OPTIMAL LOAD SHEDDING THROUGH LYAPUNOV TRANSIENT STABILITY METHOD

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ABSTRACT

A new technique is described for handing the problem of finding appropriate instants of shedding excess loads after the loose of some generations. The direct method of Lyapunov is used to specify these instants. The solution of the problem is performed in three steps :

1. A steady-state solution for the load shedding strategy is obtained using an optimization technique.
2. The transient stability associated with the load shedding is then studied using the point-by-point method.
3. The second method of Lyapunov is used at each point to determine the appropriate instants of successive load sheddings.

This new technique is applied to a 6-busbars and 12-busbars systems. Study has revealed that the best times for load sheddings are those where minimum local potential energy occurs.

INTRODUCTION

With present time interconnected power systems, conditions often arise which would result in the isolation of a part of the system with an excess of loads on it. A declining frequency situation will, therefore, develop in that part. Load shedding provides the means of arresting such frequency decline after all available generation reserves are used up. Certain percentages of the loads are dropped in succession.

The problem of optimal load shedding is essentially the problem of obtaining such steady-state which guarantees a minimum of load dropped [1-7]. In a previous paper by the author [8], the transient stability associated with load shedding strategy was studied. The main problem still to be solved is : At what instants should sheddings start while keeping system oscillations at a minimum?

To answer this question, use will be made of the Lyapunov energy functions [9-13]. The direct method of Lyapunov is mainly applied to the problem of transient power system stability to permit the study of multimachine systems. Using this method for transient stability study will only indicate whether the system is stable or not. A new use will be made of these functions to indicate the appropriate instants for successive load sheddings steps. A problem formulation is given in [3], and is given in Appendix (A).

LYAPUNOV ENERGY FUNCTION

A form of Lyapunov function that has been found to be suitable for use in power systems is given by [9]:

$$V(\omega,\delta)=\sum_{i=1}^{N_G}\frac{1}{2}M_i(\omega_i-\omega_s)^2+[\sum_{i=1}^{N_G}(E_i^2G_{ii}-P_{mi})(\delta_i-\delta_i^s)+$$
$$\sum_{i=1}^{N_G}\sum_{j=i+1}^{N_G}E_iE_j\{B_{ij}(\cos(\delta_i^s-\delta_j^s)-\cos(\delta_i-\delta_j))$$
$$+G_{ij}(\sin(\delta_i^s-\delta_j^s)-\sin(\delta_i-\delta_j))\}]$$

where,

M_i = $H_i/\pi f$, $Kw.s^2/KVA.rad$
H_i : inertia constant of ith machine, Kw.s/KVA
f : system nominal frequency, Hz
ω_i : speed of ith machine, rad/s
ω_s : synchronous speed, rad/s
δ_i : rotor angle of ith machine with respect to a

reference axis rotating at synchronous speed, rad

E_i : voltage behind transient reactance of the ith machine, p.u.

P_{mi} : net mechanical power of ith machine, p.u.

N_G : total number of machines

s : superscript indicating postfault stable equilibrium point calculated from steady-state after load shedding is performed

G_{ij}, B_{ij} : real and imaginary parts of (i,j) elements of Y_{bus} matrix reduced to the internal nodes of the synchronous machines, p.u.

Observing this function, it is possible to distinguish two parts which seem to be the kinetic energy given by the first summation term, and potential energy given by the last term between the two prackets[....]. In the post disturbance period, the system whose energy would be represented by $V(\omega,\delta)$, its kinetic energy is being converted into potential energy. In this paper load shedding will be carried out at instants of local minimum potential energy. It will be realized that shedding at these instants will improve oscillations in machines speeds.

SOLUTION STEPS

The solution of the load shedding problem proceeds as follows:

(a) Load flow analysis is used to obtain system conditions prior the disturbance. From this analysis the initial machines internal voltages as well as rotor angles are then calculated.

