AN X-ARRAY HOT-WIRE PROBE IN A HEATED TURBULENT FLOW: CAIIBRATION AND DIGITAL SIGNAL ANALYSIS

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ABSTRACT

In this paper, a comprehensive velocity/yaw angle calibration of an X-array hot-wire probe is carried out to determine and account for the yaw angle dependence in the response equation. This was achieved by creating "look-up" tables for the calibration coefficients versus the yaw angle, instead of using a yaw parameter in the traditional effective cooling velocity approach. The response equations of the hot-wire and cold wire, together with the look-up tables were then implemented in an iterative digital signal analysis scheme to resolve the instantaneous velocity vector and instantaneous temperature, rather than just calculating prechosen statistical flow quantities as is the case with available techniques. The instantaneous values can then be processed by means of standard statistical operations to yield any time-averaged or turbulent quantity for the fluid flow with relative ease. The analysis scheme has been tested and proved very sucessful, particularly for flows of low mean velocities.

NOMENCLATURE

A,B,n	Calibration coefficients in the hot-wire response equation (generally function of the yaw angle, α)
C_{t1}, C_{t2}	Calibration coefficients in the cold-wire response equation
d, l	diameter and length of the hot-wire probe sensor
E, E_t	voltage output signals of constant temperature and constant current anemometers respectively
N	sample size; number of data points sampled at a particular measuring location per second
Nu	Nusselt number of the hot-wire sensor
R_s	resistance of the hot-wire sensor
Re	Reynolds number based on the diameter of the hot wire sensor
T, $\bar{T}$	instantaneous and time-averaged temperatures respectively
t'	fluctuating part of T; $t' = T - \bar{T}$
T_a	ambient temperature
T_m	mean film temperature of the hot-wire sensor
U, V	instantaneous velocities in the flow direction and normal direction respectively
$\bar{U}$, $\bar{V}$	time-averaged velocities in the flow direction and normal direction respectively
u', v'	fluctuating parts of U and V
U_e	effective flow velocity (equation 6)
U_{tot}	total instantaneous velocity vector
$\overline{u'v'}$	turbulent shear stress
$\overline{u't'}$	turbulent heat flux in the flow direction
$\overline{v't'}$	turbulent heat flux in the normal direction
α_1, α_2	yaw angles of wire 1 and wire 2 of X-array hot-wire probe
α_e	effective yaw angle (equation 6)
k	thermal conductivity of air in the hot-wire response equation
μ	dynamic viscosity of air

ϕ angle between the total velocity vector and axis of X-wire probe

ρ air density

INTRODUCTION AND BACKGROUND

The increasing use of X-array and multiple-sensor hot-wire probes in turbulent flows has resulted in an improvement and development of the calibration and signal analysis techniques in recent years (c.f. Beuther et al, 1987; Legg et al, 1984 and Kawall et al, 1983). In all techniques the response equation of the hot-wire accounts, in some way or another, for the yaw response (directional sensivity) of the probe. The output veltage of the hot-wire is directly related to the electric power dissipated in its sensor. This in turn is directly related to the convective heat loss past the sensor. Therefore, for specific hot-wire and operating conditions, the convective heat loss is essentially a function of the Reynolds number based on the wire diameter. Consequently, the hot-wire response may be represented by a functional relationship in the form

$$Nu = f\,(Re) \qquad (1)$$

where

$$Nu = \frac{E^2}{R_s\ \pi\ l\ k(T_s - T_a)} \quad \text{and} \quad R_e = \frac{\rho U d}{\mu}$$

Relation (1), considers only the heat transfer due to forced convection, may be expressed as:

$$Nu\left(\frac{T_m}{T_a}\right)^{-0.17} = A + B\,Re^n \qquad (2)$$

Equation (2), originally suggested by Collis and Williams (1959) for 0.02 < Re < 44, or its simplified version;

$$E^2 = A + BU^n \qquad (3)$$

is the most common response equation for hot-wires. Equation (2) was, particularly, used in the present study because it represents a physical heat transfer model for the hot-wire, rather than just a regression model, which has been shown to form the basis for complete separation of temperature and velocity effects on the hot-signals (Abdel-Rahman et al, 1987).

