COMPARATIVE REVIEW ON NETWORK ADJUSTMENT

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ABSTRACT

A comparative analysis and analogue between the various adjustment methods for survey nets is presented. The criteria used for comparison is based on simulation technique by imposing random errors (angular and linear) on the typical network. The results which cover the application of 11 different approximate and rigorous least-squares methods, show important indications in the choice of method for network adjustments. The analysis of each method is discussed.

INTRODUCTION

All geodetic schemes such as secondary control survey nets (triangulation, trilateration, hybrid), levelling,...etc., require sophisticated adjustment procedures which vary in accordance to accuracy and the convenient adopted computational machine. Since in some of these methods the different calculations are tedious and lengthy, there is a pushing demand to choose the suitable one for the particular problem in hand so as to meet its needs without a deficit in accuracy, or excessive computational effort, or using dispensable equipment.

This investigation intends to compare the accuracy of different methods of adjustment for triangulation, trilateration and hybrid. The comparison made is based on the results of five methods for triangulation adjustment, five for trilateration and one for combined adjustment. For comparison purposes, simulation technique is applied on a typical network. Random errors are imposed on the network, and are considered as the real errors. On this basis the final adjusted quantities of the different methods are compared.

The relative merits of the different methods of adjustment adopted in this investigation can be assessed and categorized for use in various purposes. It should be noted that this paper is not aimed at the comparison between different types of observation of networks. Rather, it is an attempt to compare between various methods of adjustment.

I. The Tested Adjustment Methods

The various adjustment methods for triangulation are:

1. Variation of Coordinates
2. Condition Equations (Exact Method)
3. Condition Equations (General Method)
4. Equal Shift
5. Successive Approximation

and for trilateration:

6. Variation of Coordinates
7. Condition Equations (as a whole)
8. Condition Equations (separate figures)
9. Virtual Work
10. An Area Method

in addition to:

11. Hybrid

The above-mentioned methods employ either approximate procedures (equal shift and successive approximation), or least-squares techniques (condition equations and variation of coordinates). As for the minimum work approach demonstrated by the virtual work method, the measured distances are considered as elastic members in an internally redundant framework. The lengths of the distances between the fixed points remain unchanged during the process of adjustment. If the redundant members have fabrication (i.e. measuring) errors, axial forces must be applied to fit them in the framework. As a result axial forces will develop in all other members causing changes in their lengths. By applying minimum work techniques, these changes in length are determined, and which are in fact the sought corrections of the measured distances.

The suggested area method (Dr. Abdelal A.W., Assiut University) as investigated here, depends on splitting the

Table 1. Original Angles.

Angle	Value			Angle	Value			Angle	Value		
1	77°	31'	20"	11	60°	17'	01"	21	38°	07'	48"
2	93	32	04	12	44	10	15	22	35	28	21
3	55	00	56	13	65	33	59	23	69	21	29
4	61	24	57	14	50	51	46	24	36	46	22
5	72	30	43	15	13	58	23	25	37	38	26
6	67	00	19	16	29	57	11	26	63	34	15
7	51	57	06	17	45	35	36	27	90	28	50
8	34	30	50	18	62	21	30	28	62	10	38
9	26	47	45	19	42	05	43				
10	37	54	18	20	13	22	09				

network into separate figures and computing the area of each polygon twice (i.e. through different triangles). The difference between the two computed areas is distributed according to the area of each triangle. The coefficient ratio for correcting the lengths is taken as the square root of the ratio of corrected area divided by the uncorrected one. The corrected length thus equals to the measured length multiplied by the corresponding coefficient ratio. As each length is common to two triangles, the final adjusted length is taken as the average value.