(b) Simulate the system disturbance. In our case, the input mechanical power at the faulty generator busses are reduced while there is no reserve at the remaining busses. Then applying the optimal load shedding technique described in [3] to get the new steady-state loads distribution at the different load busses, and voltage magnitudes and phase angle for all busses.

(c) Calculate the post disturbance internal voltages as well as rotor angles δ_i^s of machines from steady-state conditions obtained from step (b).

(d) During the application of the point-by-point method, the different loads are represented by their respective admittances to ground. The principle mentioned in section "Lyapunov Energy Founction", is applied to check the potential energy function for a local minimum. If a minimum is detected, then a load shedding step is performed. Now, our strategy is to completely shed the excess loads which are determined from (b) in steps with a constraint that the drop in any machine frequency does not exceed 49Hz. To know the frequency of each machine at each point, equations (8)&(9) given in Appendix (A) have to be applied to calculate the derivatives. The modified Euler technique for numerical integration is applied with a time increment Δt. During this application, load flow solution has to be carried out for the modified system which contains the machine transient reactances and the loads represented with their respective admittances to ground. If after dropping all loads to be shed any speed still less than the 49Hz limit, then this means that the system is unstable.

(e) It is a well known fact that exciter control system is vital for preserving the stability of synchronous machines when subjected to sudden fault. Its action is to control the excitation emf in order to reduce the output power to reach power balance between the output and the power consumed by the system during the fault. One can look to the load shedding problem in a similar manner. After each load shedding process, the power consumed by the system reduces. This can be regarded as electrically equivalent to a fault on the system. Hence, the importance of control action during the load shedding process arises. In the following applications exciter control system is assumed to act on each machine having the following performance : After each load shedding step the internal emf of each machine is changed by a percentage equal to the percentage of the load shed.

APPLICATIONS

The method described above has been applied to two test systems:

A: 6-BUS SYSTEM

The power system shown in Figure(1) is used as a test system. The transmission line impedances in per unit on a 100MVA base are given in Table(1). The scheduled generations and loads and assumed bus voltages in per unit are given in Table(2). The inertia constants and direct axis transient reactances are given in Table (3). Table(4) shows the state prior the emergency. Also shown are the upper and lower limits of the different inequalities. A normal load flow is used to obtain that state. Emergency

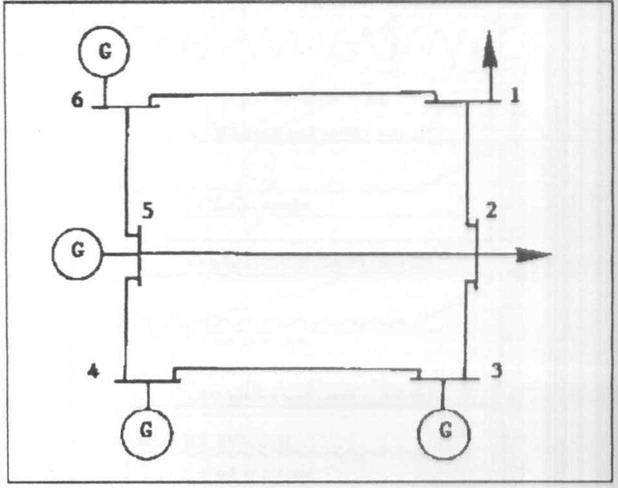

Figure 1. 6-bus system.

is simulated by a reduction in the available real generation at bus 6 from 0.45p.u. to 0.15p.u. Optimal load shedding is obtained while priorities are equal for all loads. Table (5) shows the solution for this case.

Table(6) shows shedding steps and their respective time in conjunction with the amount of load to be shed at each step for three different strategies:

1. Shedding when potential energy has a local minimum (case 1).
2. Shedding according to preselected instants (case 2).
3. Shedding actuated according to drop in frequency below 49. 5Hz (case 3).

The three strategies are performed in 10 steps where 10% of the load to be shed is shed at each instant.

Table 1. Transmission line impedances.

