# ESTIMATION OF THE STROUHAL NUMBER FOR THE FLOW THROUGH THE AORTIC VALVE, VALIDATION OF THE DOPPLER DERIVED PRESSURE GRADIENT

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## ABSTRACT

The strouhal number (St), which is the ratio between the local and the convective acceleration terms for unsteady flow through variable cross-sectional areas was estimated in 19 patients with aortic stenosis according to the equation :

$$St = \frac{\text{maximum velocity } (V_{max})}{(\text{mean velocity } (V_{mean}))^2} \times \frac{2 \text{ orifice diameter } (D)}{\text{acceleration time}(t_{acc})}$$

The values of $V_{max}$, $V_{mean}$ and $t_{acc}$ were evaluated by Doppler measurements. The orifice diameter (D) was calculated from the equation :

$$D = (4 \times \text{stroke volume} / \pi \times \text{velocity time integral})^{1/2}$$

where the stroke volume was obtained by 2-Dimensional measurements. The estimated St ranged from 0.022 to 0.156 (mean 0.06 ± 0.036). Accordingly the equation $\Delta P = 4\ V^2$ accurately predicts the pressure gradient assuming perfect alighnment between the Doppler beam and the direction of the flow. The pulsatility index $V_{max}/V_{mean}$ had a mean value of 1.34 ± 0.12 for patients with stenotic valves compared to 1.71 ± 0.16 for normal subjects. This parameter may be used to separate between subjects with increased flow without valvular obstruction e.g. hyperdynamic states, and those with mild aortic stenosis.

## INTRODUCTION

Assessment of severity of valvular stenosis is of great clinical importance because flow obstruction can be life threatening. In a stenotic valve, stiffening of the cusps restricts their movement thus reducing the valve area. The methods of assessing the severity of the stenosis are based upon the hemodynamic changes that follow this reduction of the orifice area. Because of the resistance to ejection the pressure in the left ventricle rises sometimes as high as 350 mmHg while the pressure in the aorta is still normal [1]. The cathetarization technique categorizes the degree of constriction according to the pressure drop across the stenotic valve. To estimate the valve area Gorlin's equation, which is basically the hydraulic formula for the flow through a converging nozzle or an orifice under steady state conditions with an empirical coefficient, is applied [2].

With the blood jetting at a tremendous velocity through the small opening of the valve a turbulent flow field is created and is associated with the generation of murmur which is audible through a stethoscope during the systolic phase of the cardiac cycle. The hope of developing a non-invasive diagnostic tool capable of determining the severity of the stenosis has promoted a great amount of work to correlate between the frequency spectra of the murmur and the degree of constriction [3], [4].

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*Presented at the First Pan Arab Congress of Cardiology & the 17th Annual Meeting of The Egyptian Society of Cardiology, Feb 1990, Cairo Egypt.*

Recently two dimensions Doppler echocardiography offered a new reliable non-invasive technique for evaluation of valvular stenosis based upon the measurement of blood velocity in the cardiac chamber and the great blood vessels [5], [6], [7]. By applying the steady Bernoulli's equation between two points upstream and downstream the valve and by considering the upstream velocity to be neglegible compared to the jet velocity, the instantaneous pressure difference ($\Delta P$) is evaluated by :

$$\Delta P = 4\ V^2 \ \text{mmHg}$$

where V is the instantaneous jet velocity in meters per second. This simple equation may however leads to an overestimated pressure gradient in cases of increased flow due to hyperdynamic states even in the absence of valvular constriction or to an underestimated pressure gradient in cases of myocardial dysfunction despite the presence of a stenosis [8], [9], [10].

Considering the nature of the problem a complete analytical solution present a tremendous challenge to theoreticians. The momentum equation for the flow through the valve is non-linear and the flow field downstream is that of a pulsatile (non-sinosoidal) turbulent bounded jet. During the acceleration phase turbulent flow accelerates through a convergence, seperates under the influence of an adverse pressure gradient, undergoes intense turbulent fluactuations, reattaches and eventually returns to a condition of fully developed pipe flow further downstream of the nozzle exit. Because of the complexity of the problem a more practicle approach is to extract as much information as possible from the measured velocity curve by the widely accepted Doppler echocardiography technique to improve the specificity of the clinical test .

This study was undertaken to further assess (1) the validity of the underlying assumptions and approximations in calculating the Doppler derived pressure gradient. (2) the applicability of the pulsatility index as a non-dimensional parameter that may characterizes the velocity signal.