II. Basic Network and Method of Analysis

The test-net, illustrated in Figure (1), is a typical net configuration composed of 8 stations. It comprises 10 triangles, 2 centre points, 28 angles ranging from 13° to 93° (Table 1), and 17 side lengths varying from 3036 ms. to 9338 ms. (Table 2). The net can be considered as a third-order network. The angles and lengths are obtained from coordinates of the stations, such that in their present status represent ideal quantities free of any mathematical inconsistency, i.e. these angles and lengths fulfil all geometrical conditions.

For comparison of the tested methods, the criteria used and the mathematical analysis are based on imposing random errors, whether angular or linear, on the corresponding original quantities in the typical network.

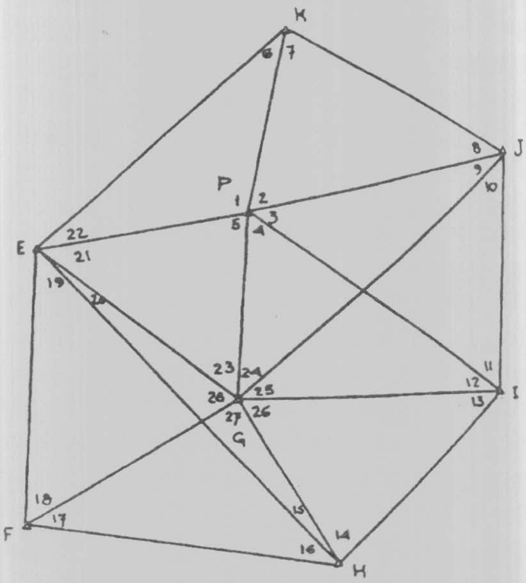

Table 1. Basic network.

Accordingly, two sets of observations are rendered (Tables 3,4). The former represents a net with observed angles, while the latter with observed distances. It is in this status that each of these two sets of observations is adjusted by the various methods used in the comparison investigation. Consequently, the resulting corrections (V) after adjustment by each method can be compared with the set of real errors (e), and the discrepancies (Δ = V - e) obtained be used for the evaluation of the tested adjustment method.

Table 2. Original lengths.

Line	Length	Line	Length
EK	5108.816	GP	3178.050
EP	4816.638	GJ	6312.686
EG	4909.175	GI	4004.997
EH	9338.094	IP	4393.177
EF	4901.020	IJ	3981.206
FG	4565.085	JP	4220.190
FH	6580.274	JK	5348.832
HG	4701.064	KP	3036.445
HI	4623.851		

III. Selection and Distribution of Random Errors

A. Angular Errors

The normal distribution is used for the selection of the above-mentioned set of real errors, so that the tested net would be very close to one which could be met in practice. Table (5) shows the error value and its corresponding frequency which are made up on the basis of this distribution. The standard deviation (σ) adopted is 2.5 sec. which is the common practice value for a one-second theodolite. These values are extracted from the accumulative normal distribution tables and distributed among the (ideal) angles as shown in the table.

Table 3. Observed angles (on basis of normal distribution).

Angle	Value	Angle	Value	Angle	Value
1	77° 31' 20.6"	11	60° 17' 02.1"	21	38° 07' 52.5"
2	93 32 00.9	12	44 10 18.7	22	35 28 19.7
3	55 00 55.7	13	65 33 56.3	23	69 21 31.4
4	61 24 55.3	14	50 51 47.4	24	36 46 22.9
5	72 30 43.4	15	13 58 25.4	25	37 38 25.7
6	67 00 20.6	16	29 57 08.8	26	63 34 14.0
7	51 57 05.4	17	45 35 30.9	27	90 28 51.8
8	34 30 53.5	18	62 21 29.2	28	62 10 36.0
9	26 47 42.0	19	42 05 43.7		
10	37 54 13.9	20	13 22 09.2		

Table 4. Observed lengths (on basis of normal distribution).