Bus Code p-q	Impedance
1-2	0.2+j0.4
1-6	0.1+j0.15
2-3	0.1+j0.3
2-5	0.2+j0.5
3-4	0.2+j0.8
4-5	0.1+j0.5
5-6	0.05+j0.2

Table 2. Scheduled generations and loads and assumed bus voltages.

Bus Code p	Assumed Bus Voltage	Generation P	Generation Q	Load C	Load D
1	1.00 + j0.0	0.0	0.0	0.3	0.13
2	1.00 + j0.0	0.0	0.0	0.6	0.22
3	1.06 + j0.0	0.2	?	0.0	0.0
4	1.08 + j0.0	0.3	?	0.0	0.0
5	1.05 + j0.0	0.1	?	0.0	0.0
6	1.05 + j0.0	?	?	0.0	0.0

Table 3. Inertia constants and direct-axis transient reactances.

Bus Code p	Inertia Constant H	Direct-Axis Transient Reactances X'_d
3	2	0.4
4	3	0.5
5	1.5	1.0
6	20	0.02

Table 4. System pre-emergency state.

Bus Code p	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	0.9	0.99051	1.05	2.032	0.0	-0.3	-0.3	0.0	-0.13	-0.13
2	0.9	0.97409	1.05	0.0	0.0	-0.6	-0.6	0.0	-0.22	-0.22
3	1.0	1.06000	1.10	4.256	0.05	0.2	0.2	-0.4	0.2152	0.30
4	1.0	1.08000	1.10	9.320	0.05	0.3	0.3	-0.4	0.0372	0.40
5	1.0	1.05000	1.10	5.117	0.05	0.1	0.1	-0.4	0.0452	0.20
6	1.0	1.05000	1.10	4.503	0.15	0.347	0.45	-0.4	0.1690	0.40

Table 5. Optimal solution, $k_1 = k_2 = 1$.

Bus Code p	V	δ	C_{min}	C	C^*	D_{min}	D	D^*	P	Q
1	0.97854	1.030	0.0	0.2370	0.3	0.0	0.1027	0.13		
2	0.98750	0.0	0.0	0.4808	0.6	0.0	0.1763	0.22		
3	1.07293	3.894							0.20	0.2384
4	1.07608	8.542							0.30	0.0260
5	1.04102	3.965							0.10	0.1209
6	1.01641	2.911							0.15	-0.0143

Table 6. Different shedding strategies for 6-bus system.

Step Number	Amount Of Load Shed	Shedding time Case 1	Shedding time Case 2	Shedding time Case 3
1	0.018225	0.78	0.20	2.68
2	0.018225	1.62	0.40	2.70
3	0.018225	2.38	0.78	2.72
4	0.018225	3.16	1.20	2.74
5	0.018225	3.98	1.62	2.76
6	0.018225	4.74	2.00	2.78
7	0.018225	5.54	2.38	2.80
8	0.018225	6.34	2.76	2.82
9	0.018225	7.14	3.16	2.84
10	0.018225	7.90	3.58	2.86

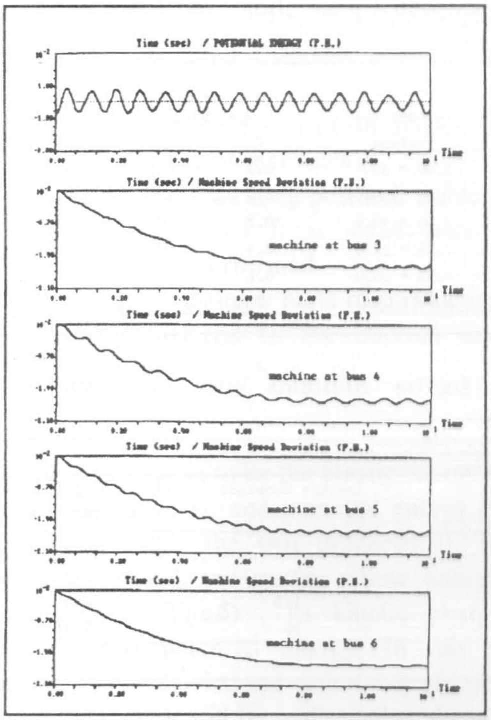

Figure 2a. Transient response for 6-bus system-case 1.

