DETERMINATION OF ERRORS DUE TO TROPOSPHERIC REFRACTIVITY IN LOW-ELEVATION-ANGLE RADAR DETECTION

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BSTRACT

The percentage relative errors in height, range and elevation angle in the radar detection of low elevation-angle targets, due to variation in tropospheric refractivity, are estimated. A technique to draw h-R-θ charts based on surface refractivity data is outlined. Expressions for the percentage relative errors in this case are also derived.

NTRODUCTION

Radar detection of low-elevation-angle targets is ampered by the variations in the refractivity of the lower oposphere. In this region, the electromagnetic waves avel in curved paths that depend on the vertical profile f the refractivity N. As a consequence, this results in rors in determining the range R, the height h and the evation angle θ of the target.

The earliest approach to this problem was to assume a ear N-profile and to adopt the effective earth's radius ansformation using the standard effective radius factor k 4/3. Based on this approach, range-height-angle charts e constructed over which location of targets can be otted. Such plots are called coverage diagrams. More cent radar stations are equipped with meteorological struments to determine the surface refractivity N_s. Using is information and assuming an exponential atmosphere, more appropriate value for k can be calculated. lternatively, the surface refractivity can be used in semi-mpirical formulae to determine directly the location of e target.

More accurate results can be obtained when the target cation is specified using dynamic ray tracing along with ofiles provided either from nearby aviation eteorological data or from sounding performed at the e of the radar itself.

A large number of radar stations in Egypt operate thout the presence of any meteorological data. Even for ose that make use of such information, one is never sure at they utilize correction factors appropriate for our type climate.

n this work, we present a simple technique to determine e relative percentage error in height, range and elevation gle of a target given either the effective earth radius factor or the surface refractivity at the radar site.

THEORETICAL APPROACH

For low-elevation-angle targets, below about 3 km, the effective earth radius concept gives fairly accurate results as compared to the ray tracing techniques. For spherical earth, the geometry of the problem is shown in Figure (1). The target height h is given by [1].

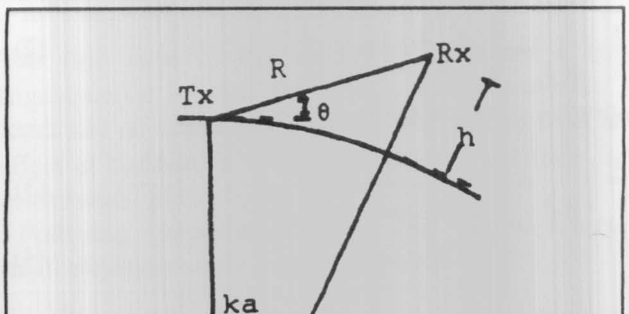

Figure 1. General geometry of the problem.

$$h = R.\sin\theta + \frac{R^2\cos^2\theta}{2ka} \qquad (1)$$

This equation is used to plot the h-R-θ chart for a given value of k. Figure (2) gives the chart for Mersa Matruh where the average k assumes the value of 1.527 [2].

Deviations from this average value will produce errors in the readings from the chart. Furthermore, the application of this chart at a radar station in another part of the country will also cause errors. Therefore, it is essential to be able to determine the relative error in h, R and θ due to variations in k.

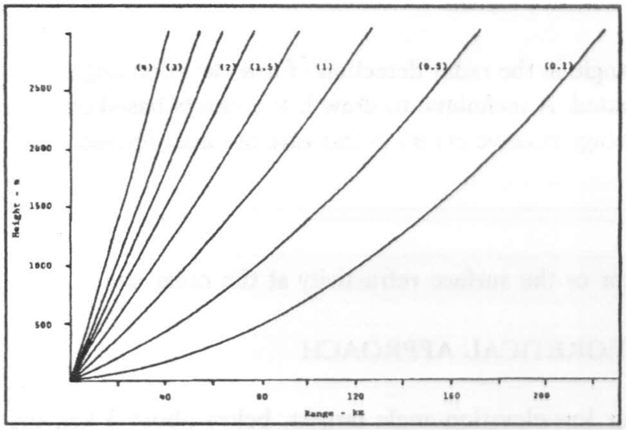

Figure 2. Radar height-range-angle chart drawn for the region of Mersa Matruh assuming k=1.527.

