MATHEMATICAL MODELING OF LOSS OF-FLOW ACCIDENT (LOFA) IN NUCLEAR POWER PLANTS

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ABSTRACT

This study presents a mathematical lumped model for loss of flow accident (LOFA) which encounters both pressure and temperature feedbacks. The main features of this model are: * The coolant mass flow rate (m°) is expressed in an analytical form during LOFA and following recovery. * The heat transfer coefficient (h) selection is based on the coolant temperature and velocity. The model was tested successfully as applied to a hypothetical accident and the results showed a good stability of PWRs against LOFA.

INTRODUCTION

Most published literatures on the analysis of LOFA and its consequences are based on experimental findings. The present work analyses this problem through a mathematical model that simulates the reactory by a number on nonlinear differential equations representing the reactor point kinetics (one equation), the delayed neutron group concentrations (up to six equations), and two equations that deal with thermal and hydraulic balances. Many computer codes are available to solve these equations numerically such as SAS-3D(1), NATDEMO(2), and HOTCHAN(2).

MATHEMATICAL MODEL

The present model utilizes a semi-numerical technique for solving all governing equations simulating the reactor. A mathematical form will be assigned to m° during the course of LOFA and following the recovery.

Basic Assumptions

i. Lumped parameter model is an adaquate representation of the core, only one node is used for the fuel and one for the coolant.

ii. The coolant inlet temperature will be considered constant for the simplicity of solution, while in fact it must be a complex function of time.

iii. The cladding temperature will not be treated explicity but the cladding effect will be lumped with fuel and will be taken into acount in the overall heat transfer coefficient between fuel and coolant.

iv. All engineered safety feature systems (ESFs) are assumed to be in a failed state. This will be the case until the coolant bulk temperature reaches certain limit, after which the ESFs will start to recover m°.

Reactor Kinetics and Delayed Neutrons Concentrations

$$\frac{dp(t)}{dt} = \frac{\rho(t)-\beta}{\Lambda} P(t) + \sum_{i=1}^{N} \lambda_i Q_i(t) \tag{1}$$

$$\frac{dQ_i(t)}{dt} = \frac{\beta_i}{\Lambda} P(t) - \lambda_i Q_i(t); \; i = 1,2,\ldots,N \tag{2}$$

where:

$Q_i(t)$ = Power equivelant of the i'th delayed neutron precursor group.
Λ = neutron generation time.
β_i = fraction of delayed neutrons by the i'th group.
N = total number of delayed neutron groups used.
λ_i = decay constant of the i'th precursor.
$\rho(t)$ = total inserted reactivity (both external and feedbacks), the feedback reactivity is the sum of fuel and coolant temperature feedbacks and pressure feedback. So,

$\rho(t)$ can be expressed as:

$$\rho(t) = \rho_{ex}(t) + \alpha_f(T_f)\,\Delta T_f + \alpha_m\,\Delta\,T_m(t) + \alpha_{pr}\,\Delta Pr(t) \tag{3}$$

where:

α_f = Doppler coefficient.
α_m = moderator feedback coefficient of reactivity.
α_{pr} = pressure feedback coefficient of reactivity.
T_f and T_m = fuel and moderator temperatures respectively.

Thermal Hydraulic Balance Equations

$$M_f C_{pf} = \frac{dT_f(t)}{dt} = \eta P(t) - \mu_{fm}(T_f - T_m) \qquad (4)$$

$$M_m C_{pm} = \frac{dT_m(t)}{dt} \quad \mu_{fm}(T_f - T_m) - m^\circ(t)(T_2 - T_1) \qquad (5)$$

where:
T_2; T_1 = coolant outlet and inlet temperatures respectively.
μ_{fm} = overall heat transfer coefficient between fuel and coolant.

T_m = average coolant temperature = $\frac{T_1 + T_2}{2}$

η = fraction of power generated in the fuel.
M_f and M_m = mass of fuel and moderator respectively
C_{pf} and C_{pm} = specific heat of fuel and moderator respectively

Mathematical Treatment

Equation from (1) to (5) can be put in a matrix form and then solved using the modified Hansen's method: [3].

According to the flow chart represented by Figure (1), the procedure allows for:

i. The appropiate value for heat transfer coefficient (h), time step (τ) and all physical parameters are selected according to the computed value of T_f and T_m.
ii. The analytical form of m° is selected based on the comparison of the computed value of T_m with T_{sat}.
iii. There is a limiting time t_L beyond which all computations are terminated.