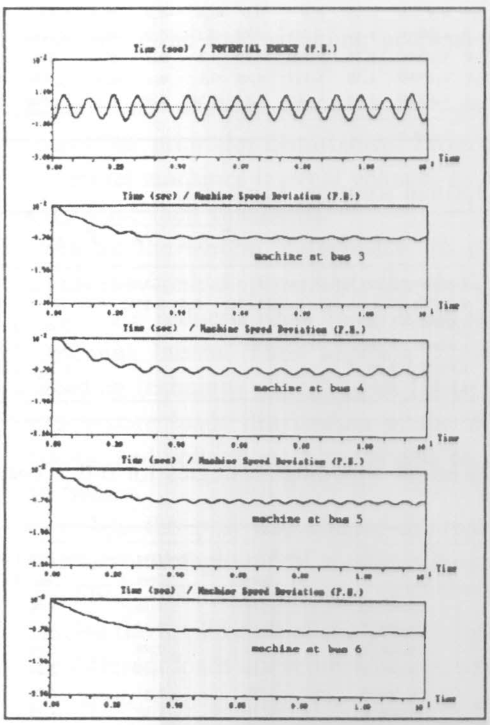

Figure 2b. Transient response for 6-bus system-case 2.

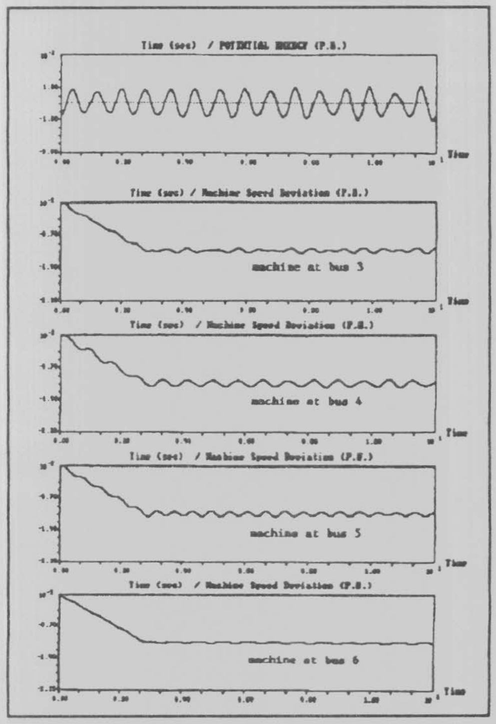

Figure 2c. Transient response for 6-bus system-case 3.

Figure (2) shows the transient response for the spe deviation from nominal system frequency (50Hz) f machines connected to busses 3,4,5 & 6 for the thr different strategies.

Note: Speed Deviation = Speed -Rated Speed /Rat Speed

B: 12-Bus system

The power system shown in Figure(3) is used as anoth test system. Tables 7,8 & 9 give the required data in p unit on a 100MVA base. Table (10) shows the state pri the emergency. Emergency is simulated by a reduction the available real generation at bus 12 from 2.4 p.u. to 1 p.u. Optimal load shedding is obtained while priorities a equal for all loads. Table (11) shows the solution for th case. Table (12) shows shedding steps and their respecti time in conjunction with the amount of load to be shed each step for the three strategies mentioned previousl The three strategies are performed in 10 steps where 10 of the load to be shed is shed at each instant.

