REINFORCED CONCRETE SLAB SUBJECTED TO CENTRAL LINE LOAD ANALYSIS BY YIELD LINE THEORY

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ABSTRACT

A theoretical investigation, applying the yield line method to the analysis, of simply supported rectangular slab under the effect of central line load is presented. The object was to study the economical arrangement of the reinforcement and the possible modes of failure. The theoretical results were compared to the recommendations of the Egyptian Code.

INTRODUCTION

The yield line theory is an acceptable method by most codes since it provides a method to estimate the ultimate loads for the slabs specially those which have irregular shapes or are subjected to non uniform loads [1]. Also the yield line method has an advantage over elastic methods where tedious mathematical computations are needed [2]. The yield line pattern is influenced by the steel arrangement and the loading pattern. The yield line theory is an upper bound solution for the collapse loads of reinforced concrete slabs [2]. The yield line method is based on relating the ultimate strength of the steel in both directions and the collapse loads utilizing idealized failure patterns based on experimental observations [3].

ANALYSIS

The possible modes of failure for simply supported rectangular slabs, subjected to central line load of value Qt/m' and orthotropically reinforced are shown in Figure 1.

Failure Mode a

The yield line pattern of the first mode is shown in Figure 1-a. Where the notation as indicated.

Let $\tan \Theta = t \qquad y = tKL/2$

The internal work D is given by

$$D = 4M\,(\mu/t + 1/K) \tag{1}$$

The external work E is given by

$$E = QL\,(1-0.5Kt) \tag{2}$$

By equating internal and external work D = E then:

$$Q = \frac{4M}{L}\,\frac{(\mu/t + 1/K)}{(1-0.5Kt)} \tag{3}$$

To obtain the minimum value of the load then

$dQ/dt = 0$ which leads to

$$(1-0.5Kt)(-\mu/t^2)-(\mu/t+1/K)(0.5K) = 0$$

$$t = \sqrt{\mu^2K^2+2\mu} - \mu K \tag{4}$$

From Eqs. 3 and 4 :

$$M = (QL/4)[K(1+\mu K)-K^2(\mu^2K^2+2\mu)^{0.5}]$$

For this mode

$$M = (QL/4)F_a$$

Where :

$$F_a = K(1+\mu K) - K^2(\mu^2K^2+2\mu)^{0.5}$$

Failure mode (b)

From the notation in Figure 1-b the internal work D is given by

$$D = 4M\,(\mu/t' + 1/\alpha) \tag{1}$$

The external work E is given by

$$E = QL\,(1-0.5\alpha t') \tag{2}$$

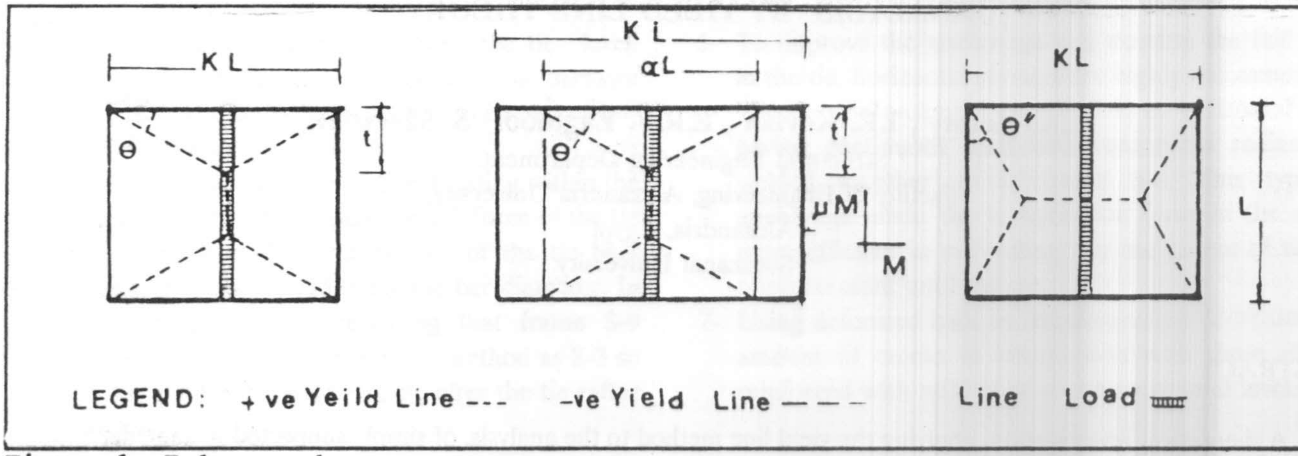

Figure 1. Falure modes.

