ANALYSIS OF REINFORCED CONCRETE SALB SUBJECTED TO CENTRAL LINE LOAD

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ABSTRACT

The main purpose of this paper was to study the economical arrangement of reinforcement according to different methods; namely; the plate elastic analysis, the strip method, the yield-line theory and the new 1989 Egyptian Code of Practice method. Comparison between the moment volume calculated by these methods is presented and discussed. The highest moment volume value was obtained by the Egyptian Code method as compared to those given by the other methods. The yield-line method of analysis proved to be the most suitable method for the prediction of the failure load.

INTRODUCTION

The elastic analysis of reinforced concrete slabs is rather complicated except in certain simple cases. It is generally accepted that reinforced concrete behaves in an inelastic manner except possibly at relatively low loads, and so design on the basis of the plastic theory is more realistic. It gives a comparatively simple approach for calculating the load-carrying capacity and the design moments with a suitable factor of safety against failure. This paper uses Hillerborg's strip method for the ultimate load design of reinforced concrete slabs under the effect of central line load. This method assumes that the flexural behaviour of a slab under load is of primary significance, and at failure no load is carried by the torsional strength of the slab. Consequently, the load is carried by bending in orthogonal directions by pure strip action. A recent review of this approach in the light of limit analysis has aroused a great deal of interest in its many aspects. Tests have shown that this approach affords a safe method of design by the ultimate load theories and also that under working loads the behaviour of the slabs is satisfactory with regard to both cracking and deflection. Considerable simplifications are achieved by the application of this method which provides a theoretical basis for intuitive ideas, and at the same time does not require more than a basic knowledge of the analysis of beams. This makes it a good design method to apply.

ANALYSIS

a. Elastic Analysis of Slabs Subjected To Partial Loading

The solution of differential Equation (1) of a plate can be carried out by applying certain mathematical expression such as fourier expansions or by numerical methods such as finite difference method and finite element method.

Whatever the method, the assumptions that the material is homogeneous, elastic and isotropic are always considered. These assumptions are not valid if the variation of reinforcement or post cracking condition are to be considered when analysing reinforced concrete slabs.

However, the use of the elastic theory provides the designer with a complete solution, where he can compute the moments at any point. Thus, an economical distribution of reinforcement is possible.

The design tables prepared by Richard Bates [2] using the elastic theory is used here, for simplicity, as follows:

Figure (1) shows a rectangular plate with dimensions (a) and (b) which is subjected to partial load (p t/m^2). The values of the moment M_x and M_y at point m is calculated

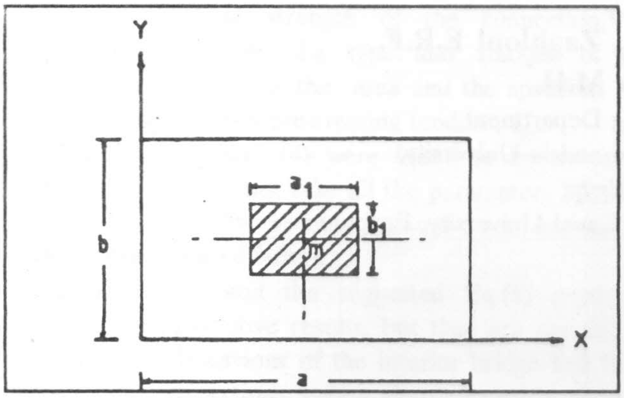

Figure 1. A rectangular plate subjected to partial load.

by using the following equations (8).

$$M_x \text{ at point } m = F_x\, P\, a_1\, b_1 \quad (1.a)$$

$$M_y \text{ at point } m = F_y\, P\, a_1\, b_1 \quad (1.b)$$

where:

F_x and F_y are multiplication factors depending on (a_1/a) and (b_1/b) ratios and can be obtained from the design tables given in Reference [2] for different slab aspect ratios (a/b).

a_1 and b_1 are the dimensions of the loaded area.