Line	Length	Line	Length
EK	5108.844	GP	3178.054
EP	4816.696	GJ	6312.704
EG	4909.258	GI	4005.021
EH	9338.067	IP	4393.225
EF	4900.978	IJ	3981.170
FG	4565.125	JP	4220.177
FH	6580.142	JK	5348.840
HG	4701.041	KP	3036.513
HI	4623.786		

Care has been taken in the distribution process so that the error values meet the Egyptian specifications for third degree triangulation:

i. Maximum triangular misclosure = 10 sec. (Figure 2)
ii. Average triangular misclosure = 5 sec.

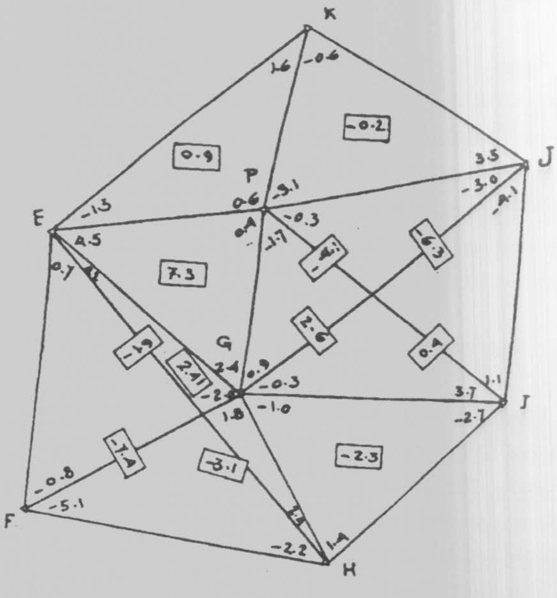

Figure 2. Distribution of random errors and triangular misclosure.

Table 5. Simulated errors (angles).

error range	probability %	frequency	selected simulated errors (sec.)
up to ± 0.6"	18.96	5.31 ~ 6	+ 0.6, + 0.4, + 0.2, -0.6, -0.3, -0.3
± 0.6 - ± 1.2	17.90	5.01 ~ 5	+ 0.7, - 0.8, + 1.1, - 1.0, + 0.9
± 1.2 - ± 1.8	16.01	4.48 ~ 4	- 1.3, + 1.4, -1.7, + 1.6
± 1.8 - ± 2.4	13.66	3.82 ~ 4	+ 1.8, - 2.0, -2.2, + 2.4
± 2.4 - ± 3.0	10.45	2.93 ~ 3	+ 2.4, - 2.7, -3.0
± 3.0 - ± 3.6	8.01	2.24 ~ 2	- 3.1, + 3.5
± 3.6 - ± 4.2	5.70	1.60 ~ 2	+ 3.7, - 4.1
± 4.2 - ± 4.8	3.83	1.07 ~ 1	+ 4.5
± 4.8 - ± 5.4	2.37	0.66 ~ 1	- 5.1
± 5.4 - ± 6.0	1.49	0.42 ~ 0	

iii. A criterion for angular accuracy in the whole net (n triangles)

$= \sqrt{[(\Delta\Delta)/3n]}$

where Δ is the triangular misclosure, lies between 3" to 5".

Linear Errors

Again, the normal distribution is used to select proportional errors to be distributed among the network sides (Table 6). The standard deviation adopted here is 1.2 x 10^{-7} l, corresponding to common practice accuracy of an EDM instrument conforming with a one-second theodolite, so that all observations would always be accuracy-wise homogeneous.

IV. Analysis and Results

After the addition of the previously mentioned simulated errors to the original quantities in each case of angles and lengths, the adjustments are made using methods 1 to 5 for angles, methods 6 to 10 for lengths and method 11 for combined trilateration-triangulation as shown in item I.

The final adjusted angles and lengths are given in Tables (7) and (8) respectively. The discrepancies between the adjusted and original quantities are then computed for each method. These discrepancies actually represent the differences between the corrections and the simulated real errors (Δ = V-e). In an ideal adjustment all these discrepancies should, in fact, be zero. They are given in tables (9), (10).

At the foot of each column the numerical average discrepancy is recorded to give an indicator for the assessment of each method. Moreover, the maximum value is shown to act as an additive indication for serious errors.