where $t' = \tan \Theta'$

Equating D = E then

$$Q = \frac{4M}{L}\frac{[\mu/t' + 1/\alpha]}{[1- 0.5\,\alpha t']}$$

For minimum value of Q, $\frac{\partial Q}{\partial t'} = 0$

and $\frac{\partial Q}{\partial \alpha} = 0$

if $\frac{\partial Q}{\partial \alpha} = 0$ then

$$t'^2/\mu + 2\alpha t' - 2 = 0 \qquad (7)$$

and if $\frac{\partial Q}{\partial \alpha} = 0$ then:

$$\alpha^2\mu + 2\alpha t' - 2 = 0.0 \qquad (8)$$

From Eqns. (7) and (8)

$$t' = \mu\alpha \qquad (9)$$

Substituting in Eqn. (7)

$$\alpha = 1/\sqrt{1.5\mu} \qquad (10)$$

From Eqns. (9) and (10)

$$t' = \sqrt{\mu/1.5} \qquad (11)$$

From Eqns. (6), (10) and (11) then

$$M = QL/4\,(1/\sqrt{13.5\mu}) = (QL/4)F_b \qquad (12)$$

Where :

$$F_b = 1/\sqrt{13.5\mu}$$

Failure mode (c) :

From the notation in Figure 1-c

The internal work D is given by

$$D = 4M\,(\mu K + 1/t'')$$

The external work E is given by

$$E = .5\,QL$$

where $t'' = \tan \Theta''$

$$Q = 8M/L[\mu K + 1/t'']$$

For minimum value of Q, $dQ/dt'' = 0$

which yields $t'' = \infty$

This means that failure mode (c) is unlikely to occur.

Thus for slabs subjected to line load only two failure modes are possible namely mode (a) and mode (b). Mode (b) occurs if $F_b > F_a$ which leads to :

$$\sqrt{\mu} > K(1+\mu K)-K^2(\mu^2K^2-2\mu)^{.5}$$

$$\sqrt{\mu} > \sqrt{\mu}\, K(1+\mu K)-\sqrt{\mu}K^2(\mu^2K^2-2\mu)^{.5}$$

$$F,\ 0.272 > F$$

where:

$$F = \sqrt{\mu}\, K(1+\mu K)-\sqrt{\mu}\, K^2(\mu^2K^2-2\mu)^{.5} \qquad (13)$$

In order to determine the possible mode of failure, for a slab of given K and μ, the value of the coefficient (F) is to be calculated first, then:

If $F \geq .272$ failure mode (a) occurs and if $F \leq .272$ failure mode (b) occurs.

The relation between α and μ has been computed and plotted in figure (1) where failure mode (a) dominates. The figure shows that as the value of μ increases the value of α decreases which means that failure mode turns to be localized close to the line of the load as μ increases.