## THEORETICAL CONSIDERATION

The momentum equation govening the pulsatile flow between point 1 and point 2 in figure (1) assuming the flow to be mainly in the longitudinal direction X is

$$\rho\, \partial V/\partial t + \rho V \partial V/\partial x = -\, \partial P/\partial x + 4\, \tau/d \qquad (1)$$

where $\rho$ is the density of fluid, d the cross sectional diameter, v, p, and $\tau$ are the instantaneous velocity, pressure and viscous stress at the wall respectively.i.e. the pressure gradient = momentum change due to local acceleration ($\rho \partial V/\partial t$) + momentum change due to convective acceleration ($\rho V \partial V/\partial x$) + friction force.

Because of the pulsatile nature of the flow, the shear stress ($\tau$) may be expressed as [11] .

$$\tau = (\pi \rho \mu \ / \ t_s)^{1/2}\ V$$

where $\mu$ is the blood viscosity and $t_s$ is the systolic ejection time. Accordingly the ratio between the friction term and the acceleration terms is of order of $10^{-3}$ and may be assumed to have an irrelevant contribution to the pressure gradient.

In order to compare the relative contribution of the local acceleration and convective acceleration terms, order of magnitude analysis is applied as follows .

$$\frac{\textbf{local acceleration term}}{\textbf{convective acceleration term}} = \frac{\rho V_{max}}{t_{acc}} \ \frac{L}{\rho V^2_{mean}} = \frac{V_{max}}{V_{mean}} \ \frac{L}{V_{mean} t_{(acc)}} \qquad (2)$$

where $t_{acc}$ is the time for the velocity to attain its maximum value $V_{max}$, and $V_{mean}$ is the time mean velocty during systole. The length scale L is the longitudinal distance over which the velocity changes significantly. It is reasonable to approximate L by twice the orifice diameter since even with severe stenosis where the mobility of the cusps is reduced the flow will actually starts to converge proximal to the aortic root in the left ventricle outflow tract.

If this ratio (Strouhal number (St)) is of order 1, the unsteadiness term is of the same importance as the convective acceleration term in determining the pressure gradient. If St is much smaller than 1 then the local acceleration term is negligible and the equation may be reduced to

$$-\, \partial P \ / \ \partial x = \rho\ V\ \partial V \ / \ \partial x$$

which upon integration over the distance x yields

$$P_1 - P_2 = 4 V^2 \quad \text{mmHg} \qquad (3)$$

In an in-vivo animal experimental study by Clark [12] the contribution of each term in equation (1) to the pressure gradient across the aortic valve were quantitatively evaluated for both normal conditions, St = 2.58, and for simulated aortic stenosis, St = 0.052. His results showed that the main contribution to the pressure difference comes from the local acceleration term for St = 2.58 and from the convective acceleration term for St = 0.052. Between this two limits we may expect an increasing role of the local acceleration term with decreasing the severity of the stenosis.

The St expressed in equation (2) suggests that the ratio $V_{max} / V_{mean}$ i.e. the pulsatility index, may provide an extra non-dimensional parameter for the diagnosis and assessment of aortic stenosis irrespective of the stroke volume. If this ratio does not change significantly with the presense of stenotic valve, then the St may be simplified to $St = L / V_{mean} t_{acc}$.

## DATA COLLECTION AND ANALYSIS

### *Patients population*

Two patient populatios were studied (a) 15 normal volunteers with no prior history of heart diseases, hypertension, murmurs or major medical problems and (b) 19 clinical patients with clinical ECG and x ray diagnosis of aortic stenosis. Only thirteen underwent cardiac catheterization. All patients were prospectively selected. Eleven of the 19 patients had bicuspid aortic valve while the other eight had rheumatic aortic stenosis.

### *Equipments*

Echocardiographic studies were performed with a combined two dimensional and Doppler echocordiographic system (OTTI 5000). Imaging transducer frequencies were between 2.5 and 3.5 MHz and the Doppler tranducer frequency was 2.0 MHz. The freezed frames were printed on a Mitsubishi video printer. Two-dimensinal and Doppler Echocardiography examinations were obtained from standard parasternal and apical views to clarify the diagnosis of aortic stenosis. A standard pulsed wave and continuous wave Doppler study were done on all subjects. All windows, right parasternal apical and suprasternal notch were attempted with multiple sampling sites at each position. Guided by the 2-D images and the audible signal, great care was taken to align the Doppler beam in the direction of the flow and to locate the position of the highest audible frequency and the maximum velocity signal with clear defined spectral envelope.