For the case of rectangular plates subjected to uniform distributed load ($a_1/a = 1$ and $b_1/b + 1$) and the moment at point (m) will be such that:

$$M_x = F_x\, P\, a\, b \quad (2.a)$$

$$M_y = F_y\, P\, a\, b \quad (2.b)$$

for the case of line load ($a_1/a = 0$ or $b_1/b = 0$) the moment at point (m) will be as follows:

$$M_x = F_x\, .\, P\, .\, (b_1 \text{ or } a_1) \quad (3.a)$$

$$M_y = F_y\, .\, P.\, (b_1 \text{ or } a_1) \quad (3.b)$$

The weight of the reinforcement for a slab of dimention a and b is approximately proportional to the moment volume (V), since the difference of the lever arm ($Y_{c.t}$) is small. Therefore:

$$V = (M_x + M_y)\, .\, a\, .\, b \quad (4)$$

put $(b/a) = K$

$$\therefore\ V = (M_x + M_y)\, .\, K\, a^2 \quad (5)$$

For the case of rectangular plate subjected to uniform distributed load (P t/m^2), the value of V is

$$V = P.\, K^2\, a^4\, (F_x + F_y) \quad (6)$$

For the case of rectangular plate subjected to a line load parallel to X-axis ($b_1/b = 0$) with length equal (a), the value of V is:

$$V = (P'\, a\, F_x + P'\, a\, F_y).\, K\, a^2$$

$$= P'\, a^3\, K\, (F_x + F_y) \quad (7)$$

where:

P' = the line load (t/m').

For the case of rectangular plate subjected to a line load parallel to Y-axis ($a_1/a = 0.0$) with length equal (b), the value of V is :

$$V = (P'\, b\, F_x + P'\, b\, F_y).\, K\, a^2$$

$$V = P'\, K^2\, a^3\, (F_x + F_y) \quad (8)$$

b. Strip analysis of reinforced concrete slabs

The treatment of line load by this method depends on the angle the line load makes with the actual main load-carrying direction. The most usual cases in practice are where the load direction is at right-angle to or parallel to the load-carrying directions, therefore there are two cases:

Case 1: The line load (Q t/m') acts along a line perpendicular to the main load carrying direction

As shown in Figure (2) the values of M_x and M_y are:

$$M_x = \frac{QKL}{4} \quad [\text{ moment per unit length}]$$

$$M_y = 0.0$$

The moment volume (V) = $(M_x + M_y).\, KL^2$

$$\therefore\ V = \frac{QK^2L^3}{4} = QL^3.\Phi \quad (9)$$

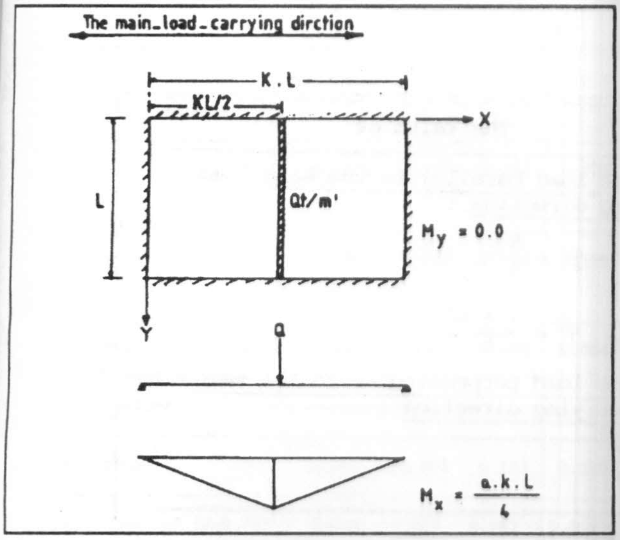

Figure 2. A line load acts along a line perpendicular to the main load-carrying direction.

where

$$\phi = \frac{K^2}{4}$$

Case 2: The Line Load Acts Along a Line Parallel to the Main-Load Carrying Direction

As shown in Figure (3) the values of M_x are:

$$M_x = (Q/C)\ .\ C/2\ .\ C/4 = \frac{Q.C}{8}$$

and

$$M_y = Q/C \frac{L^2}{8}$$

The moment volume (V) = $(M_x + M_y)\ KL^2$

$$\therefore (V) = \left[\frac{QC}{8} + \frac{Q}{8} \quad \frac{L^2}{8} \right] KL^2 \qquad (10)$$