In situations where the velocity vector of the flow makes an angle with the normal to the wire, as is the case with slanted and X-array wire probes, the sensitivity of the wire to the yaw angle α, α being the angle between the velocity vector and the normal to the wire, must be accounted for. In such a case, the functional relationship (1) may take the form

$$Nu = f\,(Re\,,\,\alpha) \qquad (4)$$

Equation (2) must, therefore, be modified so that it can account for the dependance on α. Several methods have been proposed and used to account for the yaw angle dependence of the hot-wire signal. Almost all methods depend on the assumption that the hot-wire is sensitive to the so-called effective cooling velocity U_e instead of U. U_e is normally expressed in terms of the magnitude of the flow velocity vector U through analytical/empirical function to account for the directional sensitivity. In all such techniques, the calibration coefficients A, B and n are assumed constant. Common existing examples of U_e are

$$U_e = U \cos\alpha \qquad (5)$$

$$U_e = U \cos\alpha_e \qquad (6)$$

$$U_e = U\,[\,\cos^2\alpha + k^2 \sin^2\alpha]^{1/2} \qquad (7)$$

$$U_e = U\,[\,1 - b\,(\,1 - \cos^{0.5}\ \alpha\,)]^2 \qquad (8)$$

Equation (5) is the simplset of this techniques, and is normally known by the "consine" cooling law. This is based on the assumption that the hot-wire is only sensitive to the normal velocity component and neglecting the tangential velocity component entirely. Such an assumption of the "cosine" cooling is asymptotic for an infinitely long wire (Corrsin, 1963). In practice, however, the hot-wire has a finite length and, therefore, the resulting non-uniform wire temperature and disturbances introduced by the prongs must cause a deviation from the "cosine" law. Bradshaw (1971) introduced the concept of the effective yaw angle, α_e, as an improvement over the "cosine" cooling. The angle α_e, somehow related to α, is usually determined experimentally and used instead of α (see equation 6). This method is only valid for flows of low turbulence intensity; where the instantaneous velocity vector does not deviate by more than $\pm$ 10° approximately from the probe axis (Verriopolis, 1983). Champagne et al (1967) suggested equation (7) to account for the deviation from the "cosine" law, where k in the equation was found to depend on the length-to-diameter ratio of the wire only. Equation (8) was suggested by Friehe and Schwarz (1968) as an alternative to the "cosine" law where b is a constant parameter. The parameters k and b in equations (7) and (8) were, however, shown to depend on both the total velocity U and yaw angle α (Bruun and Tropea, 1985). Beuther, Shabbir and George (1987), realizing the dependence of k on the total velocity, used equation (7) with k expressed as polynomial function of U.

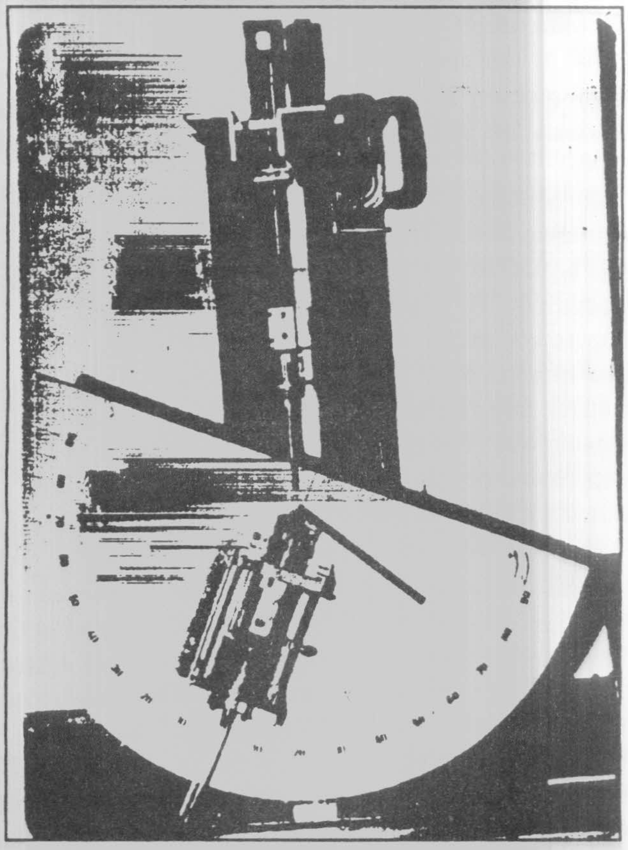
Due to the variation of the yaw parameters k and b, there does not exist a single criterion on which best value of a yaw parameter will be specified. Also, the calibration coefficients A, B and n, usually assumed constants, were shown to be strong functions of α (Abdel-Rahman, 1987; Bruun and Tropea, 1985 and Reardon, 1985). Further more, Bruun and Tropea pointed out that when flows with large turbulence intensities are to be studied or instantaneous velocities are required a digital method which includes the yaw dependence of A, B and n must be used. Therefore, due to the above stated reasons, the present work accounts for the yaw angle sensitivity by making A,B and n functions of α, instead of following the common approach of using any of the effective cooling velocity expressions, equations (5) → (8) , along with equation (2). In fact, Reardon (1985) used a very similar idea by expressing the coefficients A, B and n as general power laws of α instead of creating " look-up" tables. However, he used only the values of A, B and n in the yaw angle range of O° to 70°, where the power law representation appeared valid. In the present approach, the total range of α (0° to 90°) was considered, to be discussed later in the paper. Added to this, his idea was applied to equation (3), which is the isothermal version of equation (2).