The relative error in h for given R and θ can be expressed by

$$\left.\frac{\Delta h}{h}\right|_{R,\theta} = \frac{1}{h}\left(\frac{\partial h}{\partial k}\right)\Delta k \tag{2}$$

Substituting equation (1) in (2) we get

$$\left.\frac{\Delta h}{h}\right|_{R,\theta} = \frac{-R.\cos^2\theta}{R.\cos^2\theta + 2ak.\sin\theta}\left(\frac{\Delta k}{k}\right) \tag{3}$$

$$= REH\,(\Delta\, k/k) \tag{4}$$

Similarly, the relative errors in R and θ can be derived

$$\left.\frac{\Delta R}{R}\right|_{h,\theta} = \frac{R.\cos^2\theta}{2(R.\cos^2\theta + ak.\sin\theta\;\;)}\left(\frac{\Delta k}{k}\right) \tag{5}$$

$$= RER\,(\Delta\, k/k) \tag{6}$$

and

$$\left.\frac{\Delta\theta}{\theta}\right|_{h,R} = \frac{R.\cos\theta}{2\theta(ka-R.\sin\theta)}\left(\frac{\Delta k}{k}\right) \tag{7}$$

$$= RE\theta\,(\Delta\, k/k) \tag{8}$$

It is clear that as θ increases, all the relative errors decrease reaching zero at $\theta = \pi/2$.

For small angles, equations (3),(5) and (7) reduce to

$$\left.\frac{\Delta h}{h}\right|_{R,\theta} = \frac{-R}{2ka\,\theta + R}\left(\frac{\Delta k}{k}\right) \tag{9}$$

$$\left.\frac{\Delta R}{R}\right|_{h,\theta} = \frac{R}{2ka\,\theta + 2R}\left(\frac{\Delta k}{k}\right) \tag{10}$$

$$\left.\frac{\Delta\theta}{\theta}\right|_{h,R} = \frac{R}{2ka\,\theta - 2R\theta^2}\left(\frac{\Delta k}{k}\right) \tag{11}$$

Comparing the relative errors we can deduce that

$$\frac{\Delta\theta}{\theta} > \left|\frac{\Delta h}{h}\right| > \frac{\Delta R}{R} \tag{12}$$

That is to say, for a given target, the pointing angle of the antenna is the most sensitive parameter to variations in the refractivity gradient and subsequently variations in k.

As indicated by equations (3), (5) and (7), given the position on the h-R-θ chart drawn for a certain k, the relative errors are functions of the chart readings and the relative deviation in k. Table (1) gives a numerical example for the coefficients of the relative errors in height REH, in range RER and in angle REθ. Figures (3) and (4) show REH, RER and REθ as functions of target height and for different ranges. The number between brackets is the elevation angle. The results indicate that the relative errors decrease with target height and increase with target range. In general, as the elevation angle increases, the values of the three errors approach each other.

A single chart for the average value of k at a certain location or for the whole country can be utilized. If the correct value of k, or its average at a given location, is known, then correction factors can be obtained either from tables like Table (1) or from charts like Figures (3) and (4). On the other hand, this technique can be used to calculate an ambiguity area around the target position for given deviations of k from its average value, like for example one standard deviation.

Most radar stations equipped with meteorological instruments measure only the surface refractivity N_s. The first km gradient ΔN can be estimated from N_s using the following empirical relation

$$\Delta N = -\,A.\exp\,(B.\,N_s) \tag{13}$$

where A and B are coefficients determined statistically for each region . In Mersa Matruh they assume the values of

256 and 0.015889 respectively [3]. The value of k can

Table 1. Percentage relative error coefficients in height (REH), range (RER) and elevation angle (REθ)as functions of range and height for an elevation angle of 0.1.

Range	Height	REH	RER	REθ
10	22.59	-14.90	12.14	19.29
20	55.47	-24.28	17.71	38.58
30	98.62	-30.72	20.91	57.06
40	152.06	-35.42	22.99	77.15
50	215.77	-39.00	24.44	96.44
60	289.77	-41.82	25.52	115.73
70	374.05	-44.10	26.35	135.01
80	468.60	-45.98	27.01	154.30
90	573.45	-47.55	27.55	173.59
100	688.56	-48.89	27.99	192.88
110	813.97	-50.04	28.37	212.17
120	949.65	-51.04	28.69	231.45
130	1095.61	-51.93	28.96	250.74
140	1251.85	-52.71	29.20	270.03
150	1418.37	-53.40	29.41	289.32
160	1595.18	-54.02	29.60	308.61
170	1782.26	-54.59	29.77	327.90
180	1979.62	-55.10	29.92	347.19
190	2187.27	-55.56	30.06	366.47
200	2405.19	-55.98	30.18	385.76
210	2633.40	-56.37	30.29	405.05
220	2871.89	-56.73	30.40	424.34