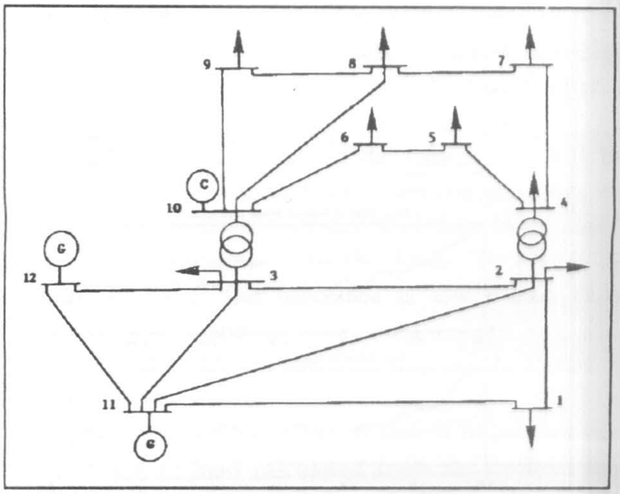

Figure 3. 12-bus system.

Table 7. Transmission line impedances and line charging.

Bus Code p-q	Impedance	Line Charging $y'_{pq}/2$
1-2	0.06701+j0.17103	0.0173
1-11	0.04699+j0.19797	0.0219
2-3*	0.01335+j0.04211	0.0064
2-4	0.00000+j0.55618	0.0
2-11*	0.05811+j0.17632	0.0187
3-10	0.00000+j0.25202	0.0
3-11	0.05695+j0.17388	0.0170
3-12	0.05403+j0.22304	0.0246
4-5	0.03181+j0.08450	0.0
4-7	0.12711+j0.27038	0.0
5-6	0.08205+j0.19207	0.0
6-10	0.09498+j0.19890	0.0
7-8	0.17093+j0.34802	0.0
8-9	0.22092+j0.19988	0.0
8-10	0.06615+j0.13027	0.0
9-10	0.12291+j0.25581	0.0
11-12	0.01938+j0.05917	0.0264

* Impedance of a transformer

Table 8. Scheduled generations and loads and assumed bus voltages.

Bus Code p	Assumed Bus Voltage	Generation P	Generation Q	Load C	Load D
1	1.0 + j0.0	0.0	0.0	0.942	0.190
2	1.0 + j0.0	0.0	0.0	0.478	0.039
3	1.0 + j0.0	0.0	0.0	0.293	0.143
4	1.0 + j0.0	0.0	0.0	0.295	0.166
5	1.0 + j0.0	0.0	0.0	0.090	0.058
6	1.0 + j0.0	0.0	0.0	0.147	0.093
7	1.0 + j0.0	0.0	0.0	0.050	0.050
8	1.0 + j0.0	0.0	0.0	0.135	0.058
9	1.0 + j0.0	0.0	0.0	0.061	0.016
10	1.07+ j0.0	0.0	?	0.0	0.0
11	1.045+j0.0	0.4	?	0.0	0.0
12	1.06+ j0.0	?	?	0.0	0.0

Table 9. Inertia constants and direct-axis transient reactances.

Bus Code p	Inertia Constant H	Direct-Axis Transient Reactances X'_d
10	0.25	1.0
11	2.00	0.5
12	20.00	0.1

Table 10. System pre-emergency state.

Bus Code p	V_{min}	V	V_{max}	δ	P_{min}	P	P_{max}	Q_{min}	Q	Q_{max}
1	0.9	0.98074	1.05	4.803	0.0	-0.942	-0.942	0.0	-0.190	-0.190
2	0.9	1.00568	1.05	6.943	0.0	-0.478	-0.478	0.0	-0.039	-0.039
3	0.9	1.01398	1.05	7.879	0.0	-0.293	-0.293	0.0	-0.143	-0.143
4	0.9	1.00575	1.05	-1.533	0.0	-0.295	-0.295	0.0	-0.166	-0.166
5	0.9	1.00458	1.05	-1.535	0.0	-0.090	-0.090	0.0	-0.058	-0.058
6	0.9	1.02040	1.05	-0.832	0.0	-0.147	-0.147	0.0	-0.093	-0.093
7	0.9	1.01137	1.05	-1.090	0.0	-0.050	-0.050	0.0	-0.050	-0.050
8	0.9	1.04470	1.05	-0.063	0.0	-0.135	-0.135	0.0	-0.058	-0.058
9	0.9	1.05244	1.05	0.000	0.0	-0.061	-0.061	0.0	-0.016	-0.016
10	1.0	1.07000	1.10	0.897	0.0	0.000	0.000	-0.4	0.566	0.800
11	1.0	1.04500	1.10	12.385	0.1	0.400	0.400	-0.4	0.487	0.800
12	1.0	1.06000	1.10	16.975	0.5	2.234	2.400	-0.4	-0.114	0.800

