknown. The third electrode pair was built on the P.V.C. traversing probe used in the internal static presure measurements. Figure (4) gives detailes of the electric impedence technique. Specifications and calibration of the system could be found in [17].

The Tube Law

In order to determine the tube law, the experimental apparatus was modified by separating the test section from the flow circuit and using it as a chamber under static test conditions.

Data for the area ratio as a function of the transmural pressure were obtained in the range of $0.2 < \alpha < 1.1$ as shown in Figure (5). The data was correlated to equation (1) using the least-squares method with non-linear dependent variable. The final form of the function with the minimum sum of square errors gave:

$$P - Pe = 0.565\ (1 - \alpha^{-0.5}) \quad N/m^2 \tag{16}$$

The above equation is also shown in the Figure. Typical cross-sectional shapes for different ranges of area ratio are also presented in the same Figure.

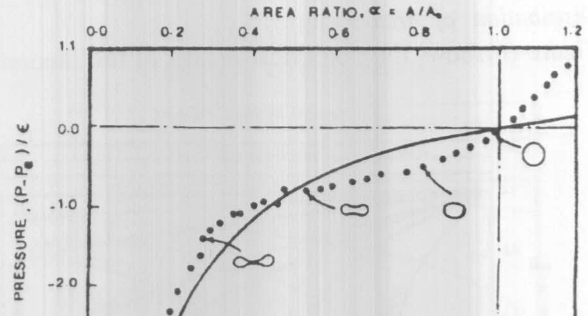

Figure 5. Experimental measurements for the cross-sectional area as a function of the transmural pressure.

RESULTS AND DISCUSSION

In this section the numerical results obtained from the computer programme are presented together with some experimental results obtained for selected parameters. The numerical results were obtained for data of a standard thin-walled latex tube which is commonly used in investigating flow through collapsible tubes. In order to facilitate the comparison between the theoretical results and the experimental results, the specifications of the latex tube used in the experimental study were input data. These were:

- The tube nominal diameter (D) = 0.9 cm
- The wall thickness of the tube = 0.05 cm
- The bending stiffness of the tube (ε) = 0.565 kN/m^2

The numerical results are given in the form of graphs representing the distribution of the dependent parameters such as the area ratio (α), the speed index (S), and the transmural pressure ($P - P_e$) along the tube axis. These results covered a wide range of flow rates from 10 to 60 cu.cm/s at different values of external pressure.

Since the experimental study could not cover the wide range for the flow parameters studied numerically, limited specific number of experiments were carried out. The purpose of these experiments was to test the validity of the theoretical model.

Distribution of Area Ratio

Figure (6)shows the numerical results of the theoretical

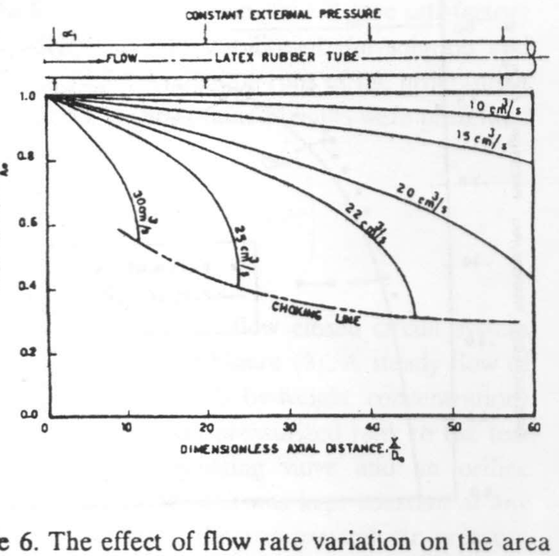

Figure 6. The effect of flow rate variation on the area ratio along a horizontal latex rubber tube.

