EFFECT OF THE INNER SCALE SIZE OF ATMOSPHERIC TURBULENCE ON THE FLUCTUATIONS OF LASER INTENSITY

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ABSTRACT

The parameters relating to the inner scale size of atmospheric turbulence are carefully studied. These parameters are the height above the ground level, the wind speed, and the stability parameter. The effect of the inner scale size on both the average intensity and the second irradiance moment of laser radiation is also investigated for both the plane and spherical waves. The effect of the emitting aperture size on the average intensity is studied by introducing different values of the effective beam radii.

INTRODUCTION

The intensity fluctuations of a light wave propagated in a turbulent medium have been investigated for more than two decades. The effect of variations in the refractive index of turbulent atmosphere on optical waves, like laser waves, propagated through it, is well characterized if the intensity fluctuations are weak, i.e., if the intensity variance normalized by the mean intensity < 0.3 [1].

The inner scale size parameter plays a major role in the turbulence phenomenon and its consequences on light propagation. The fluctuating inner scale obviously leads to fluctuating irradiance moments. Generally, a finite value of the inner scale can lead to larger values of the second irradiance moment (or scintillation index) for both the plane and spherical wave propagations.

The changing atmospheric conditions in the propagation path cause continuous fluctuations in the inner scale size, which directly affect the laser intensity through fluctuations in the refractive-index structure constant. At lower wind velocities the turbulence itself is unstable. The near-ground atmosphere is more statistically stationary at wind velocities higher than 2 m /s [2].

In this paper we perform some theoretical analysis for the effect of a finite inner scale on laser intensity or log-intensity variance, for short path lengths and weak turbulence conditions. The fluctuations of the refractive-index structure constant are ignored. It is assumed unchanged for the same height above ground level or heights less than 1 km. Another purpose of this paper is to study the effect of the emitting aperture size by introducing different effective beam radii in the analysis.

In Sec. II the factors relating to the inner scale size of atmospheric turbulence are studied. Section III deals with the apparent refractive-index structure constant at a finite inner scale size from which the normalized variance of the log-intensity is obtained. The effect of the inner scale size, on the average intensity and second irradiance moment, for both the plane wave and spherical wave, is investigated in Sec. IV. Section V is devoted to the effect of the emitting aperture size on the normalized average intensity of the plane wave.

INNER SCALE SIZE IN STABLE AND UNSTABLE CONDITIONS

The inner scale size 1_o is related to the rate of kinetic energy dissipation $\in$ by the following relationship [3]

$$1_o = 7.4\ (\upsilon^3 / \in)^{1/4}\ ,\ \mathrm{m} \tag{1}$$

where υ is the kinematic viscosity of air. $\upsilon \simeq 0.16 \times 10^{-4}$ $m^2\ s^{-1}$.

The dependence of $\in$ on the scale length of wind speed u_* has the form [4]

$$\in = \frac{u_*^3}{kh}\,\phi_\epsilon(\xi), \tag{2}$$

where $\phi_\in(\xi) = [1+0.46(-\xi)^{2/3}]^{3/2} \qquad -2\leq\xi\leq 0$

$$= [1+2.3\ \xi^{3/5}\ ]^{3/2} \qquad -\ 0\leq\xi\leq 2 \tag{3}$$

Here, k is the von Karman's constant (0.4), h is the height above the ground level, and the parameter ξ is known as the stability parameter.

The scale length of wind speed is related to the wind speed U by the following equation

$$u_* = \frac{k\overline{U}}{\ln(h/h_r) - \psi_m(\xi)}$$

where

$$\Psi_m(\xi) = 2\ \ln[(1+x)/2] + \ln[(1+x^2)/2] - \arctan(x) + \pi/2, \quad (5)$$

$$x = (1-16\xi)^{1/4} \qquad (\xi<0)\ \text{[unstable conditions]} \quad (6)$$

$$\Psi m(\xi) = -7\xi \qquad (\xi>0)\ \text{[stable conditions]} \quad (7)$$

As the roughness height of wind speed h_r has generally the values 2-4 cm, we use h_r =2 cm in our calculations.

In figure (1) we have plotted the inner scale size, for different values of wind speed, as a function of the stability parameter. It is clear that under stable conditions ($\xi>0$) the inner scale size takes larger values than under unstable conditions ($\xi<0$) for the same wind speed.