The idea of obtaining the variation of A, B and n with α required a complete velocity/yaw angle calibration of the hot-wire as will be described later. Equation (2) may now be expressed as:

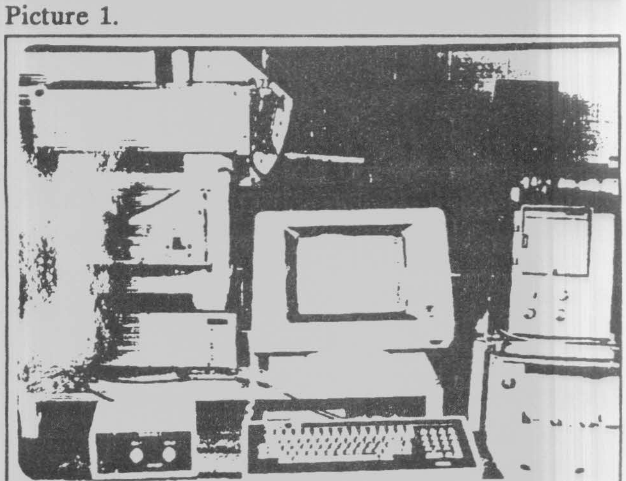
$$Nu\left(\frac{T_m}{T_a}\right)^{-0.17} = A(\alpha)+B(\alpha)Re^{n(\alpha)} \quad (9)$$

EXPERIMENTAL SET-UP

The experimental set-up and data acquisition system are shown in pictures (1) and (2). The main part of the experimental set-up was the TSI Model 1125 calibrator. The calibrator possesses four fine screens upstream of the nozzle and a 5:1 contraction ratio to an outlet jet diameter of 16.5 mm. The calibrating flow was taken from an available compressed air line which was regulated using a pressure regulator and needle valve. A careful check showed that the pressure reading of a Pitot probe placed at the nozzle exit was equal to the static pressure drop across the nozzle; the later was therefore used



Picture 1.



Picture 2.

(because its convenience) to determine the flow velocity which was measured using a multirange micromanometer (Furness Controls). The velocity could be held constant to within ± 0.5% over the entore range. To facilitate the yaw sensitivity calibration of the X-wire probe, a 40 cm diameter angle scale and support steel frame were constructed. The calibrator was then mounted on the angle scale which can rotate on the support frame about its center point. This arrangement allowed the calibrator and consequently the jet flow to be set to any angle within ± 70° to the vertical (resolution of ≈ 0.25°). Care was taken to ensure that the center-line of the calibrating flow intersects the rotation axis of the angle scale at right angles.

For sensing the velocity and temperature of the calibrating flow, a DISA 55P51 gold-plated X-array probe and a DISA 55P15 single wire probe were used respectively. To improve the frequency response of the temperature probe, its prongs were brought sufficiently close to each other (≈ 0.5 mm) and its original 5 μm diameter tungsten wire was replaced by a 1 μm diameter platinum wire. The probe supports of both the X-wire and temperature probes were carefully welded together to construct a triple-wire arrangement in such a way that the sensor of the temperature probe was offset (2 mm) as well as slightly upstream (2 mm) from the X-wire sensors to avoid their thermal wake. The X-wire was operated in the constant temperature mode using a DISA 56C01 system with two DISA 56C16 plug-in bridges. The temperature probe was operated in the constant current mode using a DISA 56C01 system incorporating a DISA 56C20 temperature bridge. The temperature/resistance characteristics of the X-wire sensors and the temperature/voltage characteristics of the temperature probe were obtained using a Digi-sense Thermistor (YSI-400) to measure the reference temperature. These characteristics were needed to determine the sensor temperature (T_s) and measure the instantaneous flow temperature (T_a) respectively. All analog signals from X-wire probe, temperature probe and micromanometer were simultaneously sampled using a multichannel. high speed 12 bit analog-to-digital converter interfaced to an IBM PC-XT.