model for the area ratio distribution along the axis of a tube in the horizontal position. The results were obtained for the same inlet area ratio (α_1 = 1.0) and different values of flow rate. For low flow rates (up to 20 cm^3/s) a slight decrease of the area ratio along the tube was observed while at higher flow rates the decrease of area ratio occured more rapidly as a result of the reduction of internal pressure, as the friction increased. This would lead the flow to reach the critical flow conditions sooner in the tube as the flow rate increased. It is expected that beyond the location of these conditions the flow will continue either as supercritical through a smooth transition or the flow rate will readjust its value so that the chocking occurs at the end of the tube. A transition to supercritical is normally associated with an increase in both area ratio and internal pressure resulting in a return to critical conditions at the same location. This means that the transition to supercritical flow is physically impossible [12]. The more probable behaviour for the flow is to readjust its rate value to that causing the critical condition to occur at the end of the tube. For a specific tube length, the choking will occur at the end of the tube at a flow rate which is maximum for that length of the tube. Since increasing the flow rate could cause choking to occur within the tube which is not physically expected, the flow rate will be limited to a maximum value attained for the critical condition at the end of the tube. This phenomenon is known as flow limitation. The choking line drawn in the Figure represents the limitation of the flow rates as a function of the axial distance (i.e. the tube length).

Other interesting results may be obtained for the area ratio distribution along the tube axis if the initial area ratio is varied. This is shown in Figure (7) where the flow rate is held constant at a value of 15 cm^3/s and the initial area ratio is varied in the range of (α_1 = 0.5 to 0.9). The results show that as the initial area ratio α_1 is decreased, the choking conditions will occur within a shorter length of tubing. The plotted curves show that although the initial area ratio may take different values, the critical area ratio (α^*) has a single value corresponding to the specified flow rate. Since choking does not occur within the tube, as explained before, the flow will probably readjust its rate value to a smaller limited value corresponding to the tube length. Experimental results are given in the Figure. For inlet area ratio (α_1) value down to 0.6, the experimental data are in good agreement with the numerical results, but as the inlet area ratio is reduced to 0.5, the experimental data deviated from the numerical values. The numerical curve representing $\alpha_1 = 0.5$ indicated that choking would

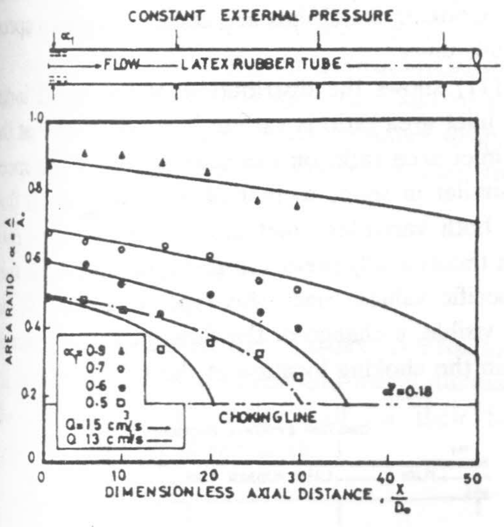

Figure 7. Comparison between theoretical and measured values of the area ratio along a horizontal latex rubber tube.

occur at a distance of 20 diameters from the inlet. Since the tube length used in our experiment is 30 diameters, it means that choking could be within the tube length which is not physically possible, as explained before. Rather, the flow will readjust its rate value to a lower value so that the choking would occur at the end of the tube. The maximum limited value of flow rate for the flow condition with $\alpha_1 = 0.5$ was 13 cm^3/s. It is interesting to compare the experimental data obtained for $\alpha_1 = 0.5$ with a numerical curve corresponding to a flow rate of 13 cm^3/s. This numerical curve is drawn on the graph; the dash-dot line. The experimental data, surprisingly, fitted the curve within the experimental error. This confirms the explanation given above for the flow limitation phenomenon that the choking must occur at the end of the tube. The length of the tube between x/D = 20, the initial location predicted for choking, and the end of the tube was observed to exhibit vibrations due to a series of stationary waves.