It should be noted that the criteria used in the analysis are the angular discrepancies (table 9) which are a measure for the strength of shape determination. On the other hand, discrepancies in side lengths (table 10) are used as a measure for the strength of scale determination.

These criteria resemble the commonly used method of relative error ellipses where the linear error represents the dimension of an error-ellipse along the side; and the angular error represents the perpendicular dimension. Unlike absolute error ellipses, these criteria used in the analysis are invariant with the adopted reference system.

CONCLUSIONS

From the foregoing, it can be readily seen that:

For triangulation:

i. As expected, the two applied rigorous methods, the condition eqs. and the variation of coordinates, give identical results within the computational accuracy (0.1 sec.)

Table 6. Simulated errors (lengths).

error range	probability %	frequency	selected simulated errors (sec.)
up to 29 * 10^{-7}. l	18.96	3.22 ~ 4	+ 29, -29, +15, -14 * 10^{-7}.l
29 - 58 * 10^{-7}. l	17.90	3.04 ~ 3	+ 55, -50, - 30
58 - 87 * 10^{-7}. l	16.01	2.72 ~ 3	+ 80, -85, + 60
87 - 116 * 10^{-7}. l	13.66	2.32 ~ 2	+ 110, -90
116 - 145 * 10^{-7}. l	10.45	1.78 ~ 2	+ 120, -140
145 - 174 * 10^{-7}. l	8.01	1.36 ~ 1	+ 170,
174 - 203 * 10^{-7}. l	5.70	0.96 ~ 1	-200,
203 - 232 * 10^{-7}. l	3.83	0.65 ~ 1	+ 225,
232 - 261 * 10^{-7}. l	2.37	0.40 ~ 0	
261 - 290 * 10^{-7}. l	1.49	0.08 ~ 0	

Table 7. Adjusted Angles.

Angle	Triangulation						
	Variation of Coord.			Cond. Eq. Exact Meth.	Cond. Eq. General Meth.	Successive Approx.	Equal Shift
1	77°	31'	18.4"	20.4"	20.8"	21.3"	21.3"
2	93	32	5.6	2.7	3.1	1.6	1.6
3	55	00	55.4	58.2	57.6	57.6	57.5
4	61	24	56.6	56.1	55.4	55.4	55.6
5	72	30	44.0	42.6	43.2	44.1	44.1
6	67	00	19.5	18.2	17.5	17.2	17.0
7	51	57	3.8	6.9	6.9	7.3	7.4
8	34	30	50.6	50.4	50.0	51.1	51.0
9	26	47	45.3	44.9	44.7	44.6	44.5
10	37	54	20.6	15.0	14.5	14.5	14.5
11	60	17	00	2.0	3.3	3.3	3.5
12	44	10	13.5	18.3	17.3	17.7	17.1
13	65	33	55.1	56.8	57.2	56.7	56.8
14	50	51	48.9	47.1	46.3	46.1	46.7
15	13	58	25.4	23.1	23.8	23.8	24.0
16	29	57	10.8	12.5	11.7	11.6	11.8
17	45	35	36.2	32.5	32.8	33.0	31.9
18	62	21	33.8	30.8	30.8	31.1	31.9
19	42	05	39.7	44.2	44.7	45.1	44.4
20	13	22	11.0	9.2	9.8	9.8	9.4
21	38	07	45.4	47.7	47.1	46.9	46.2
22	35	28	22.2	21.5	21.7	21.5	21.7
23	69	21	30.5	30.5	29.7	29.0	29.7
24	36	46	22.7	20.8	22.4	22.4	22.4
25	37	38	27.3	24.8	25.0	24.9	24.9
26	63	34	16.2	16.3	16.5	17.3	16.4
27	90	28	47.8	51.9	51.7	51.4	52.3
28	62	10	35.8	35.8	34.8	35.1	34.3

Table 7. Contd.