Table 11. Optimal solution, $k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = k_7 = k_8 = k_9 = 1$.

Bus Code p	V	δ	C_{min}	C	C^*	D_{min}	D	D^*	P	Q
1	0.97962	3.734	0.0	0.68504	0.942	0.0	0.13817	0.190		
2	1.00002	5.191	0.0	0.34997	0.478	0.0	0.02855	0.039		
3	1.00656	5.864	0.0	0.21302	0.293	0.0	0.10397	0.143		
4	1.02048	-1.178	0.0	0.21780	0.295	0.0	0.12256	0.166		
5	1.01843	-1.171	0.0	0.06639	0.090	0.0	0.04279	0.058		
6	1.02717	-0.635	0.0	0.10963	0.147	0.0	0.06936	0.093		
7	1.02198	-0.837	0.0	0.03678	0.050	0.0	0.03678	0.050		
8	1.04287	-0.056	0.0	0.10205	0.135	0.0	0.4385	0.058		
9	1.04801	0.000	0.0	0.04663	0.061	0.0	0.01223	0.016		
10	1.06071	0.686							0.0	0.437
11	1.01990	9.509							0.4	0.030
12	1.03609	12.551							1.5	-0.021

Table 12. Different shedding strategies for 12-bus system.

Step Number	Amount Of Load Shed	Shedding time Case 1	Shedding time Case 2	Shedding time Case 3
1	0.066368	0.26	0.10	0.62
2	0.066368	0.30	0.26	0.64
3	0.066368	0.38	0.38	0.66
4	0.066368	0.70	0.50	0.68
5	0.066368	1.16	0.70	0.70
6	0.066368	1.32	1.00	0.72
7	0.066368	1.64	1.16	0.74
8	0.066368	1.78	1.32	0.76
9	0.066368	2.06	1.64	0.78
10	0.066368	2.22	1.78	0.80

Figure (4) shows the transient response for the speed deviation from nominal system frequency (50Hz) for machines connected to busses 10,11 & 12 for the three different strategies mentioned previously.

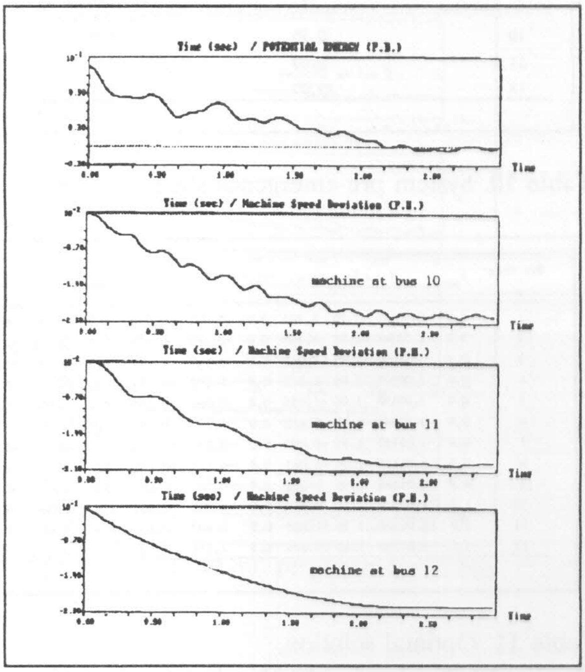

Figure 4a. Transient response for 12-bus system - case 1.