Distribution of Transmural Pressure.

Figure (8) shows the numerical results of the theoretical model for the transmural pressure distribution along the axis of a tube in the horizontal position. The results are obtained for the same inlet area ratio ($\alpha_1 = 1.0$) and different values of flow rate (Q) ranging from 10 to 30

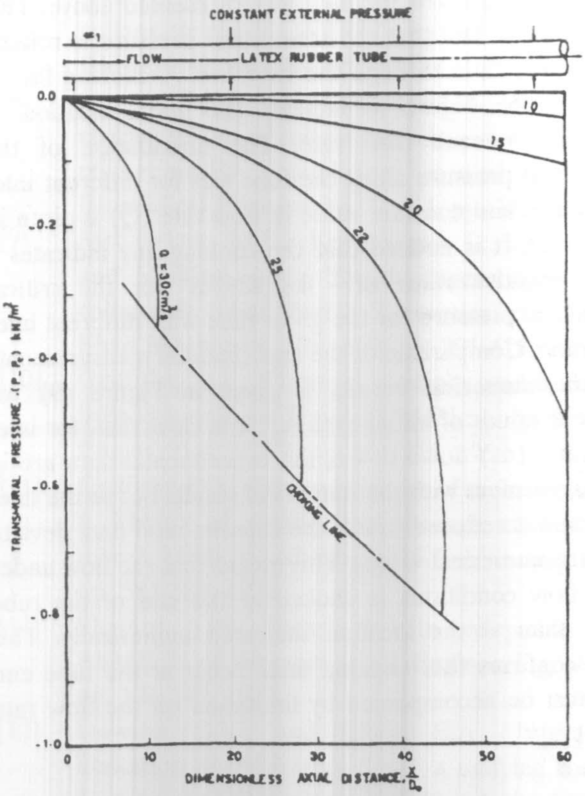

Figure 8. The effect of flow rate variation on the transmural pressure along a horizontal latex rubber tube ($\alpha_1 = 1.0$).

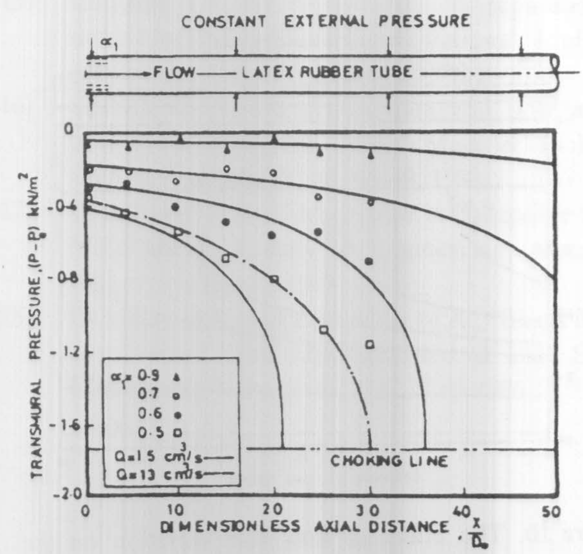

Figure 9. Comparison between theoretical and measured values of the transmural pressure along a horizontal latex rubber tube.

cm^3/s. It is obvious that transmural pressure distribution is similar to that for the area ratio presented above. This is due to the fact that the transmural pressure is related to the area ratio through the tube law. A choking line is given in the Figure to indicate the initial location of choking for each flow rate. The distribution of the transmural pressure along the tube axis for different inlet area ratios and constant value of flow rate (Q) is given in Figure (9). It is noticed that the choking line indicates a single negative value of - 0.8 KN/m^2 for the critical transmural pressure for the given tube with different inlet area ratio. Comparison of the experimental measurements and the numerical results is given in Figure (9) for different values of initial area A_0. It is clear that, for inlet area ratio (α_1) down to 0.6, the experimental data are in good agreement with the numerical results but as the inlet area ratio is reduced to 0.5, the experimental data deviate from the numerical values. This means that the flow under these flow conditions is choked at the end of the tube rather than at the location indicated numerically. This result confirms that choking must occur at the tube end and must be accompanied by limitation on the flow rate value [18].