Angle	Trilateration					Hybrid
	Variation of Coord.	Cond. Eq. net as whole	Cond. Eq. sep fig.	virtual Work	Area method	
1	18.4"	18.5"	22.2"	22.2"	16.3"	30.4"
2	5.6	6.3	10.9	11.0	1.9	3.0
3	55.4	55.7	52.7	52.7	53.8	57.0
4	56.6	56.3	50.2	50.2	53.8	56.9
5	44.0	43.3	43.8	43.8	46.4	42.7
6	19.5	19.7	20.0	20.0	19.7	18.1
7	3.9	3.6	3.8	3.7	5.3	6.8
8	50.5	50.2	45.3	45.3	52.8	50.2
9	45.3	45.2	47.7	47.7	40.2	44.9
10	20.5	20.6	19.5	19.5	13.7	17.2
11	16' 58.8	16' 58.5	0.1	0.1	16' 59.1	2.7
12	13.5	13.1	11.6	11.6	13.2	16.1
13	55.1	54.9	58.9	58.9	34' 0.1	58.0
14	48.9	48.8	45.1	45.1	46.7	46.1
15	25.4	26.2	30.6	30.6	31.7	23.1
16	10.7	11.2	8.9	8.9	9.1	11.3
17	36.2	35.3	39.6	39.6	39.7	34.8
18	33.4	33.0	34.5	34.5	34.5	30.6
19	39.8	40.6	37.1	37.1	37.5	43.8
20	11.0	11.2	16.2	16.2	17.3	9.7
21	45.6	46.1	43.7	43.7	44.7	47.7
22	22.2	21.9	17.8	17.8	24.0	21.4
23	30.5	30.6	32.5	32.5	28.8	30.0
24	22.6	22.8	29.4	29.7	15.5	21.5
25	27.3	27.8	28.9	28.8	21.0	25.5
26	16.0	16.3	16.0	16.0	13.2	16.2
27	47.8	47.5	41.0	41.0	40.8	50.8
28	35.8	35.2	32.3	32.3	32.2	36.2

ii. Again, as expected, the rigorous methods yield higher precisions than the approximate methods. This is obvious, where angular comparison is considered, since the rigorous methods are basically based on least squares in the angles themselves. But it is apparently seen that the discrepancies between all the approximate methods and the rigorous ones are relatively slight, and as a matter of fact insignificant. It follows that unless the required accuracy is particularly high, and sophisticated computational facilities are available, it is is not worth the trouble to apply rigorous methods.

iii. The approximate methods can be listed in order of accuracy as follows:

- Condition Eq. (General Method)
- Successive Approximation.
- Equal Shift

iv. As for scale, the rigorous methods do not seem to yield the best results. Generally, the applied methods can be put in the following order:

- Successive Approximation
- The Rigorous Methods
- Condition Equation (General Method)
- Equal Shift

Table 8. Adjusted Lengths.

Line	Triangulation				
	Variation of Coord.	Cond. Eq. Exact Meth.	Cond. Eq. General Meth.	Successive Approx.	Equal Shift
EK	5108.816	.816	.816	.816	.816
EP	4816.704	.633	.624	.641	.617
PK	3036.488	.438	.441	.467	.439
PJ	4220.178	.138	.178	.241	.155
KJ	5348.866	.787	.806	.880	.769
EG	4909.235	.152	.150	.183	.152
PG	3178.035	.031	.017	.029	3177.995
GI	4005.023	4004.912	4004.910	4004.969	4004.888
PI	4393.217	.077	.095	.159	.067
HI	4623.822	.725	.753	.806	.709
GH	4701.000	4700.924	4700.941	4700.992	4700.896
FH	6580.185	.199	.213	.291	.177
FG	4565.078	.079	.088	.130	.068
EF	4901.012	4900.947	4900.949	4900.983	4900.931
JI	3981.209	.173	.188	.244	.163
EH	9338.060	.042	9337.944	.035	9337.902
GJ	6312.658	.637	.651	.743	.614

Table 8. Contd.