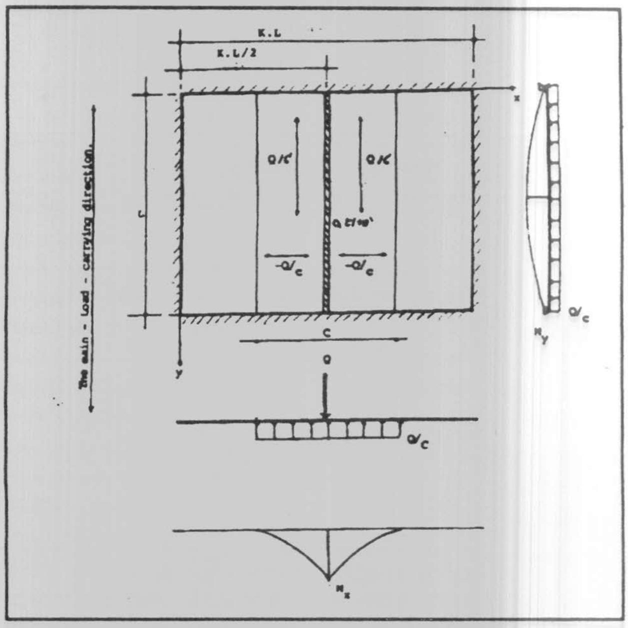

Figure 3. A line load acts along a line parallel to the main load-carrying direction.

The value of (c) is chosen to give suitable values of moments. In this investigation the value of C is calcualted by assuming:

$$\left(\frac{M_x}{M_y}\right) \text{Strip method} = \left(\frac{M_x}{M_y}\right) \text{elastic solution}$$

Where the elastic solution is for a slab with infinite length in X-direction and subjected to line load (Q t/m') parallel to Y-direction, for which:

$$\left(\frac{M_x}{M_y}\right) \text{elastic solution} \cong 1.0$$

$$\therefore \frac{QC}{8} + \frac{C}{8} \quad \frac{L^2}{8} \qquad \therefore C = L$$

$$\therefore V = QL^3 . \frac{K^2}{4} \qquad (11)$$

Also, the value of (C) can be obtained to give the most economical solution (minimum moment volume). As before in Eq. (2) $(\partial V/\partial C) = 0.0$, this gives C = L.

From the above equations, one can notice that for $K \geq 1$ the suitable value of (C) is (L), and this value gives isotopic reinforcement. For $K \leq 1$ the value of (C) is taken equal (KL).

Therefore the momeent volume in this case is as follows:

For $K \geq 1$

$$V = QL^3 . \phi \qquad (12)$$

where $\phi = K/4$

For $K \leq 1$

Table 1. The value of ϕ.

The design method	The value of
Strip method	<u>The line load parallel to the main load carrying direction</u> : $K \geq 1 \quad \phi = \frac{K}{4}$ $K \leq 1 \quad \phi = \frac{1 + K^2}{8}$ <u>The line load perpendicular to the main load carrying direction</u> : $\phi = \frac{K^2}{4}$
Orthotropic yield line method (the most economical solution)	<u>For $K \leq 0.816$</u> : $\phi = \frac{F_a k}{4} (1 + \mu_e)$ where : $\mu_e = \frac{K^2}{2 - 2K^2}$, $F_a = K(1 + \mu_e K) - K^2(\mu_e^2 K^2 + 2\mu_e)^{0.5}$ <u>For $K \geq 0.816$</u> : $\phi = 0.136\ K$
The Egyptian Code	<u>For $1 \leq K \leq 1.5$</u> : $\phi = \frac{K}{8(1+K)} \cdot \frac{(1/15+G)\ (2K-1/15+G) + K}{(1/15 + G - \frac{G^2}{2})}$ where : $G = 0.4\ (2 - \frac{1}{K})$ For $K \geq 1.5 \quad \phi = \frac{225}{552} K$ For $1 \geq K \geq 1/1.5 \quad \phi = \frac{386\ K}{1800(\frac{7}{15} + \frac{2K}{25})\ (1+K)}$ For $K < 1/1.5 \quad \phi = \frac{19K^2}{72}$
Elastic method	◊ The line load parallel to the short side $\phi = K\ (F_x + F_y)$ ◊ The line load parallel to the long side $\phi = K^2(F_x + F_y)$

Table 2. Comparison of ϕ values.