Distribution of Speed Index

Figure (10) shows the numerical results of the theoretical model for the speed index distribution along the axis of a tube in the horizontal position.

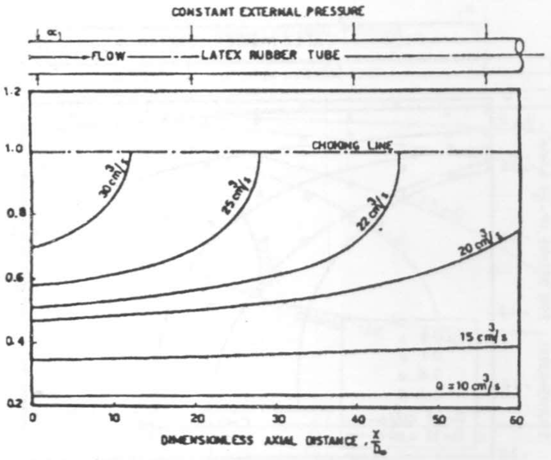

Figure 10. The effect of flow rate variation on the speed index along a horizontal latex rubber tube (α_1 = 1.0).

The results are given for an inlet area ratio (α_1 = 1.0) and different values of flow rate (Q) in the range of 10 to 30 cm^3/s. Choking conditions are corresponding to speed index equals unity.

Figure (11) shows the distribution of the speed index when the inlet area ratio is varied. It is observed that the effect of inlet area ratio on the distribution of the speed index is similar in shape to that of the effect of the flow rate. For both variables, inlet area ratio and flow rate, choking is theoretically predicted to occur within the tube and at specific values. Since this type of choking is not physically visible, a change of the flow parameters occurs to maintain the choking location at the tube end.

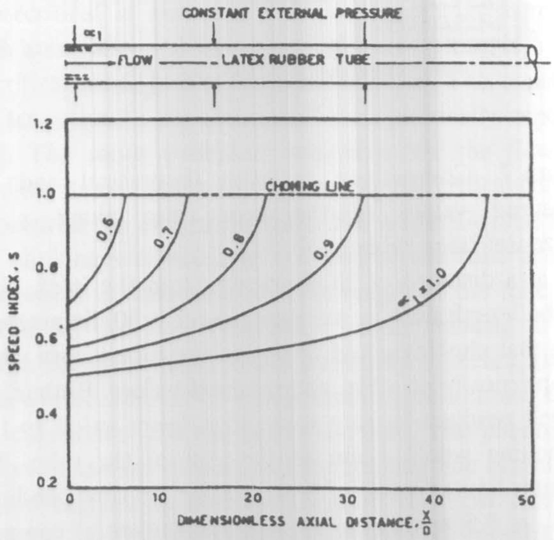

Figure 11. The effect of variation of inlet area ratio on the speed index along a horizontal latex rubber tube.

CONCLUSION

The following conclusions may be drawn from the present study:

1. The developed one-dimensional numerical model for flow in complaint tubes could be used successfully to predict the distributions of area ratio, transmural pressure and speed index along the axis of the tube.
2. The performed experiments on the latex rubber tube provided qualitative results that are in good agreement with the theoretical results for a wide range of flow parameters.
3. The used model also predicts the phenomenon of choking reported in the literature with its associated limitation of the flow rate.

4. Measurements of flow rate and qualitative observations of the change in the tube geometry confirmed the flow limitation associated with choking and confirmed the results obtained in the theoretical analysis.
5. The reduction of inlet area ratio and/or the increase of tube length increase the pressure drop across the tube and decrease the maximum flow rate passing through it.

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