Line	Trilateration					Hybrid
	Variation of Coord.	Cond. Eq. net as whole	Cond. Eq. sep. fig.	Virtual Work	Area Method	
EK	.867	.870	.927	.927	.844	.828
EP	.704	.709	.746	.746	.696	.641
PK	.488	.484	.421	.421	.519	.450
PJ	.179	.179	.240	.240	.177	.206
KJ	.866	.871	.936	.936	.840	.860
EG	.233	.225	.256	.256	.258	.186
PG	.035	.028	.013	.013	.023	.039
GI	.022	.017	4004.965	4004.965	4004.982	4004.983
PI	.217	.221	.279	.279	.225	.152
HI	.822	.831	.827	.827	.786	.831
GH	4700.995	4700.983	.039	.039	.041	.822
FH	.185	.196	.141	.141	.142	.293
FG	.079	.067	.126	.126	.128	.093
EF	.012	.022	4900.977	4900.977	4900.978	.900
JI	.209	.216	.216	.216	.182	.215
EH	.060	.057	.069	.069	.061	.858
GJ	.658	.651	.611	.611	.760	.712

Table 9. Discrepancy in Angles, Δ = V - (original aas standard)(from tables 1,7).

Angle	Variation of Coord. Δ sec.	Triangulation Cond. Eq. Exact Meth. Δ sec.	Cond. Eq. General Meth. Δ sec.	Successive Approx. Δ sec.	Equal Shift Δ sec.
1	- 1.6	+ 0.4	+ 0.8	+ 1.3	+ 1.3
2	+ 1.6	- 1.3	- 1.0	- 2.4	- 2.4
3	- 0.6	+ 2.2	+ 1.6	+ 1.6	+ 1.5
4	- 0.4	- 0.9	- 1.6	- 1.6	- 1.4
5	+ 1.0	- 0.4	+ 0.2	+ 1.1	+ 1.1
6	+ 0.5	- 0.8	- 1.5	- 1.8	- 2.0
7	- 2.2	+ 0.9	+ 0.9	+ 1.3	+ 1.4
8	+ 0.6	+ 0.4	0.0	+ 1.1	+ 1.0
9	+ 0.3	- 0.1	- 0.4	- 0.4	- 0.5
10	+ 2.6	- 3.0	- 3.5	- 3.5	- 3.5
11	- 1.0	+ 1.0	+ 2.3	+ 2.3	+ 2.5
12	- 1.5	+ 3.0	+ 2.3	+ 2.4	+ 2.1
13	- 3.9	- 2.2	- 1.8	- 2.3	- 2.2
14	+ 2.9	+ 1.1	+ 0.3	+ 0.1	+ 0.7
15	+ 2.4	+ 0.1	+ 0.8	+ 0.8	+ 1.0
16	- 0.2	+ 1.5	+ 0.7	+ 0.6	+ 0.8
17	+ 0.2	- 3.5	- 3.2	- 3.0	- 4.1
18	+ 3.8	+ 0.8	+ 0.8	+ 1.1	+ 1.9
19	- 3.3	+ 1.2	+ 1.7	+ 2.1	+ 1.4
20	+ 2.0	+ 0.2	+ 0.8	+ 0.8	+ 0.4
21	- 2.6	- 0.3	- 1.1	- 1.1	- 1.8
22	+ 1.2	+ 0.5	+ 0.7	+ 0.5	+ C.7
23	+ 1.5	+ 1.5	+ 0.7	- 0.0	+ 0.7
24	+ 0.7	- 1.2	+ 0.4	+ 0.4	+ 0.4
25	+ 1.3	- 1.2	- 1.1	- 1.1	- 1.1
26	+ 1.2	+ 1.3	+ 1.5	+ 2.3	+ 1.4
27	- 2.2	+ 1.9	+ 1.7	+ 1.4	+ 2.3
28	- 2.2	- 2.2	- 3.2	- 2.9	- 3.7
Max. Δ	3.9	3.5	3.5	3.5	4.1
Av. Δ	1.63	1.25	1.31	1.48	1.62

Table 9. Contd.