VELOCITY/YAW-ANGLE CALIBRATION OF X-WIRE PROBE

The purpose of this calibration was basically to determine the yaw dependence of the calibration coefficients A, B and n by obtaining their values for selected values of the yaw angle . In order to do so, a comprehensive velocity-voltage calibration (velocity range of 0.4 to 7.2 m/s) was performed for various values of the flow direction ϕ, used as a variable parameter (see figures (1) and (2)). The flow direction was varied in equal increments of 5° with ϕ covering the range - 45° to + 45°

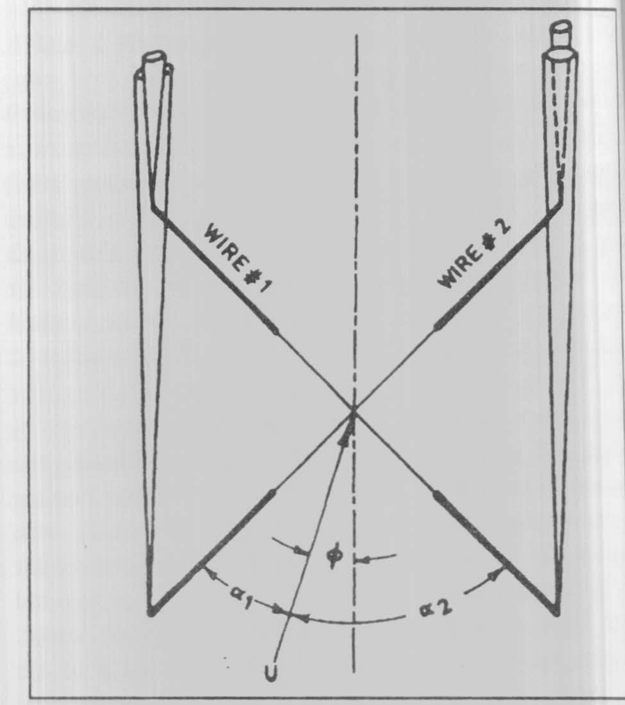


Figure 1. A sketch of X-wire probe showing velocity vector and yaw angles.

For each flow direction, a set of reference pressure and anemometer output voltages was acquired by the computer. Each reading of the calibration data points resulted from four output signals; two signals from the X-wire sensors and one signal from each of the temperature probe and the micromanometer. The real-time program, having stored the above set of readings for the flow direction under investigation, performed the necessary data reduction and then fitted the data points (using regression analysis according to equation (9), finding values for $A(\alpha)$, $B(\alpha)$, $n(\alpha)$ and the rms of the error as well. Then another flow direction was set by rotating the angle scale to carry out the same previous procedure and so on till all flow directions of interest were convered. The output from this computerized calibration program was two files, one contained all velocity calibration data for the different yaw angles convered, and another included two "look-up" tables (one for each sensor) for A,B and n versus α. Figures (3) and (4) demonstrate the dependence of the velocity calibration data on the yaw angle.

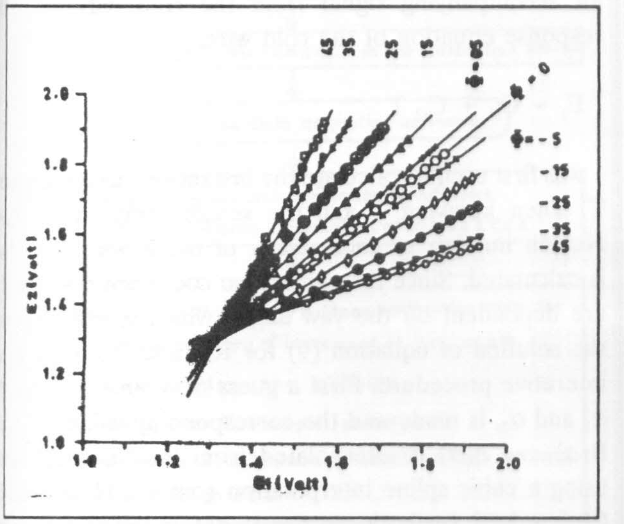


Figure 2. Calibration region as measured for X-wire probe.

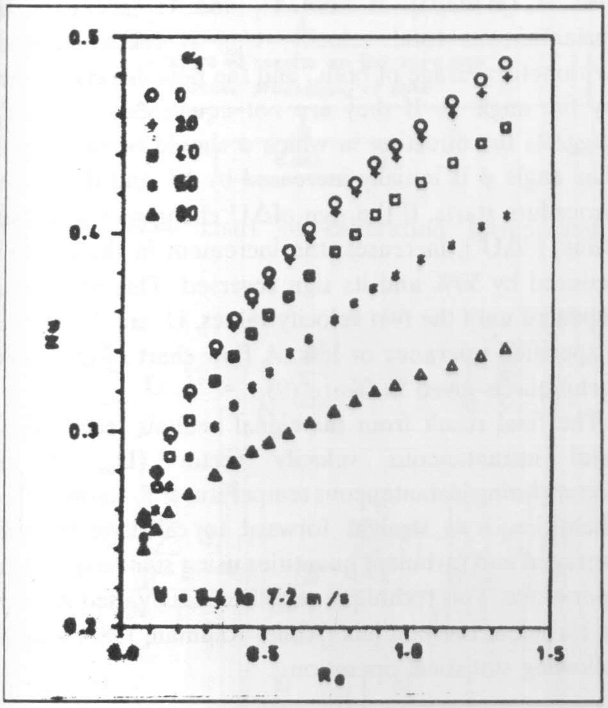


Figure 3. Velocity calibration data for different yaw angles (X-wire, wire #1)