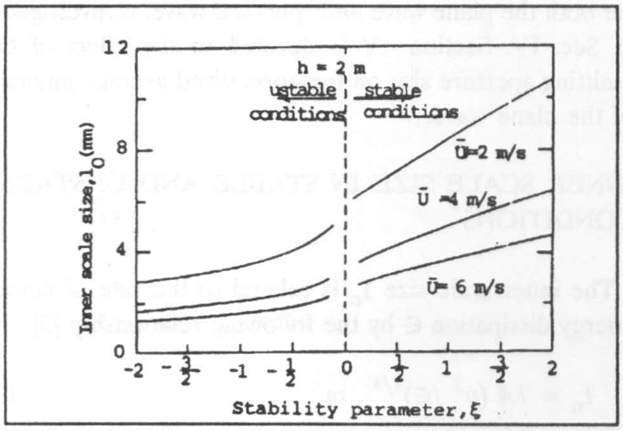

Figure 1. Variations of the inner scale size with the stability parameter.

Figure (2) shows the effect of height on the inner scale size for both the stable and unstable conditions. It is quantitatively seen that there is a general dependence of the inner scale size on both the height and the stability level.

NORMALIZED VARIANCE OF THE LOG-INTENSITY

The refractive-index structure constant is related to the inner scale size according to [5] by:

$$C^2_{na}(l_o) = C^2_n(0)\ \hat{\sigma}^2(l_o), \quad (8)$$

where $C^2_n(0)$ is the refractive-index structure constant at

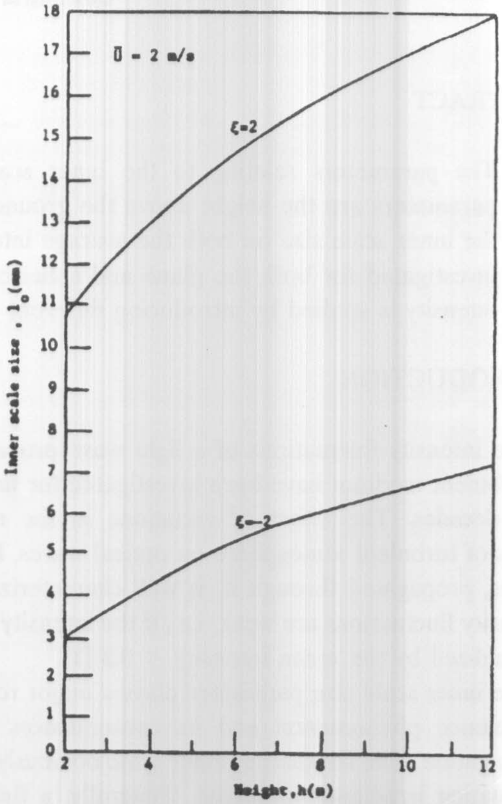

Figure 2. Variations of the inner scale size with the height.

zero inner scale size, $C^2_{na}(l_o)$ is the apparent refractive-index structure constant at a finite inner scale size, and $\hat{\sigma}^2(l_o)$ is the inner scale correction factor which is given by

$$\hat{\sigma}^2(l_o) = 3.864\left[1+\frac{1}{31.14\,\beta^2}\right]^{11/12} \times \sin\left[\frac{11}{6}\tan^{-1}(5.58\beta)\right] - \frac{1.69}{\beta^{5/6}} \quad (9)$$

The parameter β is the square of the ration of the size of the first Fresnel zone to inner scale size, i.e.,

$$\beta = \frac{\lambda L}{l_o^2} \quad (10)$$

where λ is the laser wavelength and L is the length of the propagation path. We use laser diode of wavelength (0.8486 μm), and L = 500 m.

The variance of the log-intensity in weak fluctuations at zero inner scale is given by [6]

$$\sigma^2_{\ln I}(0) = 1.23 C^2_n(0) K^{7/6} L^{11/6}, \quad (11)$$

where K is the wavenumber of the laser wave. $C^2_n(0)$ is taken as 10^{-15} $m^{-2/3}$ for all heights considered in our analysis.