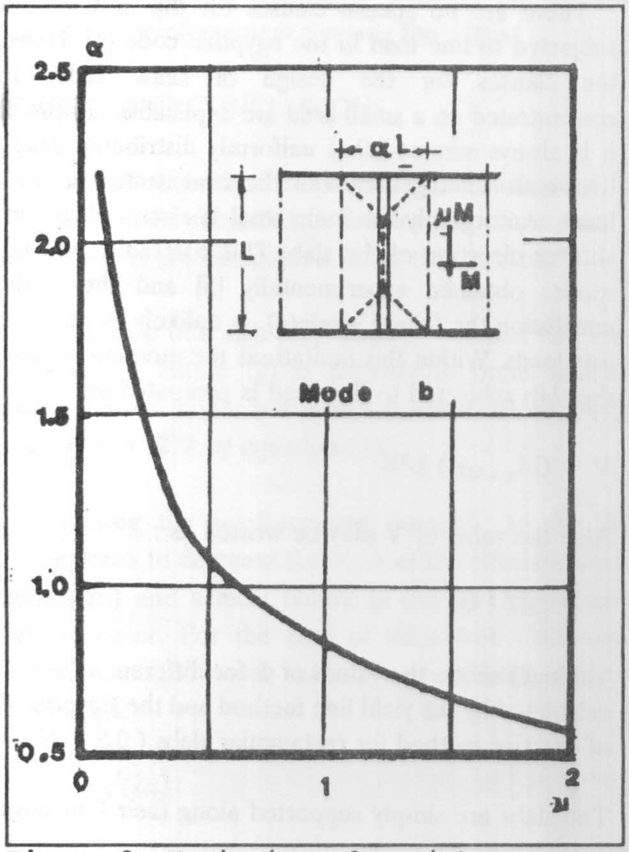

Figure 2. Variation of α with μ.

OPTIMUM ARRANGEMENT OF REINFORCEMENT:

The optimum arrangement of reinforce-ment in the two directions for rectang-ular slab will be given the symbol μ_e and defined as the value of μ which gives minimum weight of reinforcement per unit area of the slab. The weight of rein-forcement per unit area of a slab (W) is approximately proportional to (M + μM), since the difference of the lever arm of the steel in the two perpendicular directions Y_{cu} is small [4].

Therefore

$$W = C\,.\,(M + \mu M) \qquad (14)$$

where C is constant.

For minimum value of (W), $dW/d\mu = 0.0$

Failure Mode(a)

From Eqns. 3 and 4.

$$M = \frac{QL}{4}\,.\,\frac{(1-0.5Kt)}{(\mu/t+1/K)} \text{ where :}$$

$$t = \sqrt{\mu^2K^2 + 2\mu} - \mu K$$

By substituting for M in Eqn.(18) then:

$$W = C\,\frac{(1-0.5Kt)}{(\mu/t+1/K)}\,(1+\mu) \qquad (15)$$

For minimum weight of reinforcement

$$\frac{dW}{d\mu} = \frac{dW}{dt}\,.\,\frac{dt}{d\mu} = 0$$

But $\frac{dt}{d\mu} <> 0$ then $\frac{dW}{dt} = 0$

$$t = \sqrt{\mu^2K^2 + 2\mu} - \mu K, \quad \text{and}$$

$$t^2 = \mu^2K^2 + 2\mu + \mu^2K^2 - 2K\mu\sqrt{\mu^2K^2 + 2\mu}$$

$$2\mu - t^2 = 2K\mu(\sqrt{\mu^2K^2 - 2\mu} - \mu K) = 2K\mu t$$

$$\mu = \frac{t^2}{2 - 2Kt} \qquad (16)$$

$$\frac{\mu}{t} + \frac{1}{K} = \frac{t}{2 - 2Kt} + \frac{1}{K} = \frac{2 - Kt}{2K(1 - Kt)} \qquad (17)$$

$$1+\mu = 1 + \frac{t^2}{2(1-Kt)} = \frac{2-2Kt+t^2}{2(1-Kt)} \qquad (18)$$

From Eqns. 15,17 and 18

$$W = C\,.\,(2 - 2\,Kt + t^2)$$

$$dW/dt = 2t - 2K = 0 \text{ then } t = K \qquad (19)$$

From Eqns. (16) and (19)

$$\mu_e = \frac{K^2}{2(1-K^2)} \qquad (20)$$

Failure mode (b)

Similar to Mode (a)

W = constant (1+ μ) M where

$$M = \frac{Q\,L}{12\,\sqrt{1.5}\,\sqrt{\mu}}$$

$$W = \text{constant}\,.\,(1/\sqrt{\mu} + \sqrt{\mu})$$

If $dW/dt = 0$ then $\mu_e = 1$ (20)