The calibration coefficients A,B and n, normalized by the corresponding values at $\alpha = 45^\circ$ are plotted against α in figures (5) to (7). They indicate a power law dependence on α up to $\approx 70^\circ$ after which the trend appears to go to the opposite direction. Howver, to investigate wether this reversed trend is true, the entire calibration program was repeated using different approach. In this approach, a calibrating velocity was set by adjusting the needle valve. Then the angle scale was set at angles, in 5° increments, through the entire range of -45° to 45° to the vertical. Another calibrating velocity was set and the same previous procedure was followed till the entire velocity range was convered. After acquiring the data for all the calibrating velocities, the data was sorted so as to produce separate files of velocity-voltage pairs for each yaw angle, where a regression analysis was performed on the data in accordance with equation (9). The calibration coefficients resulting from this alternative calibration approach varied identically to those of figures (5), (6) and (7), thus granting more confidence in their varation with α, paricularly for $\alpha > 70^\circ$.

Up to this point, the calibration procedure of the X-wire was complete. It is worth mentioning that about 25 calibration data points were collected for every velocity/voltage calibration to obtain statistically independent calibration coefficients (Swaminathan et al, 1984). Calibration set-up and software are summarized by a flow chart given in figure (8).

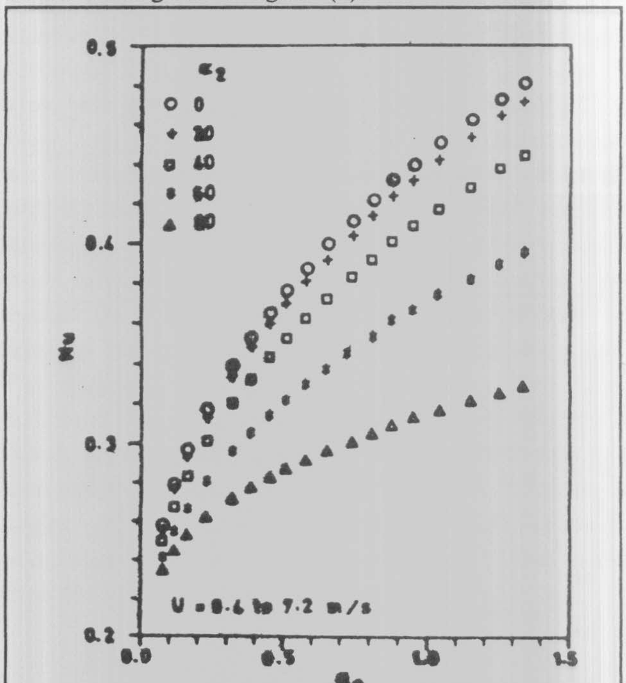


Figure 4. Velocity calibration data for different yaw angles (X-wire, # 2).

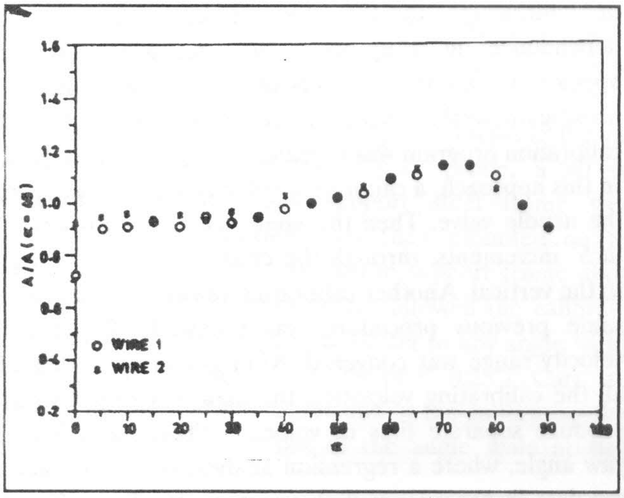


Figure 5. Variation in A with the yaw angle.