The variance of the log-intensity at a finite inner scale size is simply [5]

$$\sigma^2_{\ln I}(l_o) = 1.23\, C^2_{na}(l_o) K^{7/6} L^{11/6}. \quad (12)$$

Using equations (8), (11), and (12) we deduce the normalized variance of the log-intensity to be

$$\sigma^2_{\ln I}(l_o)/\sigma^2_{\ln I}(0) = \hat{\sigma}^2(l_o). \quad (13)$$

Figure (3) shows the variations of the normalized variance of the log-intensity with the inner scale size. It is obvious that the log-intensity variance decreases with a large rate for inner scale sizes more than 5 mm.

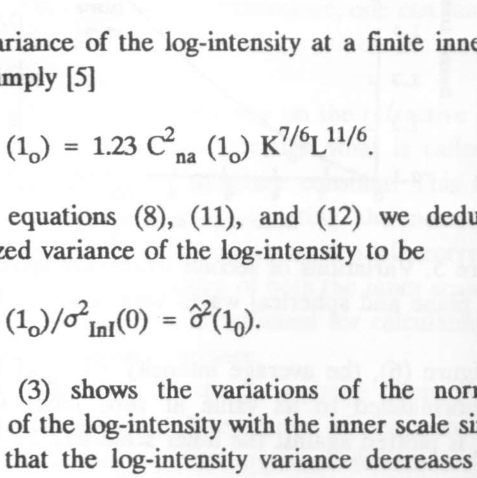

Figure 3. Variations of the normalized variance of the log-intensity with the inner scale size.

AVERAGE INTENSITY AND SECOND IRRADIANCE MOMENT

The following approximation has been proposed for the average intensity at the beam center [7]

$$<I(0)> = |U_o^2| \frac{a^2}{\rho_e^2(L)}, \quad (14)$$

where U_o is the laser field amplitude, a is the effective beam radius, and ρ_e (L) is the effective width of the beam, which in the event of focusing of radiation is given by

$$\rho_e(L) = \frac{L}{K}\left(\frac{1}{a^2} + \frac{4}{3\rho_o^2}\right)^{1/2} \quad (15)$$

Here, ρ_o is the coherence radius of the wave, which for the plane wave is given by

$$\rho_{op} = (1.45 C^2_n K^2 L)^{-3/5}, \quad (16)$$

and for the spherical wave, it takes the form

$$\rho_{os} = (0.55\, C^2_n K^2 L)^{-3/5}. \quad (17)$$

The relation between the log-intensity variancee $\sigma^2_{\ln I}$ and the normalized variance of the intensity fluctuations σ^2_I is given by [5]

$$\sigma^2_I = \exp(\sigma^2_{\ln I}) - 1 = \frac{<I^2>}{<I>^2} - 1 .$$

Hence,

$$<I^2> = <I>^2 \exp(\sigma^2_{\ln I}), \quad (18)$$

where $<I^2>$ is the second irradiance moment, and $<1>^2$ is the square of the average intensity. Thus, for plane waves, $<I>^2$ is given by

$$< I^2 >_p = <I>^2_p \exp[1.23\, C^2_{na}(l_o) K^{7/6} L^{11/6}], \quad (19)$$

while for spherical waves, it is written as

$$<I^2>_s = <I>^2_s \exp[0.5\, C^2_{na}(1_o)K^{7/6}L^{11/6}]. \quad (20)$$

In Figure (4) we have plotted the average intensity of the laser beam against the inner scale size for both the plane and spherical waves. The second irradiance moment is plotted in Figure (5) against the inner scale size for both the plane and spherical waves. In both figures we can see that the values corresponding to the spherical waves are larger than those corresponding to the plane waves. Thus, from equations (18), (19), and (20), one can deduce that the average intensity predominates the variance of the log-intensity.

EFFECT OF THE EMITTING APERTURE SIZE

Using equations (14), (15), and (16) for plane waves or (17) for spherical waves, we see that the average intensity depends on both the inner scale size and the emitting aperture size. Hence, different values of the effective beam radius a and the inner scale size 1_o are used.