Angle	Trilateration					Hybrid
	Variations of Coord. Δ sec.	Cond. Eq. net as whole Δ sec.	Cond. Eq. sep. fig. Δ sec.	Virtual Work Δ sec.	Area Method Δ sec.	Δ sec.
1	- 1.6	- 1.5	+ 2.2	+ 2.2	- 3.7	+ 0.4
2	+ 1.6	+ 2.3	+ 6.9	+ 7.0	- 2.1	- 1.0
3	- 0.6	- 0.3	- 3.3	- 3.3	- 2.2	+ 1.0
4	- 0.4	- 0.7	- 6.8	- 6.8	- 3.2	- 0.1
5	+ 1.0	+ 0.3	+ 0.8	+ 0.8	+ 3.4	- 0.3
6	+ 0.5	+ 0.7	- 1.0	+ 1.0	+ 0.7	- 0.9
7	- 2.1	- 2.4	- 2.3	- 2.3	- 0.7	+ 0.8
8	+ 0.5	+ 0.2	- 4.7	- 4.7	- 2.8	+ 0.2
9	+ 0.3	+ 0.2	+ 2.7	+ 2.7	- 4.8	- 0.1
10	+ 2.5	+ 2.6	+ 1.5	+ 1.5	- 4.4	- 0.8
11	- 2.2	- 2.5	- 0.9	- 0.9	- 1.9	+ 1.7
12	- 1.5	- 1.9	- 3.5	- 3.5	- 1.8	+ 1.1
13	- 3.9	- 4.1	- 0.1	- 0.1	+ 1.1	- 1.0
14	+ 2.9	+ 2.8	- 0.9	- 0.9	+ 0.7	+ 0.1
15	+ 2.4	+ 3.2	+ 7.6	+ 7.6	+ 8.7	+ 0.1
16	- 0.3	+ 0.2	- 2.1	- 2.1	- 1.9	+ 0.3
17	+ 0.2	- 0.7	+ 3.6	+ 3.6	+ 3.7	- 1.2
18	+ 3.4	+ 3.0	+ 4.5	+ 4.5	+ 4.5	+ 0.6
19	- 3.2	- 2.4	- 5.9	- 5.9	- 5.6	+ 0.8
20	+ 2.0	+ 2.2	+ 7.2	+ 7.2	+ 8.3	+ 0.7
21	- 2.4	- 2.0	- 4.4	- 4.4	- 3.3	- 0.3
22	+ 1.2	+ 0.9	- 3.2	- 3.2	+ 3.0	0.4
23	+ 1.5	+ 1.6	+ 3.5	+ 3.5	- 0.2	+ 1.0
24	+ 0.6	+ 0.8	+ 7.4	+ 7.4	- 6.5	- 0.5
25	+ 1.3	+ 1.8	+ 2.9	+ 2.8	- 5.0	- 0.5
26	+ 1.0	+ 1.3	+ 1.0	+ 1.0	- 1.8	+ 1.2
27	- 2.2	- 2.5	- 9.0	- 9.0	- 9.2	+ 0.8
28	- 2.2	- 2.8	- 5.7	- 5.7	- 5.8	- 1.8
Max. Δ	3.9	4.1	9.0	9.0	9.2	1.8
Av. Δ	1.63	1.72	3.77	3.76	3.61	0.7

Table 10. Discrepancy in Lengths, Δ = V - e (original as standard) (from tables 2, 8).