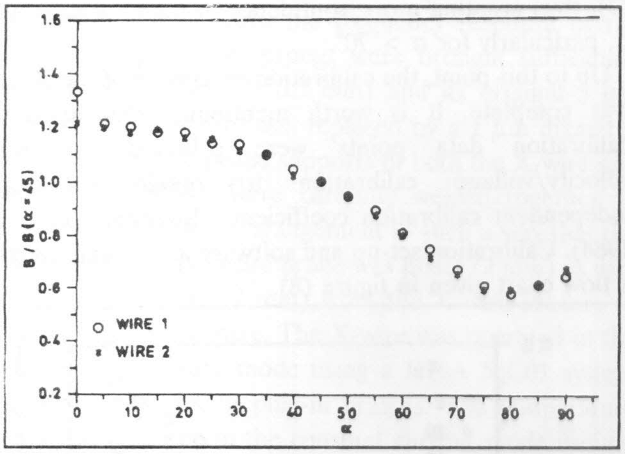


Figure 6. Variation in B with the yaw angle.

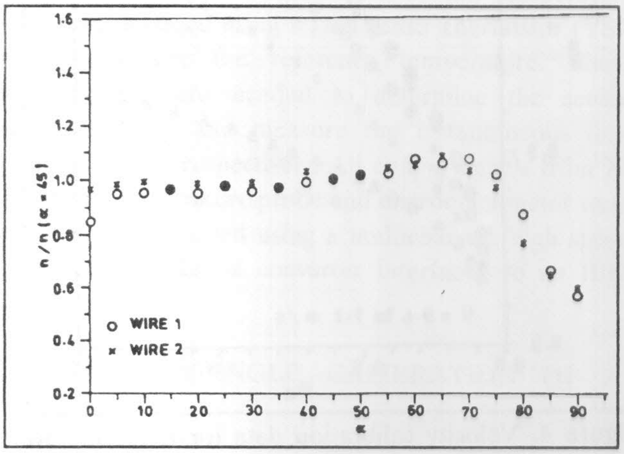


Figure 7. Variation in n with the yaw angle.

SIGNAL ANALYSIS TECHNIQUE

The analysis technique deals with three instantaneous signals at a time; two signals from the X-wire (E_1, E_2) and an accompanying signal from the cold wire (E_t). The response equation of the cold wire,

$$E_t = C_{t1} + C_{t2}\, T$$

was first used to calculate the instantaneous temperature T. Then knowing T and the sensor temperature T_s, the Nusselt number of each sensor of the X-wire (Nu_1, Nu_2) is calculated. Since the calibration coefficients A, B and n are dependent on the yaw angle, which is still unknown, the solution of equation (9) for Re_1 and Re_2 suggests an interative procedure. First a guess of ϕ, and consequently α_1 and α_2, is made and the corresponding values of $A(\alpha)$, $B(\alpha)$ and $n(\alpha)$ is interpolated from the "look-up" tables using a cubic spline interpolation routine. Then equation (9) is solved, for both sensors, to obtain the instantaneous Reynolds numbers as sensed by each sensor, Re_2 and Re_1, and subsequently the corresponding velocities U_1 and U_2 [$Re = (\rho U d/\mu)$]. If both U_1 and U_2 are equal, the instantaneous total velocity, U_{tot}, is taken to be the arithmetic average of both,, and the flow direction is given by the angle ϕ. If they are not equal, $\Delta U = U_1 - U_2$ suggests the direction in which ϕ should be incremented. The angle ϕ is initially increased by 10° and the iteration procedure starts. If the sign ofΔU changes or its absolute value, $|\Delta U|$, increases, the increment in the angle ϕ is reduced by 50% and its sign reversed. This procedure is repeated until the two velocity values, U_1 and U_2, differ by a specified tolerance or less. A flow chart of the analysis technique is given in figure (9).

The final result from the signal analysis technique is a total instantaneous velocity vector (U_{tot}, ϕ) and accompaning instantaneous temperature, T. Knowing these quantities, it is straight forward to calculate the time-averaged and turbulent quantities using standard statistical operations. This technique was successfully used in a study on turbulent buoyant jets (Abdel-Rahman, 1987) using the following statistical operations