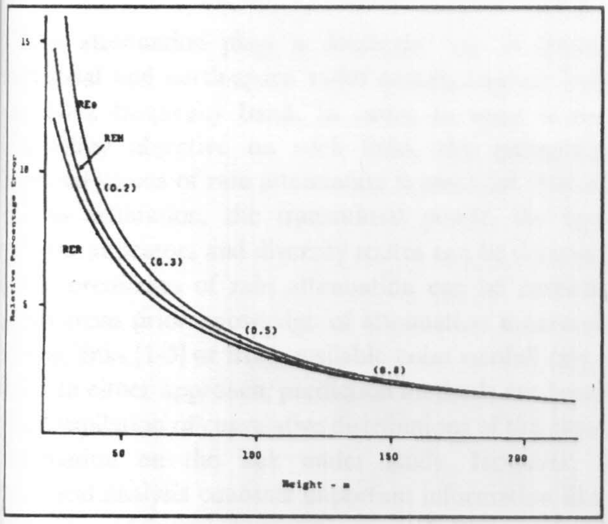

Fig. 3. Percentage relative error coefficients in height (REH), range (RER) and angle (REθ) as functions of target height for a range of 10 km. The number between brackets is the antenna pointing angle.

then be calculated from

$$k = \frac{1}{1+(\Delta N/157)} \tag{14}$$

Substituting equations (13) and (14) in (1), we get

$$h = R \sin\theta + \frac{R^2\cos^2\theta}{2a} - \frac{R^2\cos^2\theta}{a} \cdot \frac{A.\exp(B.N_s)}{314} \tag{15}$$

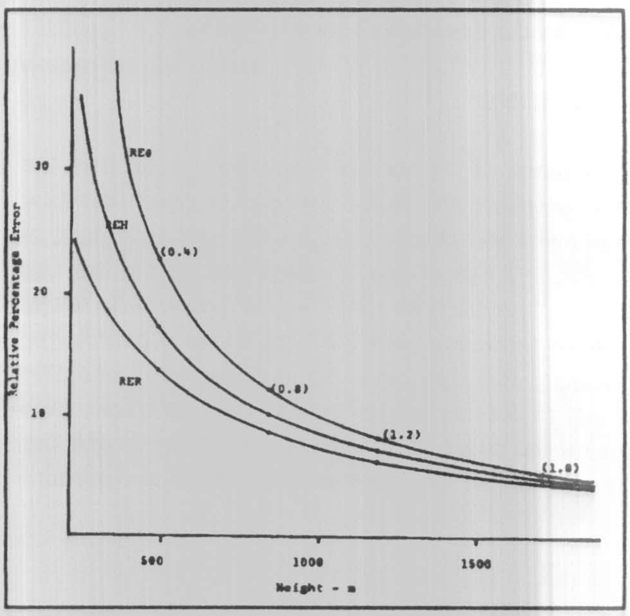

Figure 4. Percentage relative error coefficients in height (REH), range (RER) and angle (REθ) as functions of target height for a range of 50 km. The number between brackets is the antenna pointing angle.

The first term in this equation represents a simple trigonometric relation between h and R assuming flat earth and no refractivity gradient. The second term is the effect of the earth's curvature while the last term is the contribution of the variation in N.

Following the same procedure the relative errors for small elevation angles can be proven to be

$$\left.\frac{\Delta h}{h}\right|_{R,\theta} = \left(\frac{S.R^2}{ah}\right)\Delta N_s \tag{16}$$

$$\left.\frac{\Delta R}{R}\right|_{h,\theta} = \left(\frac{S.R}{R(1-2S/B)+a\theta}\right)\Delta N_s \tag{17}$$

$$\left.\frac{\Delta\theta}{\theta}\right|_{h,R} = \left(\frac{S.R}{\theta[a-R.\theta(1-2S/B)]}\right)\Delta N_s \tag{18}$$

where

$$S = \frac{AB\exp(BN_s)}{314} \tag{19}$$

Thus the h-R-θ chart can be drawn for the average value of N_s in a certain region and corrections can be made according to the deviations from this value.

CONCLUSIONS:

The variation of the vertical refractivity gradient in the lower troposphere affects the accuracy of radar detection of targets with low elevation angles. The pointing angle of the antenna is the most sensitive parameter to this variation followed by the target height and then its range. The simple technique presented in this work permits the use of any h-R-θ chart at any radar site with either the possibility of determining an ambiguity area or correction factors for the target location once proper values of k for this site are known. The approach introduced to determine the target location from only the knowledge of N_s can be utilized in any radar site, since this information can be easily obtained with simple equipments.

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