### *Signal analysis*

Utilizing the build in soft ware package, the velocity signal were analyzed to obtain the maximum velocity $V_{max}$), time to maximum velocity (acceleration time $t_{acc}$), systolic ejection time ($t_s$) and maximum pressure gradient according to equation (3). Digitizing the velocity signal and integrating with respect to time., the mean velocity and the mean pressure gradient were obtained as shown in figure (1). The orifice diameter was calculatd from the equation:

$$D = (4 \times \text{stroke volume} / \pi \times \text{velocity time integral})^{1/2}$$

where the stroke volume was estimated from 2-dimensional measurements by applying Baran's equation [13] for left ventricular volume in both systole and diastole.

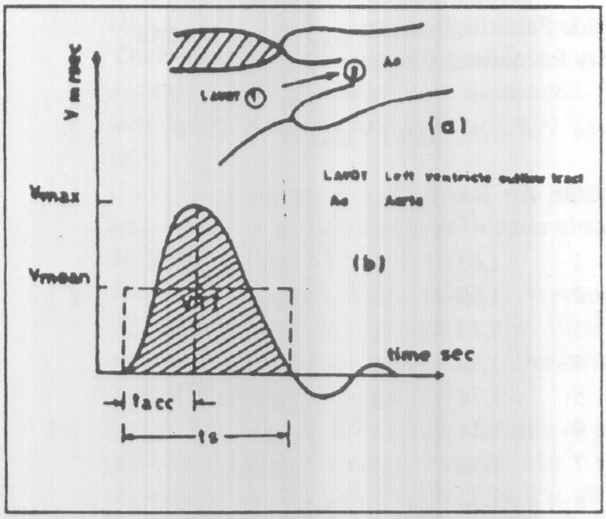

Figure 1. a) Schematics showing the left ventriclar outflow tract in the theoretical model. b) Aortic velocity signal showing flow parameters.

## RESULTS AND DISCUSSION

The strouhal number calculated according to equation (2) for patients with aortic stenosis had values much smaller than one ranging from 0.022 to 0.156 with a mean of 0.06 + 0.036. The 0.95 confidence interval for the mean is from 0.0416 to 0.078. The results are tabulated in table (1).

**Table 1.** Strouhal number for patients with aortic stenosis($\Delta P_{max}$ ranged from 35-179 mmHg mean (87.66 ± 41.4).

Strouhal number

| Case # | St | Case # | St |
|---|---|---|---|
| 1 | 0.042 | 11 | 0.022 |
| 2 | 0.079 | 12 | 0.055 |
| 3 | 0.144 | 13 | 4.040 |
| 4 | 0.049 | 14 | 0.051 |
| 5 | 0.064 | 15 | 0.030 |
| 6 | 0.056 | 16 | 0.056 |
| 7 | 0.156 | 17 | 0.030 |
| 8 | 0.080 | 18 | 0.036 |
| 9 | 0.033 | 19 | 0.064 |
| 10 | 0.046 | | |

Table 2. Pulsatility index $V_{max}/V_{mean}$ of aortic velocity for normal subjects.

$$P.I = V_{max} / V_{mean}$$

| Case # | P.I. | Case # | P.I. |
|---|---|---|---|
| 1 | 1.63 | 9 | 1.9 |
| 2 | 1.69 | 10 | 1.79 |
| 3 | 1.73 | 11 | 1.67 |
| 4 | 1.55 | 12 | 1.75 |
| 5 | 1.74 | 13 | 1.87 |
| 6 | 1.44 | 14 | 1.68 |
| 7 | 1.98 | 15 | 1.70 |
| 8 | 1.52 | | |

These results imply that the convective acceleration term is the major contributor to the pressure drop across stenotic valves and that the clinical simple formula $\Delta P = 4V^2$ accurately predicts the pressure drop providing good alighnment between the Doppler beam and the direction of the flow.

However it must be realized that the calculated pressure drop is derived from the velocity signal which is a function of the cardiac output. In hyperkinetic states the pressure drop may be over estimated. A 25 % increase in the cardiac output may results in 50 % over estimation of the pressure gradient. Peak pressure gradient derived from carachterization data in such cases would also show moderate pressure gradient even in the absence of valvular constriction. The strouhal number in these cases is of order of magnitude of one and the local acceleration term plays a more significant role in determining the pressure gradient as for case # 13 in table (1) who have high cardiac output due to active hyperthyroidism.