$$U_i = U_{tot} \cos \alpha \qquad V_i = U_{tot} \sin \alpha$$

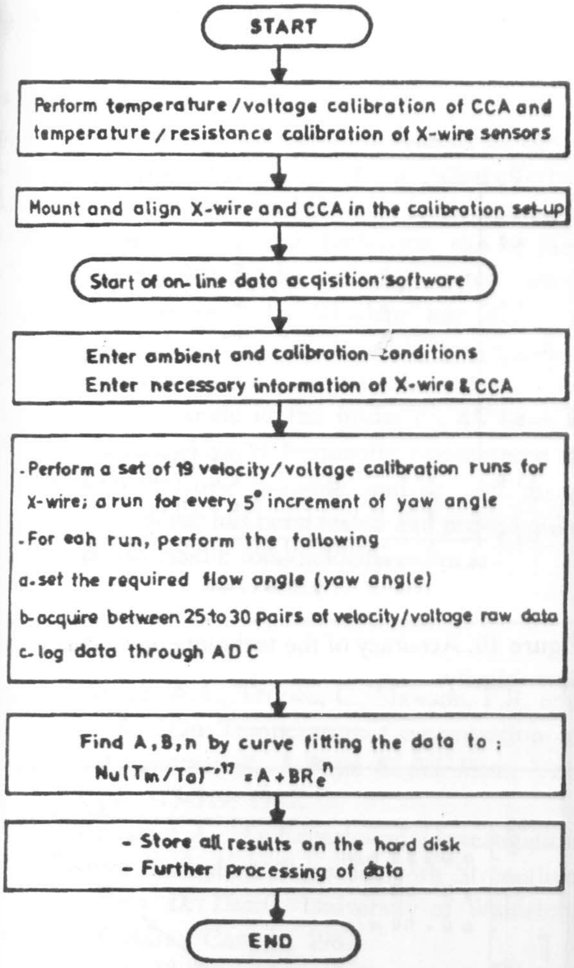


Figure 8. Flow chart of calibration set-up and software.

$$\bar{U} = \frac{1}{N}\sum_{i=1}^{N} U_i,\ \bar{V} = \frac{1}{N}\sum_{i=1}^{N} V_i,\ \bar{T} = \frac{1}{N}\sum_{i=1}^{N} T_i$$

$$\overline{u'^2} = \frac{\sum_{i=1}^{N} U_i^2 - \left[\sum_{i=1}^{N} U_i\right]^2 / N}{N-1} \qquad \overline{v'^2} = \frac{\sum_{i=1}^{N} V_i^2 - \left[\sum_{i=1}^{N} V_i\right]^2 / N}{N-1}$$

$$\overline{u'v'} = \frac{\sum_{i=1}^{N} U_i V_i}{N} - \frac{\left[\sum_{i=1}^{N} U_i\right]\left[\sum_{i=1}^{N} V_i\right]}{N^2}$$

$$\overline{u't'} = \frac{\sum_{i=1}^{N} U_i T_i}{N} - \frac{\left[\sum_{i=1}^{N} U_i\right]\left[\sum_{i=1}^{N} T_i\right]}{N^2}$$

$$\overline{v't'} = \frac{\sum_{i=1}^{N} V_i T_i}{N} - \frac{\left[\sum_{i=1}^{N} V_i\right]\left[\sum_{i=1}^{N} T_i\right]}{N^2}$$

$$\overline{t'^2} = \frac{\sum_{i=1}^{N} T_i^2 - \left[\sum_{i=1}^{N} T_i\right]^2 / N}{N-1}$$

It is worth pointing out that the above operations have the advantage of being subject to much less truncation errors as they are functions of the instantaneous values directly, rather than time-averaged values.

ACCURACY TEST OF THE ANALYSIS TECHNIQUE

To test the accuracy of the technique for solving for the total velocity magnitude U_{tot} and direction ϕ, an experiment was conducted using the experimental set-up, described before, to provide the needed jet flow. The triple wire probe arrangement was placed at approx. 1 mm away from the exit plane of the jet flow. The experiment was conducted for three calibrating velocities, namely 1,2.2 and 4.3 m/s. For each velocity, the flow angle was investigated over the range $\pm$ 40° from the probe axis. For every calibrating velocity investigated, the output voltage signals from the X-wire and cold wire probes were sampled for every increment of 10° in the yaw angle ϕ. The samples were then analyzed using the iterative technique described before. The results of this test are shown in figures (10) and (11). The measurements of the total velocity magnitude, for all the flow velocities and angles covered, are presented in figure (10) as a percentage error of the measured velocity, U(measured), from the calibrating or true velocity, U(true); i.e.

$$\%\ \text{error} = \frac{U(\text{measured}) - U(\text{true})}{U(\text{true})} \times 100$$