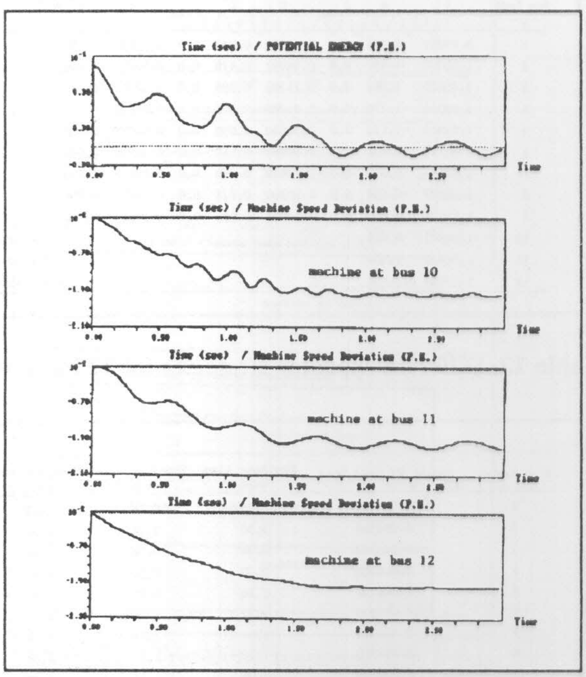

Figure 4b. Transient response for 12-bus system - case 2.

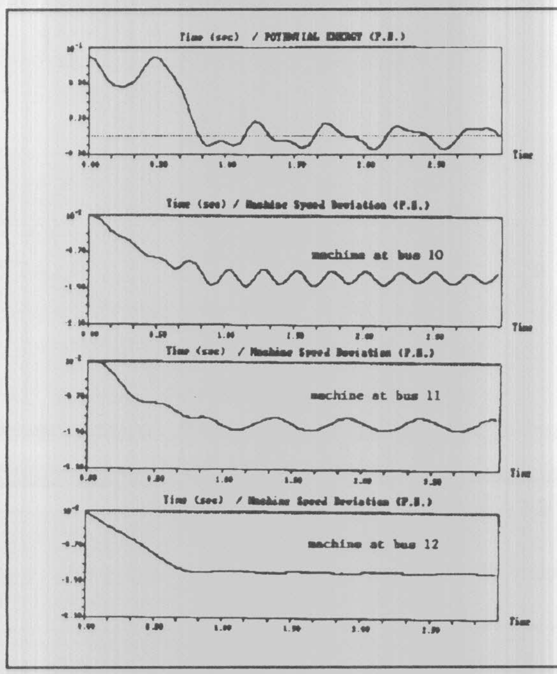

Figure 4c. Transient response for 12-bus system - case 3.

CONCLUSIONS

The transient response associated with the load sheddi process has been studied for two power systems; namel a 6-bus system and a 12-bus system. The differenti equations describing the synchronous machines respons have been solved using modified Euler technique fo numerical integration with the increment Δt = 0.0 second.

Figure (2) shows the transient response for the 6-bu system. Figure (2)-a represents the case where shedding are performed when potential energy function passe through local minimums. Figure (2)-b represents the ca where sheddings are performed according to a prespecifie instants. Figure (2)-c represents the case where shedding are performed when frequency goes below 49.5 Hz. Fo all cases the number of shedding steps are 10.

Comparing Figure (2)-a, Figure (2)-b and Figure (2)- show clearly that if excess loads are shed at instants whe the potential energy is at a local minimum, the oscillations in machines speed are kept at minimum. On would expect that Figure (2)-b be the case wher oscillations are minimum because of faster load sheddings but this was not the situation.

Figure (4) shows the transient response for the 12-bu system. Again the same conclusion is clear.

As regard to the time taken for the machines to reach constant speed: From Figure (2) and Figure (4) it is clear that there is no much difference in the time required to reach constant speed for different strategies.