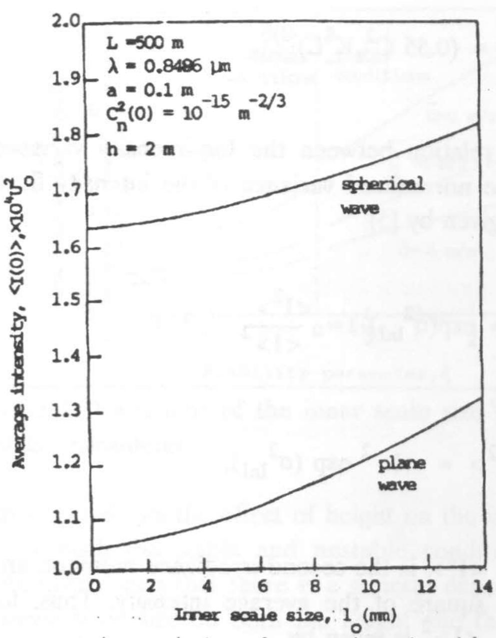

Figure 4. Variations of average intensity with inner scale size.

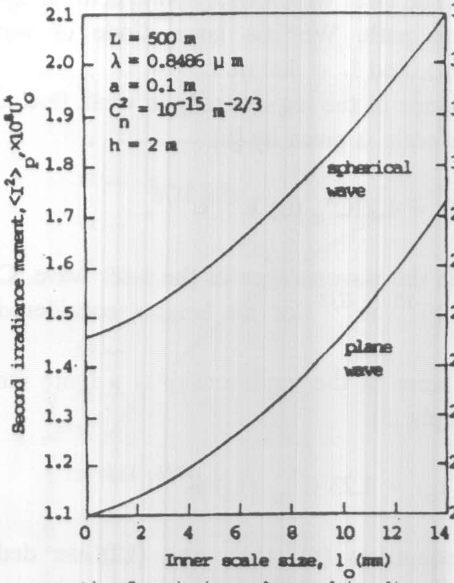

Figure 5. Variations of second irradiance moments of both plane and spherical waves with inner scale size.

In Figure (6), the average intensity $<I>_p$ of the plane wave normalized to its value at zero inner scale size $<I>^o_p$ is plotted against the inner scale size for different values of the effective beam radius. We can see from this figure that the average intensity of plane waves, having effective radii less than 1 cm, is less affected by the

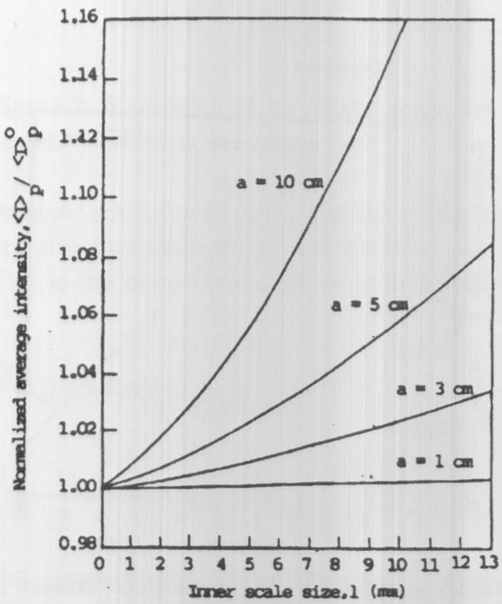

Figure 6. Variations of normalized average intensity with inner scale size.

variation of the inner scale size.

SUMMARY AND DISCUSSION

The inner scale size was first represented as a function of the kinematic viscosity of air and the rate of kinetic energy dissipation. Then, it is found that the inner scale size depends on the stability level, wind speed, and the height above the ground level. In otherwords, we found that these parameters are efficient tools for estimating the turbulence inner scale size. Furthermore, one can find that the inner scale size has some effects which are carefully studied in this paper.

The effect of the inner scale size on the refractive index structure constant is applied through what is called the apparent refractive index structure constant. This effect develops an expression for estimating the normalized variance of the log-intensity as an inner scale correction factor. Therefore, a knowledge of both the inner scale size and the Fresnel zone size, is sufficient for calculating the normalized log-intensity variance.

The inner scale size affects the average intensity of a laser wave through the dependence of the coherence radius of that wave on the apparent refractive index structure constant. On the otherhand, the inner scale size affects the second irradiance moment of a laser wave through the relation between σ^2_{lnI} and σ^2_I.

Finally the emitting aperture size plays another role besides the inner scale size in determining the average intensity. The results show that very small aperture radii (<1 cm) decrease the effect of the turbulence inner scale variations on the average intensity of the emitting waves.

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