Analytical Representation of m°

During the course of LOFA, m° will be represented by

$$m^\circ(t) = m^\circ\,[(1-\alpha) + \alpha e^{-\gamma t}] \qquad (6)$$

where
m° = initial mass flow rate
α and γ = constants determining the severity of LOFA

Through certain time step, m° is assumed constant. This constant value is calculated by averaging m° over the time interval. During the Kth time step, m° will take the value.

$$m^\circ_k = m_o^\circ\;(1-\alpha)\tau + \left\{ \frac{\alpha}{\tau\gamma}\; e^{-\gamma K\tau} \qquad 1-e^{-\gamma\tau} \right\}\left\{ \quad \right\} \qquad (7)$$

when $T_m \geq T_{sat}$, the recovery of m° is assumed to take the form

$$m^\circ(t) = m_1^\circ\,[1-e^{-\gamma_2(t-\theta)}] \qquad (8)$$

where
m_1° = value of m° at which ESF's begins to respond.
θ = time period after which flow recovery will occur.
γ_2 = parameter of flow recovery.

As stated before, m(t) will be taken as average value over the time interval, during the Kth time step (after recovery), m is computed from

$$m_k^\circ = m^\circ_1\,[\,1 - \frac{e^{-k\tau\gamma_2}}{\gamma_2\tau} \qquad (1-e^{-\gamma_2\tau})] \qquad (9)$$

For all cases studied γ_2 must be larger than γ

RESULTS

The model was tested using a typical data from the US-Yankee-PWR (4). Just before LOFA, the initial conditions are:

Thermal power = 485 MW,
m°_o = 41.9 x 10^6 lbm/hr
T_2 = 533 °F,
T_1 = 496 °F,
Operating pressure = 2000 psia, and

$$T_{sat} = 649\ ^\circ F \qquad (10)$$

The time behavior of m°, thermal power (P_{th}) and coolant outlet temperature (T_2) are plotted during the course of LOFA and after recovery. The plots are shown in Figure (1) respectively. The results illustrated by these figures are consistent and can be easily interpretted as:

- During the course of LOFA, P_{th} will decrease (mainly due to loss of flow), and T_2 will increase.
- Following recovery, P_{th} will increase and T_2 will fall off.

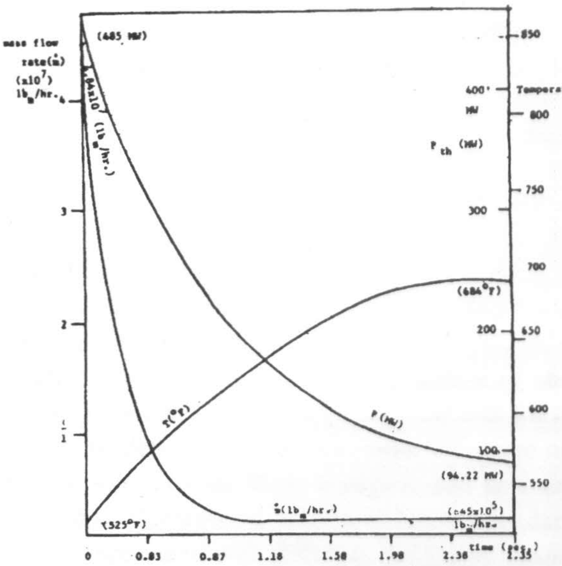
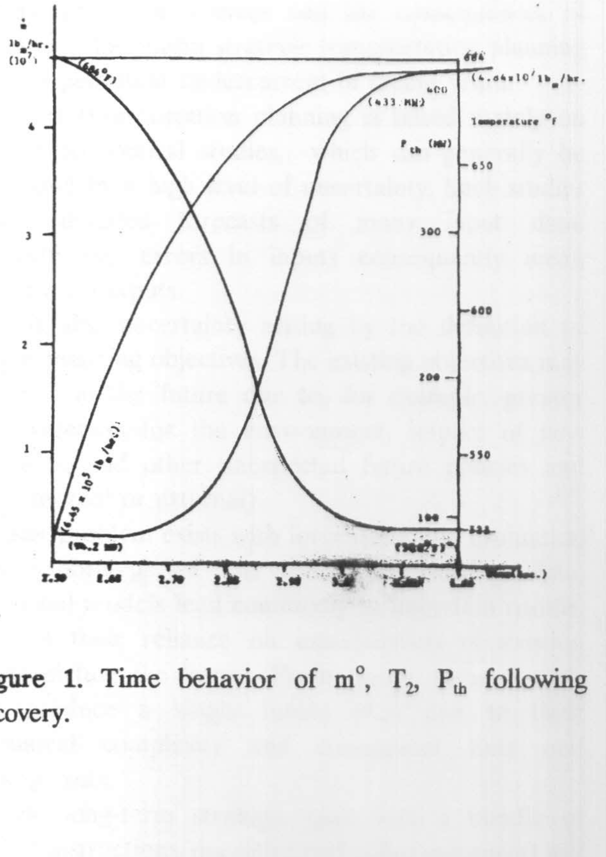


Figure 1 a.



b.

Figure 1. Time behavior of m°, T_2, P_{th} following recovery.

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