In order to increase the specificity of the Doppler examination it may be helpful to seek for another parameter that characterizes the velocity signal in the presence of a constriction irrespective of the stroke volume. By referring to the classical instantaneous pressure tracing of the left ventricle and the aorta for normal subjects, where the local acceleration term is the major contributor to the pressure gradient, it is clear that the pressure gradient is positive only during the early ejection phase. For the rest of the systole it has a negative value. This trend has been confirmed by both in-vitro, in-vivo and in patients data [12 - 14]. This adverse pressure gradient causes a rapid deceleration of the flow and the velocity signal is close to a triangular shape with early peak and rapid deceleration.

On the other hand it is known that in cases of constricted aortic valves the pressure gradient is positive during the whole systolic phase [15], [16]. The main contributor to the pressure drop is the convective acceleration term [12]. The flow is closer to quasi-steady flow and the velocity is expected to follow more closely the instantaneous pressure gradient. The pulsatility index $V_{max}/V_{mean}$ is a non-dimensional parameter which quantitatively differentiate between the two velocity signals.

Example of the velocity signals as measured in the ascending aorta just distal to the aortic valve for a patient with aortic stenosis and for a normal subjects are shown in figure (2), (3) respectively. The pulsatlity index is plotted versus the pressure gradient in figure (4) for patients with aortic stenoses. The mean value is 1.34 ± 0.12. The mean value for $V_{max}/V_{mean}$ for normal subjects is 1.7 ± 0.16 as shown in Table (2) which is significantly higher than in cases of aortic stenoses.

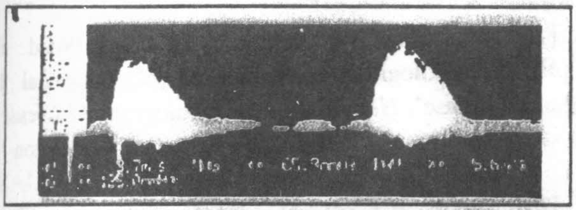

Figure 2. Continuous wave Doppler recording of the aortic velocity in a patient with aortic stenosis.

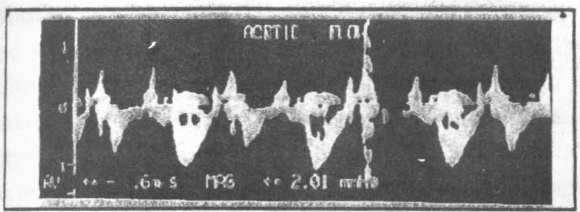

Figure 3. Pulsed wave Doppler recording of the aortic velocity in a normal subject.

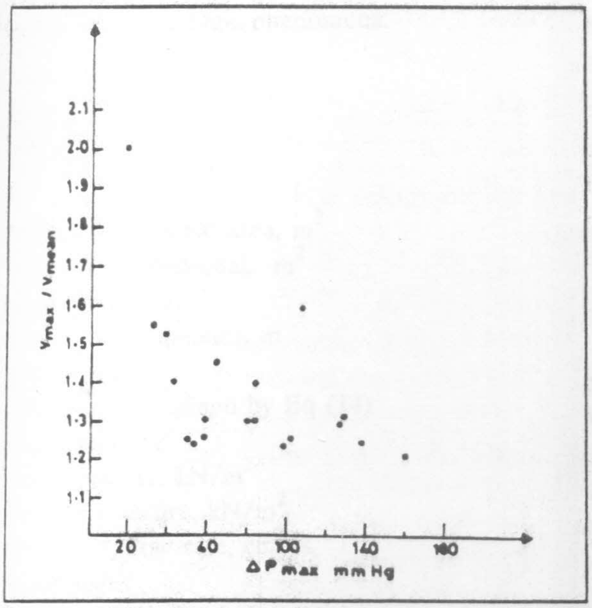

Figure 4. Pulsatility index ($V_{max}$/ $V_{mean}$) against the Dopplet derived pressure gradient for patients with aortic stenosis.

## CONCLUSION

1. The strouhal number for the flow through stenotic aortic valves is much smaller than one. This implies that the convective acceleration term is the major contributor to the pressure gradient.
2. In the absence of a constriction, the local acceleration term plays a more significant role and the velocity curve is characterized by an earlier rise to attain its maximum value and a more rapid decceleration thus giving a higher value of $V_{max}/V_{mean}$ than in cases of aortic stenoses. This parameter may be used to separate between subjects with hyperdynamic states and those with mild aortic stenoses.

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