The design methods	K= 1/2	1/1.8	1/1.6	1/1.4	1/1.2	1.0	1.2	1.4	1.6	1.8	2.0
Strip method (Perpendicular)	0.0625	0.077	0.099	0.128	0.174	0.250	0.360	0.490	0.640	0.810	1.000
Strip method (Parallel)	0.156	0.164	0.174	0.189	0.212	0.250	0.300	0.350	0.400	0.450	0.500
Orthoropic yield line method (most economical solution)	0.252	0.061	0.072	0.084	0.098	0.118	0.141	0.165	0.109	0.212	0.236
Code method	0.066	0.081	0.103	0.218	0.244	0.277	0.353	0.437	0.652	0.734	0.815
Elastic method (ν=0.15)	0.033	0.045	0.062	0.086	0.127	0.194	0.242	0.290	0.336	0.380	0.428

According to Equation (10) using C = KL gives :

$$(V) = \left[\frac{QKL}{8} + \frac{QL^2}{8KL} \right] KL^2$$

$$= \frac{QL^3}{8}(1+K^2) = QL^2 . \phi \qquad (13)$$

where

$$\phi = 1 + \frac{K^2}{8}$$

C. Yield-Line Analysis of Reinforced Concrete Slabs

The analysis of reinforced concrete slabs using yield-line theory for slabs leaded by central line load was studied in detail by the authors (7). It was concluded that:
For K $\leq$ 0.816

$$V = LQ^3 . \phi$$

where $\phi = \frac{F_a K}{4}(1+\mu_e)$

and $F_a = K(1+ \mu_e K) - K^2 (u_e^2 K^2 + 2 \mu_e)^{1/2}$

$$\mu_e = \frac{K}{2\text{-}2K^2}$$

and For K $\geq$ 0.816

$$\phi = 0.136 K$$

For comparison between the methods of analysis of slabs subjected to central line load, Table (I) was prepaired the table includes beside the results obtained by the elastic analysis (the ϕ values) the results obtained from the strip method the yield-line analysis and the value of ϕ resulting from the method adopted by 1989 Egyptian Code of Practice (8). Table (II) shows the values of ϕ obtained by the four methods of analysis for different values of K, ($0.5 \leq K \leq 2$).

RESULTS AND CONCLUSIONS

1. In the design by using the strip method, the moment volume values, calculated by assuming that the main-load carrying direction is the short side of the slab are less than those calculated by assuming that the main-load carrying direction is the long side, thus always true irrespective of M_x and M_y. As results one should choose the short side as the main load carrying direcction.

2. Accordingly the slab subjected to central line load parallel to the length (L) and peerpendicular to the other side (KL) is reinforced as follows:

For K $\leq$ 1

The slab is considered as one way slab in the short side and the value of the moments are as follows:

$$M_x = \frac{QKL}{4}$$

see Figure (2)

$$M_y = 0.0$$

For K $\geq$ 1

The slab is reinforced as isotropic slab and the value of the moment in the two direction is equal to (QL/8) and the imajinary width (C) is taken equal to (L).

3. For 0.5 $\leq$ K $\leq$ 2, the moment volume values obtained by the code are heigher than those given by the yield line method and the strip method by 245% and 63 % respectively, therefore, the code proved to be extremely conservative and not capable of predicting the true behaviour of the investigated slab. Albeit simple to apply, it does not include the variables affecting the strength.
4. To obtain the capacity of a slab under the effect of central line load with predeterminate patern of reinforcement, the strip method fails to give a unique solution while the yield-line approach is benificial in that respect.

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