Since longer time was taken for case 1 to shed the necessary amount of excess loads, then the machines will have more time to decelerate and therefore more kinetic energy is being transferred to the loads. Therefore the post fault speed of the machines is lower than those obtained in cases 2 and 3.

From the previous study the following points should be emphasized:

1. The amount of load to be shed should be greater than the amount of load calculated from the optimal load shedding solution in order to raise the system frequency to its nominal value. After the system rises above the nominal value another load restoration process should be incorporated to reach the nominal value.
2. In order to keep the system stable another action should be considered. This is the exciter action. The internal emf's for synchronous machines should be changed to achieve a stable operation.
3. The best instants for load sheddings are where the system potential energy passes through local minimums. Therefore it is recommended to shed excess loads at these instants.

APPENDIX (A)

Problem Formulation

(a): Optimal Load Shedding Problem

Consider a system of N busses with N_L pure load busses, and $N_G = N - N_L$ pure generator busses. Then the problem is:

Minimize the scalar function

$$F(\underline{V},\underline{\delta}) = \sum_{i \in S} [C_i^* - C_i(\underline{V},\underline{\delta})]^2 / 2k_i C_i^*$$

Subject to

$$D_i(\underline{V},\underline{\delta}) - \gamma_i C_i(\underline{V},\underline{\delta}) = 0 \qquad i = 1,2,\ldots,N_L \tag{2}$$

$$P_i(\underline{V},\underline{\delta}) - P_i = 0 \qquad i = N_L+1,\ldots,N \tag{3}$$

$$C_i^m \le C_i(\underline{V},\underline{\delta}) \le C_i^* \qquad i = 1,2,\ldots,N_L \tag{4}$$

$$D_i^m \le D_i(\underline{V},\underline{\delta}) \le D_i^* \qquad i = 1,2,\ldots,N_L \tag{5}$$

$$Q_i^m \le Q_i(\underline{V},\underline{\delta}) \le Q_i^M \qquad i = N_L+1,\ldots,N \tag{6}$$

$$V_i^m \le V_i \le V_i^M \qquad i = 1,2,\ldots,N \tag{7}$$

where,

S : is the set of load busses

k_i : is the priority factor assigned to that load

$$C_i(\underline{V},\underline{\delta}) = -V_i \sum_{j=1}^{N_G} V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij})$$

$$D_i(\underline{V},\underline{\delta}) = -V_i \sum_{j=1}^{N_G} V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij})$$

$$P_i(\underline{V},\underline{\delta}) = V_i \sum_{j=1}^{N_G} V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij})$$

$V_i \epsilon^{j\delta_i}$: is the voltage at bus i

$[\, Y_{ij} \epsilon^{j\theta_{ij}} \,]$: is the bus admittance matrix of the post emergency system

$\gamma_i = \tan(-\phi_i)$, where ϕ_i is the power factor angle at bus i

C_i^* & D_i^* : are active & reactive demands at load bus i prior to emergency respectively

C_i^m & D_i^m : are the minimum real & reactive demands at load bus i respectively

Q_i^m & Q_i^M : are the minimum & maximum reactive generations at generator bus i respectively

V_i^m & V_i^M : are the minimum & maximum limits of the voltage magnitude at bus i load or generator, respectively.

The solution of this problem has been described in detail in reference [3].

(b): Stability Consideration

In transient stability studies, a synchronous machine may be represented by a voltage source in back of transient reactance. With this representation for a synchronous machine, two first order differential equations are solved to obtain the change in the internal voltage angle δ and machine speed ω. Thus for a system with N_G generator busses:

$$d\delta_i/dt = \omega_i - 2\pi f \qquad i=N_L+1,\dots, N \qquad (8)$$

$$d\omega_i/dt = (\pi f/H_i)(P_{mi} - P_{ei}) \qquad i=N_L+1,\dots, N \qquad (9)$$

where P_{ei} is the electrical air gap power for machine at bus i.

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