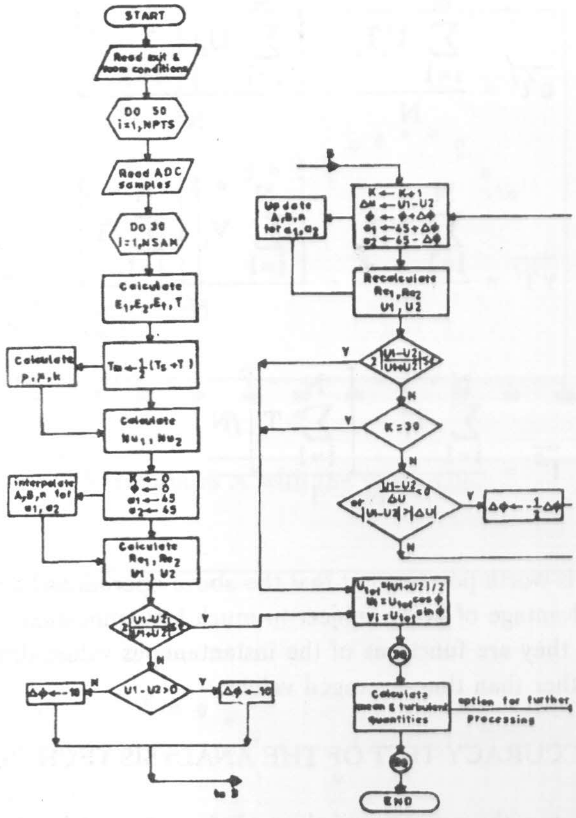


Figure 9. Flow chart of X-wire and cold wire signals'analysis.

where U(true) is the calibrating velocity, calculated from the static pressure drop across the calibrator nozzle, U(measured) is the corresponding value to U(true) which was calculated from the results of the analysis technique as:

$$U(\text{measured}) = \sqrt{\overline{U}^2 + \overline{V}^2}$$

It is clear from the figure that the deviations of the measure velocities are well within 0 to approx 1% from the true velocities for all flow angles, thereby indicating a good accuracy. Figure (11) presents the measured flow angles, ϕ(measured), versus the true flow angles, ϕ(true), for the three velocities investigated. In this figure, the true angle is the angle which was set by the angle scale of the experimental set-up, and the measured angle is the angle calculated from the output results of the analysis technique as:

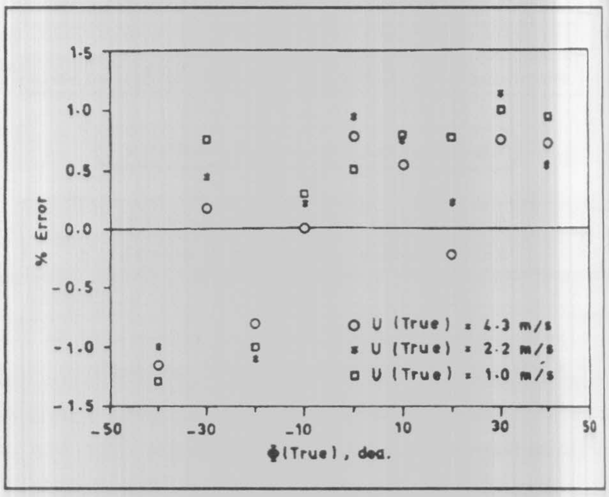
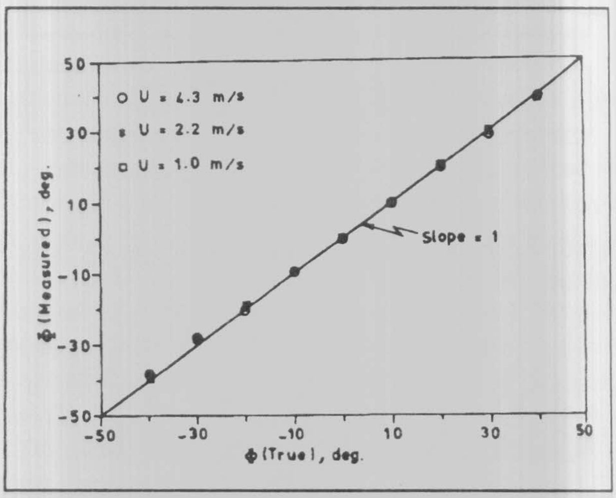


Figure 10. Accuracy of the technique in calculating the flow velocity.



Figuree 11. Accuracy of the technique in calculating the flow angle.

$$\phi(\text{measured}) = \tan^{-1}\frac{\overline{V}}{\overline{U}}$$

As the figure illustrates, the measured angle is obtained within a degree of less for - 30° < 30°. For ϕ = ± 40°, the measured angle still shows a very good agreement with the true angle (within 1° to 2°).

ONCLUDING REMARKS

It was shown that the yaw sensitivity of the X-wire probe best accounted for in the calibration coefficients of the sponse equation, rather than using the so-called effective oling velocity. The response equation, implemented in iterative digital signal analysis technique, can be used solve for the instantaneous velocity vector, which nsequently can be used to calculate any statistical antity of the fluid flow. The technique has the advantage detecting any voltage pairs from the X-wire that do not in the acceptance angle of the probe ($-45^\circ < \phi <$ °), an occurrence which is frequently encountered in ws of high turbulence intensity and/or low mean locities. The technique has been tested and proved quite ceptable in resolving the total velocity vector.

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