Equation (21) means that, in this case, the isotropic reinforcement is the most economical arrangement. Equation (20) is valid for failure mode (a). If the failure occurs as a local failure, under line load, as the case of failure mode (b), the value of μ_e is taken equal unity. The value of (K) at which the mode of failure begins to change from mode (a) to mode (b) when $\mu_e = 1$ is determined as follows :

$$\alpha = \frac{1}{\sqrt{1.5}\sqrt{\mu}} = .816\sqrt{\mu K} \quad (22)$$

$$\text{but : } \mu_e = \frac{K^2}{2(1-K^2)} \quad (23)$$

From Eqns. 22 and 23:

$$K^2 = 0.667 \quad \text{and} \quad K = 0.816 \quad (24)$$

According to the above equation

$$\mu_e = \frac{K^2}{2(1-K^2)} \quad \text{for } K \le 0.816$$

and for $K \ge 0.817$ then :

$$\mu_e = 1. \text{ and } \alpha = 0.816 \quad (25)$$

COMPARISON BETWEEN THE THEORETICAL RESULTS AND THE EGYPTIAN CODE

The moment volume is a useful parameter for comparing the economical arrangement of steel. It is used here to compare the total weight of steel reinforcement as obrained by the yield line solution and the Egyptian code.

Moment Volume by Yield Line Theory

FOR $K \le 0.816$:

$$V = (M + \mu_e M)\,KL^2 = (1+\mu_e)\frac{QL^3}{4}\,.\,F_a K$$

$$= QL_3\,.\,\frac{F_a K}{4}\,.(1 + \mu_e) = QL^3\,\Phi$$

where

$$\Phi = \frac{F_a K}{4}(1 + \mu_e) \quad \text{and} \quad \mu e = \frac{K^2}{1 - 2K^2}$$

FOR $K \ge 0.816$:

$$V = \frac{QL^3 K}{6\sqrt{1.5}} = QL^3\,.\,\Phi$$

where

$$\Phi = K/\,6\,\sqrt{1.5} = 0.136$$

Moment Volume to Egyptian Code 1989

There are no special clauses for the design of slabs subjected to line load in the egyptian code [1]. However, the clauses for the design of slabs under loads concentrated on a small area are applicable. In the code, it is always assumed that uniformly distributed dead and live loads act together with the conentrated or the line loads. Accordingly the main steel is always placed in the shorter direction of the slab. This contradicts the failure modes obtained experimentally [5] and the analytical conclusion the failure mode(c) is unlikely to occur under line loads. Within this limitations the moment volume for the slab subjected to line load is computed as

$$V = (M_{x} + M_{y})\,L^2K$$

Also the value of V may be written as :

$$V = QL^3\,.\,\Phi$$

Table (I) shows the values of Φ for different values of (K) calculated by the yield line method and the Egyptian Code of practice method for rectangular slabs ($0.5 \le K \le 2$).

The slabs are simply supported along their four edges.

Table 1. Comparison of Φ Values

K	Yield line	Egypt. Code
0.500	0.055	0.066
0.556	0.065	0.081
0.625	0.079	0.103
0.714	0.095	0.218
0.833	0.113	0.244
1.000	0.136	0.277
1.200	0.163	0.353
1.400	0.141	0.437
1.600	0.218	0.652
1.800	0.245	0.245
2.000	0.272	0.815

The Egyptian code values for Φ are obviousely high. This results from the fact that when applying the code method the main steel is placed in the shorter direction and consequently μ is greater than unity when $k>1$. Also the moment volume is computed without the possible reduction of reinforcement towards the edges.

RESULTS AND CONCLUSIONS

1-The yield line method of analysis is advantageous for the case of slabs subjected to line loading. Since the local failure is accommodated for in this method.

2- For the case of a slab subjected to central line load, the mode of failure (c) is unlikely to occur and the possible failure modes are modes (a) and (b). Mode (a) occurs if F > .272 by equation (13).

3- Increasing the reinforcement parallel to the line loading, tends to decrease the width of the effected central region (αL) and a local failure in the central region is likely to occur. For the case of slabs with orthotropic reinforce-ment,the value of α may be obtained by equation (22).

$$\alpha = 1/\sqrt{(2\mu)}$$

4-The value of μ_e which represents the economical arrangement of reinforcement in the two directions for a slab subjected to central line load may be obtained from equations (24),(25).

In this case μ_e is always less than unity, this indicates that the main load carrying direction is perpendicular to the line load.

5- The comparison between the calculated values of moment volume using the yield line theory and the egyptian code show that :

a. For $k \leq 0.817$, the calculated are value of Φ using the code slightly over estimates the values calculated using the yield line method.

b. For $K \geq 0.817$, where the local failure occurs, the code values of Φ are higher than those obtained by the yield line theory, since the code does not take the local failure into consideration.

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