Line	Triangulation				
	Variation of Coord. Δ mm.	Cond. Eq. Exact Method Δ mm.	Cond. Eq. General Method Δ mm.	Successive Approx. Δ mm.	Equal Shift Δ mm.
EK	0	0	0	0	0
EP	+ 66	- 5	- 14	+ 3	- 21
EG	+ 60	- 23	- 25	+ 18	- 23
EH	- 34	- 52	- 150	- 59	- 192
EF	- 7	- 73	- 71	- 38	- 89
FG	- 7	- 6	+ 3	+ 45	- 17
FH	- 90	+ 75	- 62	- 17	- 97
HG	- 64	- 140	- 123	- 72	- 168
HI	- 29	- 126	- 99	- 45	- 142
GP	- 15	- 19	- 33	- 21	- 55
GJ	- 28	- 49	- 35	+ 57	- 72
GI	+ 26	- 79	- 87	- 28	- 109
IP	+ 40	- 100	- 82	- 18	- 110
IJ	+ 3	- 33	- 18	+ 38	- 43
JP	- 12	- 52	- 12	+ 51	- 35
JK	+ 34	- 45	- 26	+ 48	- 63
KP	+ 43	- 7	- 4	+ 22	- 6
Max Δ	90	140	150	72	192
Av. Δ	35	55	53	36	78

Table 10. Contd.

Line	Trilateration					Hybrid
	Variation of Coord. Δ mm.	Cond Eq. net as whole Δ mm.	Cond. Eq. sep. fig. Δ mm.	Virtual Work Δ mm.	Area Method Δ mm.	
EK	+ 51	+ 54	+ 111	+ 111	+ 28	+ 12
EP	+ 66	+ 71	+ 108	+ 108	+ 58	+ 3
EG	+ 58	+ 50	+ 81	+ 81	+ 83	+ 11
EH	- 34	- 37	- 25	- 25	- 34	- 36
EF	- 7	+ 3	- 43	- 43	- 42	- 20
FG	- 6	- 10	+ 41	+ 41	+ 43	+ 8
FH	- 90	- 78	- 133	- 133	- 132	+ 19
HG	- 69	- 81	- 25	- 25	- 23	- 42
HI	- 29	- 20	- 24	- 24	- 65	- 20
GP	- 15	- 22	- 37	- 37	- 27	- 11
GJ	- 28	- 35	- 75	+ 75	+ 74	+ 26
GI	+ 25	+ 20	- 32	- 32	- 15	- 14
IP	+ 40	+ 44	+ 102	+ 102	+ 48	- 25
IJ	+ 3	+ 10	+ 10	+ 10	- 24	+ 9
JP	- 11	- 11	+ 50	+ 50	- 13	+ 16
JK	+ 34	+ 39	+ 104	+ 104	+ 8	+ 28
KP	+ 43	+ 39	- 24	- 24	+ 74	+ 5
Max Δ	90	81	133	133	132	42
Av. Δ	36	37	60	60	47	18

For Trilateration

i. Here also, and as expected, the condition eqs. and the variation of coordinates methods give identical results within the computational accuracy (0.1 sec.)

ii. Where shape is concerned, and like triangulation, the results show to be in favour of the rigorous methods which give higher precisions than the other applied ones. But unlike triangulation, the discrepancies between these methods and the approximate ones are quite significant.

iii. The area method is classified here as an approximate one due to the previously mentioned reasons in the description of the method itself in (I).

iv. In order of accuracy, the approximate methods are equal.

v. For scale comparison, the rigorous methods again furnish the best results.

vi. A noteworthy observation is that the virtual work and the condition eq. (Sep. Fig.) methods are quite identical.

vii. The applied methods are ordered as follows:

- The Rigorous Methods
- Area Method
- Virtual Work, Condition Eq. (Sep. Fig.)

For combined adjustment

Concerning both, shape and scale, the accuracy of results leans heavily towards the hybrid method of adjustment.

It is recommended for future research and to reach highly conclusive results to apply a similar procedure to that followed here employing computer simulations using numerous networks and using different permutations for the distribution of errors within each network.

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